Delay-dependent Robust Stability of Neutral Systems with Mixed Delays and Nonlinear Perturbations

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Abstract This paper concerns the delay-dependent robust stability problem of uncertain neutral systems with mixed neutral and discrete delays. Nonlinear time-varying parameter perturbations are considered. Based on the newly established integral inequalities, the neutral-delay-dependent and discrete-delay-dependent stability criterion is derived without using a fixed model transformation. The condition is presented in terms of linear matrix inequality and can be easily solved by existing convex optimization techniques. A numerical example is given to demonstrate the less conservatism of the proposed results.

Key words Neutral systems, delay-dependent criteria, nonlinear perturbations, robust stability, linear matrix inequality

1 Introduction

The problem of the stability of various neutral delay-differential systems has received considerable attention in recent years[1~5]. Current stability criteria for the neutral systems can be roughly divided into two categories, namely delay-independent criteria and delay-dependent criteria. Since the delay-dependent criteria contain information about the delay size, they are generally less conservative than the delay-independent ones as might be expected, especially when the delay is small. So, more attention has been paid to delay-dependent criteria.

Various different techniques have been proposed by many researchers to derive the delay-dependent stability criteria for a number of different neutral systems, for example, model transformation techniques[1,2,4,5], the improved bounding techniques[6,7], and the properly chosen Lyapunov-Krasovskii functionals[3,8]. The model transformation techniques have been widely used in the derivation of the delay-dependent stability criteria for neutral systems. However, these model transformations often introduce additional dynamics or require some additional assumptions, which leads to relatively conservative results (see discussions in [1, 4, 5]). Besides, for neutral systems with mixed discrete and neutral delays, most of the aforementioned methods can only provide discrete-delay-dependent and neutral-delay-independent results. In [9], a new approach was proposed to analyze the stability of neutral systems with mixed delays by incorporating some free weighting matrices, and the less conservative criteria, which were both discrete-delay-dependent and neutral-delay-dependent, were obtained without considering the model transformations. However, some of the free matrices did not serve to reduce the conservatism of the results that were obtained. Furthermore, to the best of the authors′ knowledge, few results have been reported in the literature concerning the problem of robust stability of the neutral systems with nonlinear perturbations and mixed neutral

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and discrete delays. In [10], the authors studied this problem and presented neutral-delay-independent stability criteria.

In this paper, we propose a new approach for dealing with the problem of robust stability of the neutral systems with mixed discrete and neutral delays and nonlinear perturbations. First, a new integral inequality that is particularly suitable for the analysis of the stability of the neutral systems under investigation is established. Then, a new delay-dependent criterion is derived by combining the newly established integral inequality and the new Lyapunov function used in [4]. The condition that is obtained is both discrete-delay-dependent and neutral-delay-dependent. Therefore, it is less conservative than that in [10].

2 Problem statement

Consider the following neutral system with time-varying discrete delay

$$\dot{\boldsymbol{x}}(t)-C\dot{\boldsymbol{x}}(t-\tau_2)=A\boldsymbol{x}(t)+B\boldsymbol{x}(t-\tau_1(t))+\boldsymbol{f}_1(\boldsymbol{x}(t),t)+\boldsymbol{f}_2(\boldsymbol{x}(t-\tau_1(t)),t)+\boldsymbol{f}_3(\dot{\boldsymbol{x}}(t-\tau_2),t) \quad (1)$$

where $\boldsymbol{x}(t)\in\mathbf{R}^n$ is the state vector; $A\in\mathbf{R}^n$, $B\in\mathbf{R}^n$, and $C\in\mathbf{R}^n$ are constant matrices; $\tau_1(t)$ is the time-varying discrete delay and τ_2 is the unknown but constant neutral delay, and they are assumed to satisfy

$$0\le\tau_1(t)\le\tau_{1m},\ \dot{\tau}_1(t)\le\tau_{1d},\ 0\le\tau_2\le\tau_{2m} \quad (2)$$

