Discrete-time Direct Model Reference Adaptive Control: A Systematic Approach

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Abstract For a class of discrete-time systems, the design and analysis of direct model reference adaptive control (MRAC) with normalized adaptive law are investigated. We reprove the discrete-time conclusions on the $\mathcal{L}_p$ and $\mathcal{L}_{2\delta}$ relationship properties between the input and the output, and the discrete-time swapping Lemmas 1 and 2. We also establish the properties of discrete-time adaptive law, define the normalizing signal, and relate the signal with all the signals in the closed-loop system. Thus, the stability and convergence properties of the discrete-time MRAC scheme are analyzed rigorously in a systematic fashion as in the continuous-time case.

Key words Discrete-time systems, swapping lemma, $\mathcal{L}_{2\delta}$-norm, the normalizing signal, model reference adaptive control

1 Introduction

During the past two decades, for linear continuous-time systems, the "certainty equivalence" adaptive controllers with normalized adaptive laws have dominated the literature of adaptive control due to the simplicity of the design as well as the robustness properties in the presence of modeling errors[1~5]. An important feature of this class of adaptive controllers is the use of error normalization, which allows the complete separation of the adaptive and control laws design. By using the properties of $\mathcal{L}_{2\delta}$-norm, swapping lemmas, and Bellman-Gronwall Lemma, a more elaborate but yet more systematic method is given in the analysis of adaptive control schemes.

As we know, the first analogous theoretical result for the discrete-time systems seems to be due to Ydstie[6], who used the internal model control (IMC) implementation for the extended horizon adaptive control scheme. In [7], Silva and Datta further considered the adaptive IMC in the presence of modelling errors. However, up to now, there has been no result with respect to the design and analysis of the discrete-time control schemes in a systematic fashion as in the continuous-time case.

The purpose of this paper is to solve the problem in the context of model reference adaptive control (MRAC) for the discrete-time systems. Our main work is composed of three parts. 1) For continuous-time systems, some important conclusions and mathematical tools in [2], such as lemma 3.3.2, *i.e.*, the $\mathcal{L}_p$ and $\mathcal{L}_{2\delta}$ relationship properties between the input and the output, continuous-time swapping lemmas A1 and A2, *etc.*, are often used in the analysis of adaptive controllers. Whereas for the discrete-time case, these conclusions and mathematical tools are no longer applicable, so it is much difficult to extend the existing results to the discrete-time case. Due to the utmost importance of these conclusions and mathematical tools in the analysis of the stability of adaptive control systems, proving them constitutes one objective of this paper. 2) By finding the properties of discrete-time adaptive law and defining the normalizing signal, the relationship properties between the normalizing signal and all the signals in the closed-loop system are established. 3) Stability and convergence properties of the discrete-time MRAC scheme are analyzed rigorously in a systematic fashion as in the continuous-time case.

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2 Problem statement

Let us consider the discrete-time linear time-invariant plant

$$R_p(z)y(t) = k_p Z_p(z)u(t), \quad t = 0, 1, 2, \cdots \tag{1}$$

where $u(t)$ and $y(t) \in \mathbf{R}$ are the input and output, respectively, $R_p(z) = z^n + \sum_i^{n-1} a_i z^i$, $Z_p(z) = z^m + \sum_j^{m-1} b_j z^j$ with unknown constants k_p, a_i, and b_j. z is used to denote the $\mathcal{Z}$-transform variable or time-advance operator $zx(t) = x(t+1)$, *i.e.*, z^{-1} is the time-delay operator $z^{-1}x(t) = x(t-1)$.

The reference model is chosen as

$$y_m(t) = W_m(z)r(t) = \frac{1}{P_m(z)}r(t), \quad t = 0, 1, 2, \cdots \tag{2}$$

where r is the reference input which is assumed to be uniformly bounded.

The objective of MRAC is to find an control signal $u(t)$ for (1) such that all the signals in the closed-loop plant are uniformly bounded and the tracking error $e(t) = y(t) - y_m(t) \to 0$ as $t \to \infty$.

To design and analyze the MRAC scheme for (1), one needs the following assumptions.

Plant assumptions:

A1. $Z_p(z)$ is stable, and $1/Z_p(z)$ is analytic in $|z| \geq \sqrt{\delta}$ for some given $\delta \in (0, 1]$.

A2. n, m, and the relative degree $n^* = n - m \geq 1$ are known.

A3. The sign of k_p is known, and there exists a known constant $k_p^0 > 0$ such that $|k_p| < k_p^0$.

Reference model assumption:

M1. The monic polynomial $P_m(z)$ is stable, and $W_m(z)$ is also analytic in $|z| \geq \sqrt{\delta}$ for the above δ, and the degree of $P_m(z)$ is n^*.

Remark 1. As explained in [2, 5], if the plant is expressed as the form (1), then the plant is realized as a state space plant with zero initial conditions, and the results obtained in this paper are free of initial conditions.

Notations. In the sequel, we sometimes denote the time function $x(t)$ by x, $H(z)x(t)$ ($H(z)$ denotes any $\mathcal{Z}$-transform operator polynomial) by $H(z)x$, and the discrete-time $\mathcal{L}_{2\delta}$ norm $\|\boldsymbol{x}_t\|_{2\delta} = (\sum_{i=0}^{t} \delta^{t-i}\boldsymbol{x}^{\mathrm{T}}(i)\boldsymbol{x}(i))^{1/2}$ by $\|\boldsymbol{x}_t\|$, and c denotes some positive constant. By the definition of z, obviously, $z[ab](t) = [ab](t+1) = a(t+1)b(t+1)$ for any $a(t)$ and $b(t)$.

