Joint Predictive Control of Power and Rate for Wireless Networks

KONG Shu-Lan[1] ZHANG Huan-Shui[2]
ZHANG Zhao-Sheng[1] ZHANG Cheng-Hui[2]

Abstract To mitigate the loop delay in distributed wireless networks, a predictive power and rate control scheme is proposed for the system model that also accounts for the congestion levels and input delay instead of state-delayed in a network. A measurement feedback control problem with input delay is formulated by minimizing the energy of the difference between the actual and the desired signal-to-interference-plus-noise ratio (SNR) levels, as well as the energy of the control sequence. To solve this problem, we present two Riccati equations for the control and the estimation for the time delay systems. A complete analytical optimal controller is obtained by using the separation principle and solving two Riccati equations, where one is backward equation for stochastic linear quadratic regulation and the other is the standard filtering Riccati equation. Simulation results illustrate the performance of the proposed power and the rate control scheme.

Key words Predictive power control, Kalman filter, time delay, wireless networks

1 Introduction

Recent research in cellular wireless systems has recognized power control as a flexible means of meeting different quality of service (Q∘S) constraints in an efficient manner. Power control is needed in wireless cellular communication systems for managing the co-channel interference powers. Power consumption is a key limiting factor in the performance of wireless networks because of the presence of nodes with limited power capabilities. This limitation is further compounded by the fact that the nodes need to cater to certain data rates, which in turn require the signal-to-interference-plus-noise ratio (SNR) level, and consequently the power level, to be above certain desired values. In addition, the nodes need to be responsive to congestion conditions in the network, and therefore, they should be able to adjust their transmission rates and their power levels accordingly. There have been many power control algorithms that have been investigated in the literature. Most of the initial distributed power control strategies proposed to data strive to balance SNR in a distributed way, such as [1∼4]. The Kalman filtering approach[5] uses admission control as the central Q∘S issue.

To improve the convergence properties, some stochastic power algorithms[6, 7] with very little complexity were presented. To obtain an available solution combining in an cohesive manner the requirements of power, data rate, and congestion, some distributed strategies[8, 9] were proposed for the joint control of power and data rates in a wireless network by taking into account the congestion levels as well.

Signaling and measuring take time, resulting in time delays in a power control loop, which in turn affects the dynamics of the closed-loop. The authors[10, 11] identified some problems that the loop delay might cause for various power control algorithms and proposed a few simple time compensation schemes to effectively cancel the loop delay. Some new adaptive closed-loop power control algorithms that were able to alleviate the effect of the loop delay were presented in [12]. But the global stability of the proposed algorithms could not be guaranteed. From a theoretic system perspective, the authors of [13] dealt with state-delayed models, extending the results in [8, 9] to the case when there are delayed measurements due to round trip delays. A joint rate and power control algorithm that minimizes the bound on the error variance between the desired and actual SNR is given. However, the algorithm in [13] cannot obtain a complete analytical control law.

The purpose of this article is to propose a distributed strategy for the joint control of power and data rates in a wireless network by taking into account the presence of the loop delay. A predictive power and rate control scheme is proposed for the system model with input delays instead of state-delayed in [13]. The goal of the algorithm in this article is twofold: on the one hand the aim is to mitigate the loop delay through predictive control, and on the other hand it tries to obtain a complete analytical control law minimizing the energy of the difference between the actual and the desired SNR levels, as well as the energy of the control sequence. The idea is to find a suitable linear state-space model with input delay for the wireless network dynamics considering loop delay, and then design an explicit optimal measurement-feedback controller by using the separation principle and solving two Riccati equations, where one is backward equation for control and the other is the standard filtering Riccati equation.

2 Problem formulation

2.1 System model

We consider a wireless network with nodes organized into local clusters or cells with one node acting as the master node in each cell. Any node that wishes to communicate is allowed to do so only with the master node and uses a time slot. Nodes communicating during the same time-slot in other cells cause interference in this cell. A crucial structure of the power control problem is the mapping between the power levels and the SNR.

The SNR for node i at time t on an uplink channel is defined by

$$\gamma_i(t) = \frac{G_{ii}(t)p_i(t)}{\sum\limits_{j\in\mathcal{A}} G_{ij}(t)p_j(t) + \sigma_i^2} \tag{1}$$

where G_{ij} represents the channel gain from the j-th node to the intended master node of the i-th cell, p_j is the transmitted power from the j-th node, σ_i^2 is the power of the white Gaussian noise at the receiver of the master node, to which node i is connected, and $\mathcal{A}$ is the set of all nodes interfering with node i.