where τ_{1m}, τ_{2m}, and τ_{1d} are known constants. $\boldsymbol{f}_1(\boldsymbol{x}(t),t)$, $\boldsymbol{f}_2(\boldsymbol{x}(t-\tau_1(t)),t)$, and $\boldsymbol{f}_3(\dot{\boldsymbol{x}}(t-\tau_2),t)$ are the nonlinear perturbations in the system model. They satisfy that $\boldsymbol{f}_1(0,t)=0$, $\boldsymbol{f}_2(0,t)=0$, and $\boldsymbol{f}_3(0,t)=0$. The initial condition of system (1) is described by

$$\boldsymbol{x}(t_0+\theta)=\boldsymbol{\psi}(\theta),\ \dot{\boldsymbol{x}}(t_0+\theta)=\dot{\boldsymbol{\psi}}(\theta)$$
$$\forall\,\theta\in[-\max(\tau_{1m},\ \tau_{2m}),\ 0] \quad (3)$$

where $\boldsymbol{\psi}(\cdot)$ is a vector-valued initial function. It is assumed that the nonlinear perturbations are bounded in magnitude, *i.e.*,

$$\|\boldsymbol{f}_1(\boldsymbol{x}(t),t)\|\le\alpha_1\|\boldsymbol{x}(t)\|$$
$$\|\boldsymbol{f}_2(\boldsymbol{x}(t-\tau_1(t)),t)\|\le\alpha_2\|\boldsymbol{x}(t-\tau_1(t))\|$$
$$\|\boldsymbol{f}_3(\dot{\boldsymbol{x}}(t-\tau_2),t)\|\le\alpha_3\|\dot{\boldsymbol{x}}(t-\tau_2)\|,\ \forall\,t>0 \quad (4)$$

where α_1, α_2, and α_3 are known positive scalars.

It should be noted that system (1) encompasses many natural models of time-delay systems and can be used to represent many important physical systems, for example, networks containing lossless transmission lines, vibrating masses attached to an elastic bar. In addition, if

$$\boldsymbol{f}_1(\boldsymbol{x}(t),t)=\triangle A(t)\boldsymbol{x}(t)$$
$$\boldsymbol{f}_2(\boldsymbol{x}(t-\tau_1(t)),t)=\triangle B(t)\boldsymbol{x}(t-\tau_1(t))$$
$$\boldsymbol{f}_3(\dot{\boldsymbol{x}}(t-\tau_2),t)=\triangle C(t)\dot{\boldsymbol{x}}(t-\tau_2) \quad (5)$$

then the nonlinear perturbations are reduced to be the norm-bounded uncertainties that are well known in robust control of uncertain systems.

The objective of this paper is to develop a new stability criterion for system (1) with nonlinear perturbations (4). To this end, we establish the following integral inequality, which plays a key role in the derivation of the main results.

Lemma 1. For any matrices $Z=Z^{\mathrm{T}}>0$, Y_1, Y_2, Y_3, Y_4 and positive scalars τ_1, τ_2, the following inequality holds

$$-\int_{t-\tau_1}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(\alpha)Z\dot{\boldsymbol{x}}(\alpha)\mathrm{d}\alpha\le\boldsymbol{\rho}^{\mathrm{T}}(t)\tilde{Y}\boldsymbol{\rho}(t)+\boldsymbol{\rho}^{\mathrm{T}}(t)F^{\mathrm{T}}\tau_1Z^{-1}F\boldsymbol{\rho}(t) \quad (6)$$

where

$$\boldsymbol{\rho}^{\mathrm{T}}(t)=\begin{bmatrix}\boldsymbol{x}^{\mathrm{T}}(t) & \dot{\boldsymbol{x}}^{\mathrm{T}}(t) & \boldsymbol{x}^{\mathrm{T}}(t-\tau_1) & \dot{\boldsymbol{x}}^{\mathrm{T}}(t-\tau_2)\end{bmatrix}$$
$$F=\begin{bmatrix}Y_1^{\mathrm{T}} & Y_2^{\mathrm{T}} & Y_3^{\mathrm{T}} & Y_4^{\mathrm{T}}\end{bmatrix}$$
$$\tilde{Y}=\begin{bmatrix}Y_1+Y_1^{\mathrm{T}} & Y_2^{\mathrm{T}} & -Y_1+Y_3^{\mathrm{T}} & Y_4^{\mathrm{T}}\\ * & 0 & -Y_2 & 0\\ * & * & -Y_3-Y_3^{\mathrm{T}} & -Y_4^{\mathrm{T}}\\ * & * & * & 0\end{bmatrix}$$