3 Discrete direct MRAC with normalized adaptive law

In this section, we give the design of discrete direct MRAC with normalized adaptive law. As in the continuous-time case, assume that all the parameters of plant (1) are known. Then, the model reference control structure is chosen as

$$u(t)=\boldsymbol{\theta}^{*\mathrm{T}}\boldsymbol{w} \tag{3}$$

where $\boldsymbol{\theta}^{*\mathrm{T}}=[\boldsymbol{\theta}_1^{*\mathrm{T}},\boldsymbol{\theta}_2^{*\mathrm{T}},\theta_3^*,\theta_4^*]$, $\boldsymbol{\omega}=[\boldsymbol{\omega}_1^{\mathrm{T}},\boldsymbol{\omega}_2^{\mathrm{T}},y,r]^{\mathrm{T}}$, $\boldsymbol{\omega}_1=\dfrac{\boldsymbol{\alpha}(z)}{\Lambda(z)}u$, $\boldsymbol{\omega}_2=\dfrac{\boldsymbol{\alpha}(z)}{\Lambda(z)}y$, $\boldsymbol{\alpha}(z)=[z^{n-2},\cdots,z,1]^{\mathrm{T}}$, $\Lambda(z)$ is arbitrary Hurwitz polynomial, and its eigenvalue is in $|z|\le\sqrt{\delta}$ for the above $\delta>0$. Using the matching equations

$$\begin{aligned}&\theta_4^*=k_p^{-1}\\ &\boldsymbol{\theta}_1^{*\mathrm{T}}\boldsymbol{\alpha}(z)R_p(z)+\left(\boldsymbol{\theta}_2^{*\mathrm{T}}\boldsymbol{\alpha}(z)+\boldsymbol{\theta}_3^*\Lambda(z)\right)k_pZ_p(z)=\\ &\qquad\Lambda(z)\left(R_p(z)-\theta_4^*P_m(z)k_pZ_p(z)\right)\end{aligned} \tag{4}$$

$y=y_m$ can be easily achieved. The existence of $\boldsymbol{\theta}^*$ can be guaranteed as in [5]. Using (4), $r=P_m(z)y_m$, and $e=y-y_m$, one obtains the parametric model on $\boldsymbol{\theta}^*$:

$$e=W_m(z)\frac{1}{\theta_4^*}\left(u-\boldsymbol{\theta}^{*\mathrm{T}}\boldsymbol{\omega}\right) \tag{5}$$

For (5), the certain equivalence adaptive control law is chosen as

$$u=\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\omega} \tag{6}$$

where $\boldsymbol{\theta}=[\boldsymbol{\theta}_1^{\mathrm{T}},\boldsymbol{\theta}_2^{\mathrm{T}},\theta_3,\theta_4]^{\mathrm{T}}$ is the estimate of θ^*. Since θ_4^* is constant, (5) can be rewritten as

$$\varpi=W_m(z)u=\boldsymbol{\theta}^{*\mathrm{T}}\boldsymbol{\phi}_p \tag{7}$$

where $\boldsymbol{\phi}_p=W_m(z)\boldsymbol{\omega}_p$, $\boldsymbol{\omega}_p=[\boldsymbol{\omega}_1^{\mathrm{T}},\boldsymbol{\omega}_2^{\mathrm{T}},y,W_m^{-1}(z)y]^{\mathrm{T}}$. Choosing the estimate $\hat{\varpi}$ of ϖ as $\hat{\varpi}=\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\phi}_p$, the normalized estimation error can be constructed as

$$\varepsilon=\frac{\varpi-\hat{\varpi}}{m^2}=-\frac{\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\phi}_p}{m^2},\quad m^2=1+\boldsymbol{\phi}_p^{\mathrm{T}}\boldsymbol{\phi}_p \tag{8}$$

where $\tilde{\boldsymbol{\theta}}=\boldsymbol{\theta}-\boldsymbol{\theta}^*$. For $t=0,1,\cdots$, the adaptive law for chosen as

$$\begin{aligned}&\boldsymbol{\theta}(t+1)=\boldsymbol{\theta}^p(t+1)+\Delta(t+1),\quad\boldsymbol{\theta}(0)=\boldsymbol{\theta}_0\\ &\boldsymbol{\theta}^p(t+1)=\boldsymbol{\theta}(t)+\Gamma\boldsymbol{\phi}_p(t)\varepsilon(t),\\ &\Delta(t+1)=\begin{cases}0 & \theta_4^p(t+1)\mathrm{sgn}(k_p)\ge c_0\\ \dfrac{\boldsymbol{\tau}_1}{\tau_2}\left(c_0\mathrm{sgn}(k_p)-\theta_4^p(t+1)\right) & \text{otherwise}\end{cases}\end{aligned} \tag{9}$$

where $\boldsymbol{\theta}^p=[\boldsymbol{\theta}_1^{p\mathrm{T}},\boldsymbol{\theta}_2^{p\mathrm{T}},\theta_3^p,\theta_4^p]^{\mathrm{T}}$, $\Gamma=\mathrm{diag}\{\lambda_1,\cdots,\lambda_{2n}\}$ is a gain matrix with $0<\lambda_i<2$, $i=1,\cdots,2n$, $\boldsymbol{\theta}(0)$ is an initial estimate of $\boldsymbol{\theta}^*$, $c_0=1/k_p^0>0$, $\boldsymbol{\tau}_1$ is the last column of Γ, and τ_2 is the last element of $\boldsymbol{\tau}_1$. The estimation algorithm (9) has the following properties.

Lemma 1. The adaptive update law (9) guarantees that for all $t=0,1,\cdots$,

1) $|\theta_4(t)|\ge c_0$; 2) $\boldsymbol{\theta}(t)$ and $\epsilon(t)m(t)\in\mathcal{L}_\infty$; 3) $\epsilon(t)m(t)$ and $\boldsymbol{\theta}(t+1)-\boldsymbol{\theta}(t)\in\mathcal{L}_2$.