Let $f_i(k+1)$ denote the flow rate at node i at time t, for any node in the network, we adopt the following flow-rate control algorithm[14]

$$f_i(t+1) = f_i(t) + \mu[d(t) - c(t)f_i(t)] \tag{2}$$

where $\mu > 0$ is a step-size parameter and $c(t)$ is a measure of the amount of congestion in the network at time t. $c(t)$ is estimated based on the SNR estimation as well and one method for estimating $c(t)$ is presented in [8]. $d(t)$ controls

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1. School of Mathematics Science, Qufu Normal University, Qufu 273165, P. R. China 2. School of Control Science and Engineering, Shandong University, Jinan 250061, P. R. China
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the amount of rate increase per iteration. Equation (2) is a typical rate control strategy incorporated in computer networks. $c(t)$ is assumed to be independent of the flow rates at different nodes, the parameter $d(t)$ is a zero-mean random variable with variance σ_d^2.

According to Shannon capacity formula, the flow rate $f_i(t)$ demands an SNR level $\gamma_i'(t)$ that is given by $f_i(t) = \frac{1}{2}\log_2[1+\gamma_i'(t)]$, namely, the SNR level should be at least at a value $\gamma_i'(t)$.

Usually, during normal network operation, $\gamma_i'(t) \gg 1$. Thus, $f_i(t)$ is proportional to $\log\gamma_i'(t)$. Using the fact and (2), the desired SNR varies in dB scale according to the rule

$$\bar{\gamma}_i'(t+1) = [1-\mu c(t)]\bar{\gamma}_i'(t) + \mu' d(t) \tag{3}$$

where $\mu' = 20\mu/\log_2(10)$ and $\bar{\gamma}_i'(t) = 10\log\gamma_i'(t)$.

We assume that each node in the network adjusts its power control algorithm

$$\bar{p}_i'(t+1) = \bar{p}_i'(t) + \alpha_i[\gamma_i'(t) - \gamma_i(t)] \tag{4}$$

where α_i is a step-size parameter that is allowed to vary from one node to another and $\gamma_i(t)$ is a measurement of the actual SNR that is achieved by $p_i(t)$. Now let $\beta_i(t) = \frac{G_{ii}(t)}{\sum_{j\in\mathcal{A}} G_{ij}(t)p_j(t)+\sigma_i^2}$ denote the scaling factor that determines how $p_i(t)$ affects the achieved $\gamma_i(t)$ in (1), *i.e.*, $\gamma_i(t) = \beta_i(t)p_i(t)$ or, equivalently, in dB scale,

$$\bar{\gamma}_i(t) = \bar{\beta}_i(t) + \bar{p}_i(t) \tag{5}$$

$\bar{\beta}_i(t)$ is referred to as the effective channel gain. We introduce the random walk model in [9] for $\bar{\beta}_i(t)$ as: $\bar{\beta}_i(t+1) = \bar{\beta}_i(t) + n_i(t)$, where $n_i(t)$ is a zero-mean disturbance of variance σ_n^2 and is independent of $\bar{p}_i(t)$. Substituting this model for $\bar{\beta}_i(t)$ into (5), we find that the achieved $\bar{\gamma}_i(t)$ varies according to the rule

$$\bar{\gamma}_i(t+1) = (1-\alpha_i)\bar{\gamma}_i(t) + \alpha_i\bar{\gamma}_i'(t) + n_i(t) \tag{6}$$