Proof. Define $N=\begin{bmatrix}N_1^{\mathrm{T}} & N_2^{\mathrm{T}} & N_3^{\mathrm{T}} & N_4^{\mathrm{T}}\end{bmatrix}$ and $Y_i=-N_i$ for $i=1,\cdots,4$, where N_i are any matrices with appropriate dimensions. Then, by the well-known inequality $2\boldsymbol{a}^{\mathrm{T}}\boldsymbol{b}\le\boldsymbol{a}^{\mathrm{T}}Q\boldsymbol{a}+\boldsymbol{b}^{\mathrm{T}}Q^{-1}\boldsymbol{b}$, we have

$$-\int_{t-\tau_1}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(\alpha)Z\dot{\boldsymbol{x}}(\alpha)\mathrm{d}\alpha\le$$
$$-2\int_{t-\tau_1}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(\alpha)N\boldsymbol{\rho}(t)\mathrm{d}\alpha+\int_{t-\tau_1}^{t}\boldsymbol{\rho}^{\mathrm{T}}(t)N^{\mathrm{T}}Z^{-1}N\boldsymbol{\rho}(t)\mathrm{d}\alpha=$$
$$2[\boldsymbol{x}^{\mathrm{T}}(t)-\boldsymbol{x}^{\mathrm{T}}(t-\tau_1)](-N)\boldsymbol{\rho}(t)+$$
$$\boldsymbol{\rho}^{\mathrm{T}}(t)(-N)^{\mathrm{T}}\tau_1Z^{-1}(-N)\boldsymbol{\rho}(t)=$$
$$2[\boldsymbol{x}^{\mathrm{T}}(t)-\boldsymbol{x}^{\mathrm{T}}(t-\tau_1)]F\boldsymbol{\rho}(t)+\boldsymbol{\rho}^{\mathrm{T}}(t)F^{\mathrm{T}}\tau_1Z^{-1}F\boldsymbol{\rho}(t)=$$
$$2\boldsymbol{\rho}^{\mathrm{T}}(t)[I\ \ 0\ \ -I\ \ 0]^{\mathrm{T}}[Y_1^{\mathrm{T}}\ \ Y_2^{\mathrm{T}}\ \ Y_3^{\mathrm{T}}\ \ Y_4^{\mathrm{T}}]\boldsymbol{\rho}(t)+$$
$$\boldsymbol{\rho}^{\mathrm{T}}(t)F^{\mathrm{T}}\tau_1Z^{-1}F\boldsymbol{\rho}(t)=$$
$$\boldsymbol{\rho}^{\mathrm{T}}(t)\tilde{Y}\boldsymbol{\rho}(t)+\boldsymbol{\rho}^{\mathrm{T}}(t)F^{\mathrm{T}}\tau_1Z^{-1}F\boldsymbol{\rho}(t)$$ □

Remark 1. An integral-inequality method was proposed in [11] for the robust stabilization of a class of uncertain time-delay systems. Inspired by this idea, we establish the above inequality that is suitable for the analysis of the stability of the neutral system (1).

3 Main results

In this section, a sufficient condition for the stability of system (1) with nonlinear perturbations (4) is derived based on the integral inequality (6). The main result is given as the following theorem.

Theorem 1. Given positive scalars α_1, α_2, α_3, τ_{im}, and τ_{1d}, system (1) is asymptotically stable if $\|C\|+\alpha_3<1$ and there exist symmetric positive-definite matrices P_1, Q, S, R_1, R_2, Z_1, Z_2, matrices P_2, P_3, Y_{i1}, Y_{i2}, Y_{i3}, Y_{i4} and positive scalars ε_j, $i=1,2$, $j=1,\cdots,6$, satisfying the LMI (7), as shown at the top of next page, where