Proof. See Section 5.

4 Main results

Before giving the main results, we need some preliminaries.

Lemma 2[5,8]**.** If $u\in\mathcal{L}_{2e}$ and $H(z)$ is analytic in $|z|\ge1$, then $\|y_t\|_2\le\|H(z)\|_\infty\|u_t\|_2$, where $\|H(z)\|_\infty=\sup_{\omega\in[0,2\pi]}|H(\mathrm{e}^{j\omega})|$.

Lemma 3. Consider a discrete linear time-invariant plant $y(t)=H(z)u(t)$ (See equation (2.235) in [5] for more details of the expression), where $H(z)$ is a rational transfer function in which z denotes the z-transform variable. If $H(z)$ is analytic in $|z|\ge\sqrt{\delta}$ for some $\delta\in(0,1]$ and $u\in\mathcal{L}_{2e}$, then for all $t=0,1,2,\cdots$

$$\|y_t\|_{2\delta}\le\|H(z)\|_{\infty\delta}\|u_t\|_{2\delta}$$

Furthermore, if $H(z)$ is strictly proper, then

$$|y(t)|\le\|zH(z)\|_{2\delta}\|u_{t-1}\|_{2\delta}$$

where $\|H(z)\|_{\infty\delta}=\sup_{\omega\in[0,2\pi]}|H(\sqrt{\delta}\mathrm{e}^{j\omega})|$, $\|zH(z)\|_{2\delta}=\frac{1}{\sqrt{2\pi}}(\int_0^{2\pi}|\sqrt{\delta}\mathrm{e}^{j\omega}H(\sqrt{\delta}\mathrm{e}^{j\omega})|^2\mathrm{d}\omega)^{1/2}$.

Proof. See Section 5.

Lemma 4 (Discrete-time swapping Lemma 1). Let $\tilde{\boldsymbol{\theta}},\boldsymbol{\omega}:\mathbf{Z}^+\mapsto\mathbf{R}^n$ and $W(z)$ be a proper stable rational transfer function with a minimal realization (A,B,C,d), *i.e.*, $W(z)=C^{\mathrm{T}}(zI-A)^{-1}B+d$, $d\in\mathbf{R}$. Then, for any $t=0,1,2,\cdots$

$$\begin{aligned}&W(z)[\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega}](t)=\\ &\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(t)W(z)[\boldsymbol{\omega}](t)+W_1(z)[(W_2(z)z[\boldsymbol{\omega}^{\mathrm{T}}])((z-1)[\tilde{\boldsymbol{\theta}}])](t)\end{aligned}$$

where $W_1(z)=-C^{\mathrm{T}}(zI-A)^{-1}$, $W_2(z)=(zI-A)^{-1}B$.

Proof. See Section 5.

Lemma 5 (Discrete-time swapping Lemma 2). Let $\tilde{\boldsymbol{\theta}},\boldsymbol{\omega}:\mathbf{Z}^+\mapsto\mathbf{R}^n$. Then, for any $t\in\{0,1,2,\cdots\}$

$$[\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega}](t)=F_1(z,a_0)[\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega}](t-1)+F(z,a_0)[\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega}](t)$$

where $F(z,a_0)=a_0^k/(z+a_0)^k$, $F_1(z,a_0)=(1-F(z,a_0))z$, $k\ge1$, and a_0 is any constant with $|a_0|<1$. Furthermore, for $|a_0|\le\sqrt{\delta}/2$, $F_1(z,a_0)$ satisfies $\|F_1(z,a_0)\|_{\infty\delta}\le ca_0$ for a finite positive constant c which is independent of a_0 and any given constant $\delta\in(0,1]$, where $\|(\cdot)\|_{\infty\delta}$ is defined in Lemma 3.

Proof. See Section 5.

The fictitious normalizing signal m_f is defined by

$$m_f^2(t)=1+\|u_{t-1}\|_{2\delta}^2+\|y_{t-1}\|_{2\delta}^2 \tag{10}$$

The relationship properties between m_f and all the signals in the closed-loop plant are established by the following lemma.

Lemma 6. Consider the closed-loop plant output $y=W_m(z)(r+\frac{1}{\theta_4^*}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega})$ and the control law (6). For any $t\ge0$, one has

1) $\boldsymbol{\omega}_i(t)/m_f(t)$, $\|(\boldsymbol{\omega}_i)_{t-1}\|_{2\delta}/m_f(t)$, $\|\boldsymbol{\omega}_{t-1}\|_{2\delta}/m_f(t)\in\mathcal{L}_\infty$, i=1,2;

2) If $\boldsymbol{\theta}(t)\in\mathcal{L}_\infty$, then $u(t)/m_f(t)$, $y(t)/m_f(t)$, $\boldsymbol{\omega}(t)/m_f(t),\boldsymbol{\omega}_p(t)/m_f(t),\|(\boldsymbol{\omega}_p)_{t-1}\|_{2\delta}/m_f(t),\boldsymbol{\phi}_p(t)/m_f(t)$, $m(t)/m_f(t)$, $W(z)\boldsymbol{\omega}(t)/m_f(t)$, $W(z)\boldsymbol{\omega}_p(t)/m_f(t)\in\mathcal{L}_\infty$, where $W(z)$ is any proper function that is analytic in $|z|\ge\sqrt{\delta}$ for the given δ in A1.

Proof. See Section 5.

Remark 2. By Lemma 1, $\boldsymbol{\theta}\in\mathcal{L}_\infty$ can be guaranteed, hence conclusion 2) of Lemma 6 holds.

We are now in a position to state the main results.

Theorem 1. Consider the direct MRAC scheme consisting of (1), (2), (6), and (9). If assumptions A1 $\sim$ A3 and M1 hold, then

1) All the signals of the closed-loop plant are uniformly bounded;

2) The tracking error $e(t)$ converges to zero as $t \to \infty$.