In the following, the index identifying the node is omitted for notational clarity. Second, we introduce the two-dimensional state vector: $\boldsymbol{x}_t^{\mathrm{T}} = [\bar{\boldsymbol{\gamma}}(t)^{\mathrm{T}}, \bar{\boldsymbol{\gamma}}'(t)^{\mathrm{T}}]$. Then combining (3) and (6) we arrive at the state-space model

$$\boldsymbol{x}_{t+1} = \begin{bmatrix} 1-\alpha & \alpha \\ 0 & 1-\mu c(t) \end{bmatrix} \boldsymbol{x}_t + \begin{bmatrix} n(t) \\ \mu' d(t) \end{bmatrix}$$

or, more compactly,

$$\boldsymbol{x}_{t+1} = A_t\boldsymbol{x}_t + \boldsymbol{w}_t \tag{7}$$

where the 2×2 coefficient matrix A_t is given by $A_t = \begin{bmatrix} 1-\alpha & \alpha \\ 0 & 1-\mu c(t) \end{bmatrix}$ and $\boldsymbol{w}_t = \begin{bmatrix} n(t) \\ \mu' d(t) \end{bmatrix}$, where $\boldsymbol{w}_t$ is a 2×1 zero-mean random vector with covariance matrix

$$Q_t^w = E\boldsymbol{w}_t\boldsymbol{w}_t^{\mathrm{T}} = \begin{bmatrix} \sigma^2_{n(t)} & \\ & \mu'^2\sigma^2_{d(t)} \end{bmatrix} \tag{8}$$

2.2 Cost function

To mitigate the loop delay and drive $\gamma_i(t)$ towards the desired level $\gamma_i'(t)$ we employ a predictive control sequence $u_{t-\tau}$ in (7) as follows

$$\boldsymbol{x}_{t+1} = A_t\boldsymbol{x}_t + B\boldsymbol{u}_{t-\tau} + \boldsymbol{w}_t \tag{9}$$

for some given 2×2 matrix B and 2×1 control sequence $\boldsymbol{u}_{t-\tau}$, where $\boldsymbol{u}_t = \boldsymbol{0}, t = -\tau, \cdots, 0$. And τ is a nonzero integer that incorporates round trip delay time. For example, let $\boldsymbol{u}_t = \begin{bmatrix} u_p(t) \\ u_f(t) \end{bmatrix}$ denote the individual entries of $\boldsymbol{u}_t$. In general, for arbitrary choices of B, the control signals that are added into the updates for $\{\bar{p}_i(t), f_i(t)\}$ are combinations of $\{u_p(t), u_f(t)\}$.

In addition to employing a control sequence $\boldsymbol{u}_t$, we assume for generality that we have access to output measurements that are related to the state vector as follows

$$\boldsymbol{y}_t = C\boldsymbol{x}_t + \boldsymbol{v}_t \tag{10}$$

for some known matrix C, where $\boldsymbol{v}_t$ denotes measurement noise with covariance matrix R_t^v

$$R_t^v = E\boldsymbol{v}_t\boldsymbol{v}_t^{\mathrm{T}} \tag{11}$$

We then seek a control strategy $\boldsymbol{u}_t = \mathcal{F}_t\{y_0, \cdots, y_t\}$ that satisfies the following stochastic quadratic cost function

$$\min_{\{\mathcal{F}_t\}} E\left[\boldsymbol{x}_{N+1}^{\mathrm{T}} P_{N+1}^c \boldsymbol{x}_{N+1} + \sum_{t=0}^{N} \boldsymbol{x}_t^{\mathrm{T}} L^{\mathrm{T}} L\boldsymbol{x}_t + \sum_{t=0}^{N-\tau} \boldsymbol{u}(t)^{\mathrm{T}}\boldsymbol{u}(t)\right] \tag{12}$$

with $L = [1 - 1]$, where E denotes the expectation operation. This choice of L results in $L\boldsymbol{x}_t = \bar{\boldsymbol{\gamma}}(t) - \bar{\boldsymbol{\gamma}}'(t)$ so that $\|L\boldsymbol{x}_t\|^2$ is a measure of the energy of the difference between $\{\bar{\boldsymbol{\gamma}}(t), \bar{\boldsymbol{\gamma}}'(t)\}$. In this way, the cost function defined above is such that it seeks to minimize the error between the successive actual and desired SNR levels, as well as the energy of the control sequence itself. Moreover, the solution $\{\boldsymbol{u}_t\}$ should be a function of the available measurements $\{\boldsymbol{y}_t\}$ only.