$$\varphi_{11}=A^{\mathrm{T}}P_2+P_2^{\mathrm{T}}A+QA+A^{\mathrm{T}}Q+R_1+R_2+Y_{11}+Y_{11}^{\mathrm{T}}+Y_{21}+Y_{21}^{\mathrm{T}}+(\varepsilon_1+\varepsilon_4)\alpha_1^2I$$
$$\varphi_{12}=P_1-P_2^{\mathrm{T}}+A^{\mathrm{T}}P_3+Y_{12}^{\mathrm{T}}+Y_{22}^{\mathrm{T}}$$
$$\varphi_{13}=P_2^{\mathrm{T}}B+QB-Y_{11}+Y_{13}^{\mathrm{T}}$$
$$\varphi_{14}=-A^{\mathrm{T}}QC-Y_{21}+Y_{23}^{\mathrm{T}},\quad \varphi_{15}=P_2^{\mathrm{T}}C+Y_{14}^{\mathrm{T}}+Y_{24}^{\mathrm{T}}$$
$$\varphi_{22}=-P_3-P_3^{\mathrm{T}}+S+\tau_{1m}Z_1+\tau_{2m}Z_2$$
$$\varphi_{23}=P_3^{\mathrm{T}}B-Y_{12},\quad \varphi_{24}=-Y_{22}$$
$$\varphi_{33}=-(1-\tau_{1d})R_1-Y_{13}-Y_{13}^{\mathrm{T}}+(\varepsilon_2+\varepsilon_5)\alpha_2^2I$$
$$\varphi_{34}=-B^{\mathrm{T}}QC,\quad \varphi_{44}=-R_2-Y_{23}-Y_{23}^{\mathrm{T}}$$
$$\varphi_{55}=-S+(\varepsilon_3+\varepsilon_6)\alpha_3^2I$$
$$\Psi_{a1}=[P_2^{\mathrm{T}}\ \ P_2^{\mathrm{T}}\ \ P_2^{\mathrm{T}}],\quad \Psi_{a2}=[P_3^{\mathrm{T}}\ \ P_3^{\mathrm{T}}\ \ P_3^{\mathrm{T}}]$$
$$\Psi_{b1}=[Q\ \ Q\ \ Q],\quad \Psi_{b2}=[-C^{\mathrm{T}}Q\ \ -C^{\mathrm{T}}Q\ \ -C^{\mathrm{T}}Q]$$
$$\triangle_a=\mathrm{diag}\{-\varepsilon_1I,\ -\varepsilon_2I,\ -\varepsilon_3I\}$$

$$\begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} & \varphi_{14} & \varphi_{15} & \tau_{1m}Y_{11} & \tau_{2m}Y_{21} & \Psi_{a1} & \Psi_{b1} \\ * & \varphi_{22} & \varphi_{23} & \varphi_{24} & P_3^{\mathrm{T}}C & \tau_{1m}Y_{12} & \tau_{2m}Y_{22} & \Psi_{a2} & 0 \\ * & * & \varphi_{33} & \varphi_{34} & -Y_{14}^{\mathrm{T}} & \tau_{1m}Y_{13} & 0 & 0 & 0 \\ * & * & * & \varphi_{44} & -Y_{24}^{\mathrm{T}} & 0 & \tau_{2m}Y_{23} & 0 & \Psi_{b2} \\ * & * & * & * & \varphi_{55} & \tau_{1m}Y_{14} & \tau_{2m}Y_{24} & 0 & 0 \\ * & * & * & * & * & -\tau_{1m}Z_1 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\tau_{2m}Z_2 & 0 & 0 \\ * & * & * & * & * & * & * & \triangle_a & 0 \\ * & * & * & * & * & * & * & * & \triangle_b \end{bmatrix} < 0 \tag{7}$$

$$\triangle_b = \mathrm{diag}\{-\varepsilon_4 I,\ -\varepsilon_5 I,\ -\varepsilon_6 I\}$$

Proof. Define $\boldsymbol{v}(t) = \boldsymbol{x}(t) - C\boldsymbol{x}(t-\tau_2)$, and choose the candidate Lyapunov-Krasovskii functional to be