Proof. This theorem is proved in four steps.

Step 1. Express the input and output of the closed-loop plant in terms of $\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega}$.

From (1), (2), (5), (6), and assumptions A1, A2, and M1, it follows that

$$y = W_m(z)\big(r + \frac{1}{\theta_4^*}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega}\big) \tag{11}$$

$$u = \frac{R_p(z)}{k_p Z_p(z)} W_m(z)\big(r + \frac{1}{\theta_4^*}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega}\big) \tag{12}$$

and $\frac{R_p(z)}{k_p Z_p(z)} W_m(z)$ is stable and proper, where $\tilde{\boldsymbol{\theta}} = \boldsymbol{\theta} - \boldsymbol{\theta}^*$. Applying Lemma 3 to (11) and (12), it follows that $\|y_{t-1}\| \le c + c\|(\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega})_{t-1}\|$, $\|u_{t-1}\| \le c + c\|(\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega})_{t-1}\|$, and then substituting them in (10) results in

$$m_f^2(t) \le c + c\|(\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega})_{t-1}\|^2 \tag{13}$$

Step 2. Using the discrete-time swapping lemmas and Lemma 6 to bound $\|\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega}\|$ mentioned from above.

Define $\boldsymbol{\omega}_0 = [\boldsymbol{\omega}_1^{\mathrm{T}}, \boldsymbol{\omega}^{\mathrm{T}}, y]^{\mathrm{T}}$, $\boldsymbol{\theta}_0^* = [\boldsymbol{\theta}_1^{*\mathrm{T}}, \boldsymbol{\theta}_2^{*\mathrm{T}}, \theta_3^*]^{\mathrm{T}}$, $\boldsymbol{\theta}_0 = [\boldsymbol{\theta}_1^{\mathrm{T}}, \boldsymbol{\theta}_2^{\mathrm{T}}, \theta_3]^{\mathrm{T}}$, and $\tilde{\boldsymbol{\theta}}_0 = \boldsymbol{\theta}_0 - \boldsymbol{\theta}_0^*$. Obviously, $\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\omega} = \tilde{\boldsymbol{\theta}}_0^{\mathrm{T}}\boldsymbol{\omega}_0 + \tilde{\theta}_4 r$. From (11), by some calculation, it follows that

$$\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega}_p = \frac{\theta_4}{\theta_4^*}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega} \tag{14}$$

where $\boldsymbol{\omega}_p$ is defined in (7). Using $\boldsymbol{\phi}_p = W_m(z)\boldsymbol{\omega}_p$ and Lemma 4, we can get

$$W_m(z)[\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega}_p](t-1) = [\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\phi}_p](t-1) + W_c(z)[(W_b(z)z[\boldsymbol{\omega}_p^{\mathrm{T}}])(z-1)[\tilde{\boldsymbol{\theta}}]](t-1) \tag{15}$$

where $W_c(z)$ and $W_b(z)$ are strictly proper and have the same poles as those of $W_m(z)$. From Lemma 5, by choosing a_0 to satisfy $|a_0| \le \sqrt{\delta}/2$, it is easy to obtain

$$[\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\phi}_p](t-1) = F_1(z, a_0)[\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\phi}_p](t-2) + F(z, a_0)[\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\phi}_p](t-1) \tag{16}$$

and $\|F_1(z, a_0)\|_{\infty\delta} \le ca_0$, $\|F(z)W_m^{-1}(z)\|_{\infty\delta} \le cf(a_0)$, where c is a constant independent of a_0, and $f(\cdot)$ is a known polynomial with the degree n^*, $F(z, a_0) = a_0^{n^*}/(z + a_0)^{n^*}$, and $F_1(z, a_0) = (1 - F(z, a_0))z$. Substituting (8), (14) and (15) in (16) leads to

$$[\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega}_p](t-1) = F_1(z, a_0)\Big[\frac{\theta_4}{\theta_4^*}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega}\Big](t-2) + F(z, a_0)W_m^{-1}(z)\Big[-\varepsilon m^2 + W_c(z)[(W_b(z)z[\boldsymbol{\omega}_p^{\mathrm{T}}])(z-1)[\tilde{\boldsymbol{\theta}}]]\Big](t-1) \tag{17}$$

which implies that

$$\big\|(\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega}_p)_{t-1}\big\| \le \|F_1(z, a_0)\|_{\infty\delta}\|z^{-1}\|_{\infty\delta}\Big\|\Big(\frac{\theta_4}{\theta_4^*}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega}\Big)_{t-1}\Big\| + \|F(z, a_0)W_m^{-1}(z)\|_{\infty\delta}\big(\|(\varepsilon m^2)_{t-1}\| + \|W_c(z)\|_{\infty\delta}\|((W_b(z)z[\boldsymbol{\omega}_p^{\mathrm{T}}])(z-1)[\tilde{\boldsymbol{\theta}}])_{t-1}\|\big) \le ca_0\Big\|\Big(\frac{\theta_4}{\theta_4^*}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega}\Big)_{t-1}\Big\| + cf(a_0)\Big(\|(\varepsilon m^2)_{t-1}\| + \|((W_b(z)z[\boldsymbol{\omega}_p^{\mathrm{T}}])(z-1)[\tilde{\boldsymbol{\theta}}])_{t-1}\|\Big) \tag{18}$$