3 Measurement-feedback controller

3.1 The separation principle

Let us begin by defining

$$J_N = \boldsymbol{x}_{N+1}^{\mathrm{T}} P_{N+1}^c \boldsymbol{x}_{N+1} + \sum_{t=0}^{N} \boldsymbol{x}_t^{\mathrm{T}} L^{\mathrm{T}} L\boldsymbol{x}_t + \sum_{t=0}^{N-\tau} \boldsymbol{u}(t)^{\mathrm{T}}\boldsymbol{u}(t)$$

The expectation in (12) is taken over the uncorrelated random variables, x_0 and $\{w_0, \cdots, w_N, v_0, \cdots, v_N\}$, then the cost function can be written as $E_{\{x_0,\{w_i\}_{i=0}^N,\{v_i\}_{i=0}^N\}} J_N = E_{\{x_0,\{w_i\}_{i=0}^N,\{v_i\}_{i=0}^N\}}\left(\left[\sum_{t=0}^{N-1} \boldsymbol{x}_t^{\mathrm{T}} L^{\mathrm{T}} L\boldsymbol{x}_t + \sum_{t=0}^{N-\tau-1} \boldsymbol{u}_t^{\mathrm{T}}\boldsymbol{u}_t\right] + \left[\boldsymbol{x}_{N+1}^{\mathrm{T}} P_{N+1}^c \boldsymbol{x}_{N+1} + \boldsymbol{x}_N^{\mathrm{T}} L^{\mathrm{T}} L\boldsymbol{x}_N + \boldsymbol{u}_{N-\tau}^{\mathrm{T}}\boldsymbol{u}_{N-\tau}\right]\right)$. We focus on the second term on the right-hand side of the above equation, and perform a completion of squares. Then, we have

$$\begin{aligned} & E_{\{x_0,\{w_i\}_{i=0}^N,\{v_i\}_{i=0}^N\}}[\boldsymbol{x}_{N+1}^{\mathrm{T}} P_{N+1}^c \boldsymbol{x}_{N+1} + \boldsymbol{x}_N^{\mathrm{T}} L^{\mathrm{T}} L\boldsymbol{x}_N + \\ & \qquad \boldsymbol{u}_{N-\tau}^{\mathrm{T}}\boldsymbol{u}_{N-\tau}] = \\ & E(\boldsymbol{u}(N-\tau) - \bar{\boldsymbol{u}})^{\mathrm{T}} R_N (\boldsymbol{u}(N-\tau) - \bar{\boldsymbol{u}}) + \\ & E_{\{x_0,\{w_i\}_{i=0}^N,\{v_i\}_{i=0}^N\}}[\boldsymbol{x}_N^{\mathrm{T}} P_N^c \boldsymbol{x}_N] + \mathrm{trace}(Q_N^w \triangle_N) \end{aligned} \tag{13}$$

where

$$\begin{aligned} \bar{u} &= -K_N^{\mathrm{T}}\boldsymbol{x}_N - R_N^{-1}B^{\mathrm{T}}P_{N+1}^c\boldsymbol{w}_N \\ K_N &= A_N^{\mathrm{T}}P_{N+1}^c B R_N^{-1} \\ R_N &= B^{\mathrm{T}}P_{N+1}^c B + I \\ P_N^c &= L^{\mathrm{T}}L + A_N^{\mathrm{T}}P_{N+1}^c A_N - K_N R_N K_N^{\mathrm{T}} \\ \triangle_N &= P_{N+1}^c - P_{N+1}^c B R_N^{-1} B^{\mathrm{T}} P_{N+1}^c \end{aligned}$$

Equation (13) implies that we must choose $u_{N-\tau}$ so as to minimize $E[\boldsymbol{u}_{N-\tau}+K_N^{\mathrm{T}}\boldsymbol{x}_N+R_N^{-1}B^{\mathrm{T}}P_{N+1}^c\boldsymbol{w}_N]^{\mathrm{T}}R_N[\boldsymbol{u}_{N-\tau}+K_N^{\mathrm{T}}\boldsymbol{x}_N+R_N^{-1}B^{\mathrm{T}}P_{N+1}^c\boldsymbol{w}_N]$. Here, since $\boldsymbol{u}_{N-\tau}$ is only allowed to be a linear function of $\{\boldsymbol{y}_0,\boldsymbol{y}_1,\cdots,\boldsymbol{y}_{N-\tau}\}$, we are confronted with the following problem

$$\min_{\boldsymbol{u}_{N-\tau}\in\mathcal{L}\{\boldsymbol{y}_0,,\cdots,\boldsymbol{y}_{N-\tau}\}} E\boldsymbol{a}_N^{\mathrm{T}}R_N\boldsymbol{a}_N$$