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) \tag{8}$$

where

$$V_1(t) = \boldsymbol{v}^{\mathrm{T}}(t)Q\boldsymbol{v}(t) + \begin{bmatrix} \boldsymbol{x}(t) \\ \dot{\boldsymbol{x}}(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t) \\ \dot{\boldsymbol{x}}(t) \end{bmatrix}$$

$$V_2(t) = \int_{t-\tau_1(t)}^{t} \boldsymbol{x}^{\mathrm{T}}(\alpha)R_1\boldsymbol{x}(\alpha)\mathrm{d}\alpha + \int_{t-\tau_2}^{t} \boldsymbol{x}^{\mathrm{T}}(\alpha)R_2\boldsymbol{x}(\alpha)\mathrm{d}\alpha$$

$$V_3(t) = \int_{t-\tau_2}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(\alpha)S\dot{\boldsymbol{x}}(\alpha)\mathrm{d}\alpha$$

$$V_4(t) = \sum_{i=1}^{2} \int_{-\tau_{im}}^{0} \int_{t+\beta}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(\alpha)Z_i\dot{\boldsymbol{x}}(\alpha)\mathrm{d}\alpha\mathrm{d}\beta$$

Define $\boldsymbol{\chi}(t) = A\boldsymbol{x}(t) - \dot{\boldsymbol{x}}(t) + B\boldsymbol{x}(t-\tau_1(t)) + C\dot{\boldsymbol{x}}(t-\tau_2) + \boldsymbol{f}_1(\boldsymbol{x}(t),t) + \boldsymbol{f}_2(\boldsymbol{x}(t-\tau_1(t)),t) + \boldsymbol{f}_3(\dot{\boldsymbol{x}}(t-\tau_2),t)$. It follows from (1) that $\boldsymbol{\chi}(t) = 0$. By (4) and the well-known inequality $2\boldsymbol{a}^{\mathrm{T}}\boldsymbol{b} \le \varepsilon\boldsymbol{a}^{\mathrm{T}}\boldsymbol{a} + \varepsilon^{-1}\boldsymbol{b}^{\mathrm{T}}\boldsymbol{b}$, where ε is a positive scalar, and the derivative of $V_1(t)$ along any solution of system (1) is given by

$$\begin{aligned} \dot{V}_1(t) = {} & 2\boldsymbol{v}^{\mathrm{T}}(t)Q\dot{\boldsymbol{v}}(t) + 2\begin{bmatrix} \boldsymbol{x}(t) \\ \dot{\boldsymbol{x}}(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \dot{\boldsymbol{x}}(t) \\ 0 \end{bmatrix} = \\ & 2\boldsymbol{v}^{\mathrm{T}}(t)Q[A\boldsymbol{x}(t) + B\boldsymbol{x}(t-\tau_1(t)) + \boldsymbol{f}_1(\boldsymbol{x}(t),t) + \\ & \boldsymbol{f}_2(\boldsymbol{x}(t-\tau_1(t)),t) + \boldsymbol{f}_3(\dot{\boldsymbol{x}}(t-\tau_2),t)] + \\ & 2\begin{bmatrix} \boldsymbol{x}(t) \\ \dot{\boldsymbol{x}}(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \dot{\boldsymbol{x}}(t) \\ \boldsymbol{\chi}(t) \end{bmatrix} \le \\ & 2[\boldsymbol{x}^{\mathrm{T}}(t) - \boldsymbol{x}^{\mathrm{T}}(t-\tau_2)C^{\mathrm{T}}]Q[A\boldsymbol{x}(t) + B\boldsymbol{x}(t-\tau_1(t))] + \\ & (\sum_{i=1}^{3}\varepsilon_i^{-1}) \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{x}(t-\tau_2) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Q \\ -C^T Q \end{bmatrix} \begin{bmatrix} Q \\ -C^T Q \end{bmatrix}^{\mathrm{T}} \times \\ & \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{x}(t-\tau_2) \end{bmatrix} + \varepsilon_2\alpha_2^2\boldsymbol{x}^{\mathrm{T}}(t-\tau_1(t))\boldsymbol{x}(t-\tau_1(t)) + \\ & \varepsilon_1\alpha_1^2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{x}(t) + \varepsilon_3\alpha_3^2\dot{\boldsymbol{x}}^{\mathrm{T}}(t-\tau_2)\dot{\boldsymbol{x}}(t-\tau_2) + \\ & 2\begin{bmatrix} \boldsymbol{x}(t) \\ \dot{\boldsymbol{x}}(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}^{\mathrm{T}} \times \\ & \begin{bmatrix} \dot{\boldsymbol{x}}(t) \\ A\boldsymbol{x}(t) - \dot{\boldsymbol{x}}(t) + B\boldsymbol{x}(t-\tau_1(t)) + C\dot{\boldsymbol{x}}(t-\tau_2) \end{bmatrix} + \\ & (\sum_{i=4}^{6}\varepsilon_i^{-1}) \begin{bmatrix} \boldsymbol{x}(t) \\ \dot{\boldsymbol{x}}(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} P_2^{\mathrm{T}} \\ P_3^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} P_2^{\mathrm{T}} \\ P_3^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \boldsymbol{x}(t) \\ \dot{\boldsymbol{x}}(t) \end{bmatrix} + \\ & \varepsilon_4\alpha_1^2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{x}(t) + \varepsilon_5\alpha_2^2\boldsymbol{x}^{\mathrm{T}}(t-\tau_1(t))\boldsymbol{x}(t-\tau_1(t)) + \\ & \varepsilon_6\alpha_3^2\dot{\boldsymbol{x}}^{\mathrm{T}}(t-\tau_2)\dot{\boldsymbol{x}}(t-\tau_2) \end{aligned} \tag{9}$$