by using Lemma 3 and (16), where c is a constant independent of a_0. By the definition of $W_b(z)$ and Lemma 6, it is known that $W_b(z)z[\boldsymbol{\omega}_p^{\mathrm{T}}](t-1)/m_f(t-1) \in \mathcal{L}_\infty$, therefore, $\|((W_b(z)z[\boldsymbol{\omega}_p^{\mathrm{T}}])(z-1)[\tilde{\boldsymbol{\theta}}])_{t-1}\| \le c\|(((z-1)[\tilde{\boldsymbol{\theta}}])m_f)_{t-1}\|$. Because $\theta_4, \tilde{\boldsymbol{\theta}} \in \mathcal{L}_\infty$ by Lemma 1, it follows from Lemma 6 that $\|(\frac{\theta_4}{\theta_4^*}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega})_{t-1}\| \le c\|(\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega})_{t-1}\| \le c\|\boldsymbol{\omega}_{t-1}\| \le cm_f(t)$. According to the conclusion 2) in Lemma 6, we have $\|(\varepsilon m^2)_{t-1}\| \le c\|(\varepsilon m m_f\|$. Hence $\|(\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega}_p)_{t-1}\| \le ca_0 m_f(t) + cf(a_0)\|(\varepsilon m m_f)_{t-1}\| + cf(a_0)\|(((z-1)[\tilde{\boldsymbol{\theta}}])m_f)_{t-1}\|$, by which together with (14) and Lemma 1 gives

$$\|(\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega})_{t-1}\| \le c\|(\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega}_p)_{t-1}\| \le ca_0 m_f(t) + cf(a_0)\|(\tilde{g}m_f)_{t-1}\| \tag{19}$$

where c is a constant independent of a_0, and $\tilde{g}^2 = |(z-1)[\tilde{\boldsymbol{\theta}}]|^2 + (\varepsilon m)^2$, which means that $\tilde{g} \in \mathcal{L}_2$ by the conclusion 2) in Lemma 1.

Step 3. Using discrete-time Bellman-Gronwall lemma to establish signal boundedness.

Using (19) in (13) yields

$$m_f^2(t) \le c + cf^2(a_0)\|(\tilde{g}m_f)_{t-1}\|^2 + ca_0^2 m_f^2(t) \tag{20}$$

where the coefficient c of the third term on the right-hand side of (21) is independent of a_0. By choosing appropriately small a_0 such that $ca_0^2 < 1/2$, we have $m_f^2(t) \le c + cf^2(a_0)\|(\tilde{g}m_f)_{t-1}\|^2$. Using the discrete-time Bellman-Gronwall lemma in [5] and $\tilde{g} \in L_2$, and following the similar discussion in [7], it is easy to conclude that $m_f(t) \in L_\infty$, from which conclusion 1) holds for Lemma 6.

Step 4. Establish the convergence of the tracking error e.

By (6) $\sim$ (8) and $\hat{\varpi} = \boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\phi}_p$, we have $\varepsilon m^2 = W_m(z)[\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\omega}] - \boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\phi}_p$. Applying Lemma 4 to $W_m(z)\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\omega}$, one has

$$\begin{aligned}\varepsilon m^2 &= \boldsymbol{\theta}^{\mathrm{T}}(W_m(z)\boldsymbol{\omega} - \boldsymbol{\phi}_p) + W_c(z)\Big[(W_b(z)z[\boldsymbol{\omega}^{\mathrm{T}}])(z-1)\boldsymbol{\theta}\Big] \\ &= -\theta_4 e + W_c(z)\Big[(W_b(z)z[\boldsymbol{\omega}^{\mathrm{T}}])(z-1)\boldsymbol{\theta}\Big]\end{aligned} \tag{21}$$

by using $W_m(z)\boldsymbol{\omega} - \boldsymbol{\phi}_p = [0, \cdots, 0, -e]^{\mathrm{T}}$; therefore

$$e = \frac{1}{\theta_4}\Big(-\varepsilon m^2 + W_c(z)\Big[(W_b(z)z[\boldsymbol{\omega}^{\mathrm{T}}])(z-1)\boldsymbol{\theta}\Big]\Big) \tag{22}$$

By Lemma 1, $1/\theta_4 \in \mathcal{L}_\infty$, $\varepsilon m(t)$, $\boldsymbol{\theta}(t+1) - \boldsymbol{\theta}(t) \in \mathcal{L}_2$. Noting that $W_c(z)$ and $W_b(z)$ are strictly proper and have the same poles as those of $W_m(z)$, from m, $\boldsymbol{\omega} \in \mathcal{L}_\infty$, it follows that $e \in \mathcal{L}_2$, which implies that $\lim_{t\to\infty} e(t) = 0$. □

5 Proofs of Lemmas 1, 3 $\sim$ 6

Proof of Lemma 1. 1) Let us prove it for two cases.

a) If $k_p > 0$, we have $\theta_4(t) \geq c_0$. In fact, when $\theta_4^p \mathrm{sgn}(k_p) \geq c_0$, we have $\Delta(t) = 0$ and then $\theta_4(t) = \theta_4^p \geq c_0$; when $\theta_4(t) < c_0$, it follows that $\theta_4(t) = c_0$ from the definitions of $\boldsymbol{\tau}_1, \tau_2$ and (9).

b) If $k_p < 0$, similarly, $\theta_4(t) \leq -c_0$. Hence, conclusion 1) holds.