where we have defined $\boldsymbol{a}_N = \boldsymbol{u}_{N-\tau}+K_N^{\mathrm{T}}\boldsymbol{x}_N+R_N^{-1}B^{\mathrm{T}}P_{N+1}^c\boldsymbol{w}_N$. But the above problem is simply a linear least-mean-squares estimation problem, given the observations $\{\boldsymbol{y}_0,\boldsymbol{y}_1,\cdots,\boldsymbol{y}_{N-\tau}\}$. Thus, the minimizing solution is

$$\boldsymbol{u}_{N-\tau}=-K_N^{\mathrm{T}}\hat{\boldsymbol{x}}_{N|N-\tau}-R_N^{-1}B^{\mathrm{T}}P_{N+1}^c\hat{\boldsymbol{w}}_{N|N-\tau}$$

where $\hat{\boldsymbol{x}}_{N|N-\tau}$ and $\hat{\boldsymbol{w}}_{N|N-\tau}$ are the linear least-mean-squares estimates of $\boldsymbol{x}_N$ and $\boldsymbol{w}_N$, given $\{\boldsymbol{y}_0,\boldsymbol{y}_1,\cdots,\boldsymbol{y}_{N-\tau}\}$. Moreover, the state-space model (9) implies that $\hat{\boldsymbol{w}}_{N|N-\tau}=\boldsymbol{0}$, since $\boldsymbol{w}_N$ is uncorrelated with $\{\boldsymbol{y}_0,\boldsymbol{y}_1,\cdots,\boldsymbol{y}_{N-\tau}\}$, $\boldsymbol{u}_{N-\tau}=-K_N^{\mathrm{T}}\hat{\boldsymbol{x}}_{N|N-\tau}$. Then, we have

$$\boldsymbol{u}_{N-\tau}=-K_N^c\hat{\boldsymbol{x}}_{N-\tau|N-\tau}-\sum_{j=0}^{\tau-1}\mathit{\Gamma}_{N,j}B\boldsymbol{u}_{N-\tau-j-1} \tag{14}$$

where

$$K_N^c=R_N^{-1}B^{\mathrm{T}}P_{N+1}^c\left[\prod_{j=0}^{\tau}A_{N-j}\right]$$
$$\boldsymbol{\Phi}_N=\sum_{j=0}^{\tau-1}\left(R_N^{-1}B^{\mathrm{T}}P_{N+1}^c\left[\prod_{i=0}^{j}A_{N-i}\right]\boldsymbol{w}_{N-j-1}\right)+R_N^{-1}B^{\mathrm{T}}P_{N+1}^c\boldsymbol{w}_N$$
$$\mathit{\Gamma}_{N,j}=R_N^{-1}B^{\mathrm{T}}P_{N+1}^c\left[\prod_{i=0}^{j}A_{N-i}\right]$$

With this choice of optimum control signal, the corresponding minimum mean-square error becomes

$$\begin{aligned}&E[K_N^{c\mathrm{T}}\tilde{\boldsymbol{x}}_{N-\tau|N-\tau}+\boldsymbol{\Phi}_N]^{\mathrm{T}}R_N[K_N^{c\mathrm{T}}\tilde{\boldsymbol{x}}_{N-\tau|N-\tau}+\boldsymbol{\Phi}_N]=\\&E[\tilde{\boldsymbol{x}}_{N-\tau|N-\tau}^{\mathrm{T}}K_N^cR_NK_N^{c\mathrm{T}}\tilde{\boldsymbol{x}}_{N-\tau|N-\tau}]+E[\boldsymbol{\Phi}_N^{\mathrm{T}}\boldsymbol{\Phi}_N]=\\&\mathrm{trace}(P_{N-\tau|N-\tau}K_N^cR_NK_N^{c\mathrm{T}})+\\&\mathrm{trace}(Q_N^wP_{N+1}^cBR_N^{-1}B^{\mathrm{T}}P_{N+1}^c)+\\&\sum_{j=0}^{\tau-1}\mathrm{trace}(Q_{N-j-1}^w\mathit{\Gamma}_j^{\mathrm{T}}R_N\mathit{\Gamma}_j)\end{aligned} \tag{15}$$

where we have defined $\tilde{\boldsymbol{x}}_{t|t}=\boldsymbol{x}_t-\hat{\boldsymbol{x}}_{t|t}$ and $P_{t|t}=E\tilde{\boldsymbol{x}}_{t|t}\tilde{\boldsymbol{x}}_{t|t}^{\mathrm{T}}$.