Computing $\dot{V}_2(t)$ and $\dot{V}_3(t)$, we have

$$\begin{aligned} \dot{V}_2(t) \le {} & \boldsymbol{x}^{\mathrm{T}}(t)(R_1+R_2)\boldsymbol{x}(t) - \boldsymbol{x}^{\mathrm{T}}(t-\tau_2)R_2\boldsymbol{x}(t-\tau_2) - \\ & \boldsymbol{x}^{\mathrm{T}}(t-\tau_1(t))(1-\tau_{1d})R_1\boldsymbol{x}(t-\tau_1(t)) \end{aligned} \tag{10}$$

$$\dot{V}_3(t) = \dot{\boldsymbol{x}}^{\mathrm{T}}(t)S\dot{\boldsymbol{x}}(t) - \dot{\boldsymbol{x}}^{\mathrm{T}}(t-\tau_2)S\dot{\boldsymbol{x}}(t-\tau_2) \tag{11}$$

By Lemma 1, we obtain

$$\begin{aligned} \dot{V}_4(t) = {} & \dot{\boldsymbol{x}}^{\mathrm{T}}(t)(\tau_{1m}Z_1 + \tau_{2m}Z_2)\dot{\boldsymbol{x}}(t) - \\ & \sum_{i=1}^{2}\int_{t-\tau_{im}}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(\alpha)Z_i\dot{\boldsymbol{x}}(\alpha)\mathrm{d}\alpha \le \\ & \dot{\boldsymbol{x}}^{\mathrm{T}}(t)(\tau_{1m}Z_1 + \tau_{2m}Z_2)\dot{\boldsymbol{x}}(t) - \int_{t-\tau_2}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(\alpha)Z_2\dot{\boldsymbol{x}}(\alpha)\mathrm{d}\alpha - \\ & \int_{t-\tau_1(t)}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(\alpha)Z_1\dot{\boldsymbol{x}}(\alpha)\mathrm{d}\alpha \le \\ & \dot{\boldsymbol{x}}^{\mathrm{T}}(t)(\tau_{1m}Z_1 + \tau_{2m}Z_2)\dot{\boldsymbol{x}}(t) + \boldsymbol{\xi}_1^{\mathrm{T}}(t)\tilde{Y}_1\boldsymbol{\xi}_1(t) + \\ & \boldsymbol{\xi}_2^{\mathrm{T}}(t)\tilde{Y}_2\boldsymbol{\xi}_2(t) + \boldsymbol{\xi}_1^{\mathrm{T}}(t)F_1^{\mathrm{T}}\tau_1(t)Z_1^{-1}F_1\boldsymbol{\xi}_1(t) + \\ & \boldsymbol{\xi}_2^{\mathrm{T}}(t)F_2^{\mathrm{T}}\tau_2 Z_2^{-1}F_2\boldsymbol{\xi}_2(t) \le \\ & \sum_{i=1}^{2}(\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\tau_{im}Z_i\dot{\boldsymbol{x}}(t) + \boldsymbol{\xi}_i^{\mathrm{T}}(t)\tilde{Y}_i\boldsymbol{\xi}_i(t) + \\ & \boldsymbol{\xi}_i^{\mathrm{T}}(t)F_i^{\mathrm{T}}\tau_{im}Z_i^{-1}F_i\boldsymbol{\xi}_i(t)) \end{aligned} \tag{12}$$