2) We define $I(t) = \Delta^{\mathrm{T}}(t+1)\boldsymbol{\phi}(t)\varepsilon(t) + \tilde{\boldsymbol{\theta}}^{\mathrm{T}}(t)\Gamma^{-1}\Delta(t+1) + \Delta^{\mathrm{T}}(t+1)\Gamma^{-1}\Delta(t+1)$. When $\theta_4^p(t+1)\mathrm{sgn}(k_p) \geq c_0$, obviously, $I(t) = 0$; otherwise, $I(t) = (c_0\mathrm{sgn}(k_p) - \theta_4^p(t+1))\lambda_{2n}^{-1}(c_0\mathrm{sgn}(k_p) - \theta_4^*) < 0$ from (4), (9), and the definitions of $\boldsymbol{\tau}_1, \tau_2$, and $\Delta(t+1)$. Choose $V(\tilde{\boldsymbol{\theta}}(t)) = \tilde{\boldsymbol{\theta}}^{\mathrm{T}}(t)\Gamma^{-1}\tilde{\boldsymbol{\theta}}(t)$, whose time increment along (9) satisfies

$$V(\tilde{\boldsymbol{\theta}}(t+1)) - V(\tilde{\boldsymbol{\theta}}(t)) \leq -\Big(2 - \frac{\boldsymbol{\phi}_p^{\mathrm{T}}(t)\Gamma\boldsymbol{\phi}_p(t)}{m^2(t)}\Big)\varepsilon^2(t)m^2(t) + 2I(t) \leq -\alpha_1\varepsilon^2(t)m^2(t) \tag{23}$$

where $\alpha_1 = 2 - \max_{i=1,\cdots,2n}(\lambda_i) > 0$. The conclusion 2) holds from (8) and (23).

3) Let $J(t) = \Delta^{\mathrm{T}}(t+1)((\boldsymbol{\theta}^p(t+1) - \boldsymbol{\theta}(t)) + \Delta(t+1))$. When $\theta_4^p(t+1)\mathrm{sgn}(k_p) \geq 0$, obviously, otherwise, $J(t) = (c_0\mathrm{sgn}(k_p) - \theta_4^p(t+1))(c_0\mathrm{sgn}(k_p) - \theta_4(t)) < 0$ from conclusion 1) and the definitions of $\boldsymbol{\tau}_1$, τ_2 and $\Delta(t+1)$. Then

$$\begin{aligned}&\sum_{t=0}^{\infty}\Delta\boldsymbol{\theta}^{\mathrm{T}}(t)\Delta\boldsymbol{\theta}(t) \leq \\ &\sum_{t=0}^{\infty}\Big(\big(\frac{\boldsymbol{\phi}_p^{\mathrm{T}}(t)\Gamma\boldsymbol{\phi}_p(t)}{m^2(t)}\big)\varepsilon^2(t)m^2(t) + 2J(t)\Big) \leq \\ &\max_{i=1,\cdots,2n}\{\lambda_i\}\sum_{t=0}^{\infty}\varepsilon^2(t)m^2(t)\end{aligned} \tag{24}$$

which implies that $\Delta\boldsymbol{\theta}(t) = \boldsymbol{\theta}(t+1) - \boldsymbol{\theta}(t) \in \mathcal{L}_2$ by (9) and conclusion 2). □

Proof of Lemma 3. 1) Define $y_\delta(t) = \delta^{-t/2}y(t)$, $h_\delta(t) = \delta^{-t/2}h(t)$, $u_\delta(t) = \delta^{-t/2}u(t)$. Then, $y_\delta(t) = \delta^{-t/2}y(t) = \delta^{-t/2}\sum_{i=0}^{t}h(t-i)u(i) = \sum_{i=0}^{t}\delta^{-(t-i)/2}h(t-i)\delta^{-i/2}u(i) = h_\delta * u_\delta$. Now $u \in \mathcal{L}_{2e}$ implies that $u_\delta \in \mathcal{L}_{2e}$. Since $H(z)$ is analytic in $|z| \geq \sqrt{\delta}$ and $H_\delta(z) = \sum_{t=0}^{\infty}h_\delta(t)z^{-t} = \sum_{t=0}^{\infty}\delta^{-t/2}h(t)z^{-t} = \sum_{t=0}^{\infty}h(t)(\sqrt{\delta}z)^{-t} = H(\sqrt{\delta}z)$, which imply that $H(\sqrt{\delta}z)$ is analytic in $|z| \geq 1$, thus by Lemma 2, $\|(y_\delta)_t\|_2 \leq \|H_\delta(z)\|_\infty\|(u_\delta)_t\|_2 = \|H(\sqrt{\delta}z)\|_\infty\|(u_\delta)_t\|_2$. Because $\|(y_\delta)_t\|_2 = (\sum_{k=0}^{t}|y_\delta(k)|^2)^{1/2} = (\sum_{k=0}^{t}\delta^{-k}|y(k)|^2)^{1/2} = \delta^{-t/2}\|y_t\|_{2\delta}$, $\|(u_\delta)_t\|_2 = \delta^{-t/2}\|u_t\|_{2\delta}$ and $\|H(\sqrt{\delta}z)\|_\infty = \sup_{\omega\in[0,2\pi]}|H(\sqrt{\delta}\mathrm{e}^{j\omega})| = \|H(z)\|_{\infty\delta}$, 1) follows directly.

2) If $H(z)$ is strictly proper, $H(z)z$ is at least proper. Defining $H(z)z$ is $\mathcal{Z}$ transform of the function $h_1(t)$, and using the Schwartz inequality and Parseval's theorem[9], we get

$$\begin{aligned}|y(t)| &= |H(z)zu(t-1)| = \Big|\sum_{i=0}^{t-1}h_1(t-i-1)u(i)\Big| \leq \\ &\sum_{i=0}^{t-1}\delta^{-\frac{t-i-1}{2}}|h_1(t-i-1)|\delta^{\frac{t-i-1}{2}}|u(i)| \leq \\ &\Big(\sum_{i=0}^{\infty}|\delta^{-\frac{t-i}{2}}h_1(t-i)|^2\Big)^{\frac{1}{2}}\|u_{t-1}\|_{2\delta} = \\ &\frac{1}{\sqrt{2\pi}}\left\{\int_{-\pi}^{\pi}|\sqrt{\delta}\mathrm{e}^{j\omega}H(\sqrt{\delta}\mathrm{e}^{j\omega})|^2\mathrm{d}\omega\right\}^{\frac{1}{2}}\|u_{t-1}\|_{2\delta}\end{aligned} \tag{25}$$