where

$$\boldsymbol{\xi}_1^{\mathrm{T}}(t) = [\boldsymbol{x}^{\mathrm{T}}(t)\ \ \dot{\boldsymbol{x}}^{\mathrm{T}}(t)\ \ \boldsymbol{x}^{\mathrm{T}}(t-\tau_1(t))\ \ \dot{\boldsymbol{x}}^{\mathrm{T}}(t-\tau_2)]$$

$$\boldsymbol{\xi}_2^{\mathrm{T}}(t) = [\boldsymbol{x}^{\mathrm{T}}(t)\ \ \dot{\boldsymbol{x}}^{\mathrm{T}}(t)\ \ \boldsymbol{x}^{\mathrm{T}}(t-\tau_2)\ \ \dot{\boldsymbol{x}}^{\mathrm{T}}(t-\tau_2)]$$

$$\tilde{Y}_i = \begin{bmatrix} Y_{i1} + Y_{i1}^{\mathrm{T}} & Y_{i2}^{\mathrm{T}} & -Y_{i1} + Y_{i3}^{\mathrm{T}} & Y_{i4}^{\mathrm{T}} \\ * & 0 & -Y_{i2} & 0 \\ * & * & -Y_{i3} - Y_{i3}^{\mathrm{T}} & -Y_{i4}^{\mathrm{T}} \\ * & * & * & 0 \end{bmatrix}$$

$$F_i = [Y_{i1}^{\mathrm{T}}\ \ Y_{i2}^{\mathrm{T}}\ \ Y_{i3}^{\mathrm{T}}\ \ Y_{i4}^{\mathrm{T}}], \quad i = 1, 2$$

Combining (9)∼(12) yields $\dot{V}(t) \le \boldsymbol{\eta}^{\mathrm{T}}(t)\Omega\boldsymbol{\eta}(t)$, where

$$\boldsymbol{\eta}^{\mathrm{T}}(t) = [\boldsymbol{x}^{\mathrm{T}}(t)\ \dot{\boldsymbol{x}}^{\mathrm{T}}(t)\ \boldsymbol{x}^{\mathrm{T}}(t-\tau_1(t))\ \boldsymbol{x}^{\mathrm{T}}(t-\tau_2)\ \dot{\boldsymbol{x}}^{\mathrm{T}}(t-\tau_2)]$$

$$\Omega = \Omega_0 + \sum_{i=1}^{2} F_{i0}^{\mathrm{T}}\tau_{im}Z_i^{-1}F_{i0} + L_a(-\Delta_a)^{-1}L_a^{\mathrm{T}} + L_b(-\Delta_b)^{-1}L_b^{\mathrm{T}}$$

$$L_a^{\mathrm{T}} = \begin{bmatrix} \Psi_{a1}^{\mathrm{T}} & 0 & 0 & \Psi_{a2}^{\mathrm{T}} & 0 \end{bmatrix}$$

$$\Omega_0 = \begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} & \varphi_{14} & \varphi_{15} \\ * & \varphi_{22} & \varphi_{23} & \varphi_{24} & P_3^{\mathrm{T}}C \\ * & * & \varphi_{33} & \varphi_{34} & -Y_{14}^{\mathrm{T}} \\ * & * & * & \varphi_{44} & -Y_{24}^{\mathrm{T}} \\ * & * & * & * & \varphi_{55} \end{bmatrix}$$

$$F_{10}^{\mathrm{T}} = \begin{bmatrix} Y_{11} \\ Y_{12} \\ Y_{13} \\ 0 \\ Y_{14} \end{bmatrix}, \quad F_{20}^{\mathrm{T}} = \begin{bmatrix} Y_{21} \\ Y_{22} \\ 0 \\ Y_{23} \\ Y_{24} \end{bmatrix}, \quad L_b = \begin{bmatrix} \Psi_{b1} \\ \Psi_{b2} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Omega < 0$ implies that the time derivative of $V(t)$ is negative. Therefore, it follows from Theorem 1.6 in [12] that system (1) is asymptotically stable under the assumption $\|C\| + \alpha_3 < 1$. By Schur complements, $\Omega < 0$ is equivalent to the LMI (7). □

Remark 2. Since the condition in Theorem 1 is both neutral-delay-dependent and discrete-delay-dependent, it is less conservative than some existing results of the stability for the systems with mixed neutral and discrete delays. Furthermore, instead of using the model transformations, which usually leads to conservative results and complex derivation procedure, we combine the newly established integral inequalities with the new Lyapunov functional method used in [4] to derive the delay-dependent condition. Therefore, the derivation procedure is much simpler, and the result is less conservative.

Remark 3. The method proposed in this paper can be easily extended to the neutral systems with multiple mixed neutral and discrete delays and nonlinear perturbations.

Remark 4. The norm-bounded uncertainties can be treated as a special case of nonlinear perturbations. Therefore, the stability criterion for system (1) with norm-bounded uncertainties can be obtained by following a similar line as in Theorem 1.

4 Illustrative example

A numerical example is provided in this section to illustrate the effectiveness of our results.

Example. Consider the system (1) with

$$A=\begin{bmatrix} -1.2 & 0.1 \\ -0.1 & -1 \end{bmatrix}, \ B=\begin{bmatrix} -0.6 & 0.7 \\ -1 & -0.8 \end{bmatrix}, \ C=\begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}$$

$$\|\boldsymbol{f}_1(\boldsymbol{x}(t),t)\| \le \alpha_1\|\boldsymbol{x}(t)\|$$
$$\|\boldsymbol{f}_2(\boldsymbol{x}(t-\tau_1(t)),t)\| \le \alpha_2\|\boldsymbol{x}(t-\tau_1(t))\|$$
$$\|\boldsymbol{f}_3(\dot{\boldsymbol{x}}(t-\tau_2),t)\| \le \alpha_3\|\dot{\boldsymbol{x}}(t-\tau_2)\| \tag{13}$$

where $\alpha_1 \ge 0$, $\alpha_2 \ge 0$, $\alpha_3 \ge 0$, and $0 \le |c| < 1$.

We now consider the effect of the bound α_3 on the maximal allowable value τ_{1m}. For $c = 0.1$, $\tau_{2m} = 1$, $\tau_{1d} = 0.5$, $\alpha_2 = 0.1$, and different values of α_3, we apply Theorem 1 to calculate the maximal allowable value τ_{1m} that guarantees the asymptotical stability of the system. Table 1 shows the comparison of our results with those in [10]. This example demonstrates that the stability criterion in Theorem 1 in this paper gives a less conservative result than that in [10].

Table 1 Bound τ_{1m} for different values of α_3

α_3	0	0.10	0.20	0.30
Method in [10] ($\alpha_1 = 0$)	0.9328	0.7402	0.5637	0.4042
Proposed Method ($\alpha_1 = 0$)	0.9488	0.7695	0.6087	0.4667
Method in [10] ($\alpha_1 = 0.1$)	0.8148	0.6439	0.4864	0.3433
Proposed Method ($\alpha_1 = 0.1$)	0.8408	0.6841	0.5420	0.4144

5 Conclusion

This paper has proposed a new approach for dealing with the problem of robust stability of the uncertain neutral systems with mixed neutral and discrete delays and nonlinear perturbations. A new delay-dependent stability criterion with reduced conservatism is obtained. Based on the newly established integral inequalities, it is much simpler to derive the delay-dependent results. An example has also been given to show the significant improvements over some existing results in the literature.

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