which concludes the proof. □

Proof of Lemma 4. Let $\boldsymbol{x}(t+1) = A\boldsymbol{x}(t) + Bu(t)$, $\boldsymbol{x}(0) = 0$, $y = C^{\mathrm{T}}\boldsymbol{x}(t) + du(t)$ be a minimal realization of $W(z)$, which implies that $\boldsymbol{x}(t) = (zI - A)^{-1}Bu(t)$, $y(t) = C^{\mathrm{T}}(zI - A)^{-1}Bu(t) + du(t)$ or $\boldsymbol{x}(t) = \sum_{i=0}^{t-1}A^{t-i-1}Bu(i)$, $y(t) = C^{\mathrm{T}}\sum_{i=0}^{t-1}A^{t-i-1}Bu(i) + du(t)$. Hence, we can obtain that

$$W_1(z) = -C^{\mathrm{T}}\sum_{i=0}^{t-1}A^{t-i-1}z^{i-t}, \quad W_2(z) = \sum_{i=0}^{t-1}A^{t-i-1}Bz^{i-t},$$

$$\text{and } C^{\mathrm{T}}(zI - A)^{-1}B = C^{\mathrm{T}}\sum_{i=0}^{t-1}A^{t-i-1}Bz^{i-t},$$

from which it follows that

$$\begin{aligned}\sum_{i=0}^{j}A^{j-i}B\boldsymbol{\omega}^{\mathrm{T}}(i) &= \sum_{i=0}^{(j+1)-1}A^{(j+1)-i-1}Bz^{i-(j+1)}\boldsymbol{\omega}^{\mathrm{T}}(j+1) = \\ &W_2(z)\boldsymbol{\omega}^{\mathrm{T}}(j+1) = W_2(z)z\boldsymbol{\omega}^{\mathrm{T}}(j)\end{aligned} \tag{26}$$

and then

$$\begin{aligned}&W(z)[\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega}](t) = \\ &d[\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega}](t) + C^{\mathrm{T}}\sum_{i=0}^{t-1}A^{t-i-1}B[\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega}](i) = \\ &d[\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega}](t) + C^{\mathrm{T}}\sum_{i=0}^{t-1}A^{t-i-1}B[\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(i)\boldsymbol{\omega}(i) - \\ &\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(t)\boldsymbol{\omega}(i) + \tilde{\boldsymbol{\theta}}^{\mathrm{T}}(t)\boldsymbol{\omega}(i)] = \\ &d[\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega}](t) + \tilde{\boldsymbol{\theta}}^{\mathrm{T}}(t)C^{\mathrm{T}}\sum_{i=0}^{t-1}A^{t-i-1}Bz^{i-t}\boldsymbol{\omega}(t) + \\ &C^{\mathrm{T}}\sum_{i=0}^{t-1}A^{t-i-1}B\sum_{j=i}^{t-1}((1-z)\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(j))\boldsymbol{\omega}(i)) = \\ &\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(t)W(z)\boldsymbol{\omega}(t) - C^{\mathrm{T}}\sum_{i=0}^{t-1}\sum_{j=i}^{t-1}A^{t-i-1}B[(z-1)\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(j)]\boldsymbol{\omega}(i) = \\ &\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(t)W(z)\boldsymbol{\omega}(t) - C^{\mathrm{T}}\sum_{j=0}^{t-1}\sum_{i=0}^{j}A^{t-i-1}B[(z-1)\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(j)]\boldsymbol{\omega}(i) = \\ &\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(t)W(z)\boldsymbol{\omega}(t) - \\ &C^{\mathrm{T}}\sum_{j=0}^{t-1}A^{t-j-1}\Big[\Big(\sum_{i=0}^{j}A^{j-i}B\boldsymbol{\omega}^{\mathrm{T}}(i)\Big)[(z-1)\tilde{\boldsymbol{\theta}}(j)]\Big] = \\ &\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(t)W(z)\boldsymbol{\omega}(t) - \\ &C^{\mathrm{T}}\sum_{j=0}^{t-1}A^{t-j-1}z^{j-t}\big[(W_2(z)z\boldsymbol{\omega}^{\mathrm{T}}(t))[(z-1)\tilde{\boldsymbol{\theta}}(t)]\big] = \\ &\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(t)W(z)\boldsymbol{\omega}(t) + W_1(z)\big[(W_2(z)z\boldsymbol{\omega}^{\mathrm{T}}(t))[(z-1)\tilde{\boldsymbol{\theta}}(t)]\big]\end{aligned} \tag{27}$$

□

Remark 3. The proof is considered when $\boldsymbol{x}(0) = \mathbf{0}$. When the effect of initial conditions is taken into account, we find that the above identical operator relations still hold except for $C^{\mathrm{T}}(zI - A)^{-1}\boldsymbol{x}(0)$ and $C^{\mathrm{T}}A^t\boldsymbol{x}(0)$[1] which decay to zero term exponentially, and then the conclusion is

$$\begin{aligned}W(z)[\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega}] &= \tilde{\boldsymbol{\theta}}^{\mathrm{T}}(t)W(z)[\boldsymbol{\omega}](t) + \\ &W_1(z)[(W_2(z)z[\boldsymbol{\omega}^{\mathrm{T}}])((z-1)[\tilde{\boldsymbol{\theta}}])](t) + \epsilon(t)\end{aligned}$$

[1] When $\boldsymbol{x}(0) \neq \mathbf{0}$, $y(t) = C^{\mathrm{T}}A^t\boldsymbol{x}(0) + C^{\mathrm{T}}\sum_{i=0}^{t-1}A^{t-i-1}Bu(i) + du(t) = C^{\mathrm{T}}(zI - A)^{-1}\boldsymbol{x}(0) + C^{\mathrm{T}}\sum_{i=0}^{t-1}A^{t-i-1}Bu(i) + du(t)$

where $\epsilon(t)$ denotes exponentially decaying to zero term.

Proof of Lemma 5. From $F(z,a_0)=a_0^k/(z+a_0)^k$, it follows that

$$F_1(z,a_0) = \frac{(z+a_0)^k-a_0^k}{(z+a_0)^k}z = a_0\sum_{i=1}^{k}C_k^i\Big(\frac{z}{z+a_0}\Big)^{i+1}\Big(\frac{a_0}{z+a_0}\Big)^{k-i-1} \quad (28)$$

Thus

$$\begin{aligned}&\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(t)\boldsymbol{\omega}(t) = \\ &F_1(z,a_0)z^{-1}(\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(t)\boldsymbol{\omega}(t)) + F(z,a_0)(\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(t)\boldsymbol{\omega}(t)) = \\ &F_1(z,a_0)(\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(t-1)\boldsymbol{\omega}(t-1)) + F(z,a_0)(\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(t)\boldsymbol{\omega}(t))\end{aligned} \quad (29)$$

Obviously,

$$\Big\|\frac{z^{i+1}}{(z+a_0)^{i+1}}\Big\|_{\infty\delta} = \Big\|\frac{z}{z+a_0}\Big\|_{\infty\delta}^{i+1} = \sup_{\omega\in[0,2\pi]}\Big|\frac{\sqrt{\delta}\mathrm{e}^{j\omega}}{\sqrt{\delta}\mathrm{e}^{j\omega}+a_0}\Big|^{i+1} = \Big|\frac{\sqrt{\delta}}{a_0-\sqrt{\delta}}\Big|^{i+1} \quad (30)$$

$$\Big\|\frac{a_0^{k-i-1}}{(z+a_0)^{k-i-1}}\Big\|_{\infty\delta} = \Big\|\frac{a_0}{z+a_0}\Big\|_{\infty\delta}^{k-i-1} = \sup_{\omega\in[0,2\pi]}\Big|\frac{a_0}{\sqrt{\delta}\mathrm{e}^{j\omega}+a_0}\Big|^{k-i-1} \quad (31)$$

Therefore, for $|a_0|\le\sqrt{\delta}/2$

$$\begin{aligned}\|F_1(z,a_0)\|_{\infty\delta} =& a_0\sum_{i=1}^{k}C_k^i\Big\|\frac{z}{z+a_0}\Big\|_{\infty\delta}^{i+1}\Big\|\frac{a_0}{z+a_0}\Big\|_{\infty\delta}^{k-i-1} = \\ & a_0\sum_{i=1}^{k}C_k^i\Big|\frac{(\sqrt{\delta})^{i+1}a_0^{k-i-1}}{(a_0-\sqrt{\delta})^k}\Big| \le \\ & a_0\sum_{i=1}^{k}C_k^i\Big|\frac{(\sqrt{\delta})^{i+1}(\frac{\sqrt{\delta}}{2})^{k-i-1}}{(\frac{\sqrt{\delta}}{2})^k}\Big| = \\ & a_0\sum_{i=1}^{k}C_k^i2^{i+1} = ca_0\end{aligned} \quad (32)$$

where $c=\sum_{i=1}^{k}C_k^i2^{i+1}$ is a constant independent of a_0. □

Proof of Lemma 6. By using Lemma 3, the proof of the lemma is similar to that of continuous case Lemma 6.8.1 in [2]. □

6 Conclusion

For a class of discrete-time systems, the design and analysis of direct MRAC with normalized adaptive laws are investigated in this paper. The stability and convergence properties of the discrete-time MRAC scheme are analyzed rigorously in a systematic fashion as in the continuous-time case. There are some problems that are yet to be investigated, for example, how to design and analyze discrete-time indirect adaptive control schemes, how to deal with discrete-time MRAC with the unmodeled dynamics and disturbances[10], how to generalize the result to MIMO systems[8], and how to treat with dual-rate and multi-rate discrete adaptive control schemes[11].

References

1 Goodwin G C, Sin K S. *Adaptive Filtering, Prediction and Control*. New Jersey: Prentice-Hall, 1984. 98~162

2 Ioannou P A, Sun J. *Robust Adaptive Control*. New Jersey: Prentice-Hall, 1996. 313~397

3 Narendra K S, Annaswamy A M. *Stable Adaptive Systems*. New Jersey: Prentice-Hall, 1989

4 Sastry S, Bodson M. *Adaptive Control: Stability, Convergence, and Robustness*. New Jersey: Prentice-Hall, 1989

5 Tao Gang. *Adaptive Control Design and Analysis*. New York: Wiley-Interscience, 2003

6 Ydstie B E. Auto tuning of the time horizon. In: Proceedings of the 2nd IFAC Workshop on Adaptive Control. Lund, Sweden, 1986. 215~219

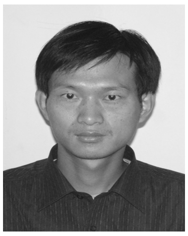
7 Silva G J, Datta A. Adaptive internal model control: the discrete-time case. *International Journal of Adaptive Control and Signal Processing*, 2001, **15**(1): 15~36

8 Xie X J, Zhang Z Q. Robust direct model reference adaptive control using factorization for multivariable plants. *International Journal of Control*, 2005, **78**(10): 762~772

9 Desoer C A, Vidyasagar M. *Feedback System: Input-Output Properties*. New York: Academic Press, 1975

10 Xie X J, Wu Y Q. Robust model reference adaptive control with hybrid adaptive law. *International Journal of Systems Science*, 2002, **33**(14): 1109~1119

11 Ding F, Chen T. Least squares based self-tuning control of dual-rate system. *International Journal of Adaptive Control and Signal Processing*, 2004, **18**(8): 697~714



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