Now using (13) and (15) for the minimizing choice of $\boldsymbol{u}_{N-\tau}$ we may write

$$\begin{aligned}&\min E_{\{\boldsymbol{x}_0,\{\boldsymbol{w}_i\}_{i=0}^N,\{\boldsymbol{v}_i\}_{i=0}^N\}}J_N=\\&\min E_{\{\boldsymbol{x}_0,\{\boldsymbol{w}_i\}_{i=0}^{N-1},\{\boldsymbol{v}_i\}_{i=0}^{N-1}\}}J_{N-1}+\mathrm{trace}(Q_N^wP_{N+1}^c)+\\&\mathrm{trace}(P_{N-\tau|N-\tau}K_N^cR_NK_N^{c\mathrm{T}})+\sum_{j=0}^{\tau-1}\mathrm{trace}(Q_{N-j-1}^w\Gamma_j^{\mathrm{T}}R_N\Gamma_j)\end{aligned}$$

3.2 Solution

We are thus left with the problem of choosing $\{\boldsymbol{u}_0,\cdots,\boldsymbol{u}(N-\tau-1)\}$ so as to minimize the expected value of J_{N-1}. But since our choice of endpoint, $N-\tau$, was arbitrary, the same arguments can be used for any intermediate time i. We thus have

$$\boldsymbol{u}_{t-\tau}=-K_t^c\hat{\boldsymbol{x}}_{t-\tau|t-\tau}-\sum_{j=0}^{\tau-1}\mathit{\Gamma}_{t,j}B\boldsymbol{u}_{t-\tau-j-1},$$
$$t=\tau,\tau+1,\cdots,N$$

or

$$\boldsymbol{u}_t=-K_{t+\tau}^c\hat{\boldsymbol{x}}_{t|t}-\sum_{j=0}^{\tau-1}\mathit{\Gamma}_{t+\tau,j}B\boldsymbol{u}_{t-j-1},\ t=0,\cdots,N-\tau$$

where

$$K_t^c=R_t^{-1}B^{\mathrm{T}}P_{t+1}^c\prod_{j=0}^{\tau}A_{t-j},\mathit{\Gamma}_{t,j}=R_t^{-1}B^{\mathrm{T}}P_{t+1}^c\prod_{i=0}^{j}A_{t-i} \tag{16}$$

$$P_t^c=L^{\mathrm{T}}L+A_t^{\mathrm{T}}P_{t+1}^cA_t-K_tR_tK_t^{\mathrm{T}},\ P_{N+1}^c$$
$$K_t=A_t^{\mathrm{T}}P_{t+1}^cBR_t^{-1},\quad R_t=B^{\mathrm{T}}P_{t+1}^cB+I$$

All that remains is to find the estimates, $\hat{x}_{t|t}$. We start with $\hat{\boldsymbol{x}}_{0|-1}=\boldsymbol{0},P_0=E\boldsymbol{x}_0\boldsymbol{x}_0^{\mathrm{T}}=\pi_0$. But these are readily given by the Kalman filter recursions corresponding to the state-space model (9) and (10), *i.e.*,

$$\begin{cases}\hat{\boldsymbol{x}}_{t+1|t}=A_t\hat{\boldsymbol{x}}_{t|t-1}+B\boldsymbol{u}_{t-\tau}+K_{p,t}(\boldsymbol{y}_t-C\hat{\boldsymbol{x}}_{t|t-1}),\\ \qquad\hat{\boldsymbol{x}}_{0|-1}=0\\ \hat{\boldsymbol{x}}_{t|t}=\hat{\boldsymbol{x}}_{t|t-1}+P_tC^{\mathrm{T}}R_{e,t}^{-1}(\boldsymbol{y}_t-C\hat{\boldsymbol{x}}_{t|t-1}),\\ \qquad t=0,1,\cdots,N\end{cases} \tag{17}$$

where $K_{p,t}=A_tP_tC^{\mathrm{T}}R_{e,t}^{-1},R_{e,t}=CP_tC^{\mathrm{T}}+R$, where P_t satisfies the Riccati recursion $P_{t+1}=A_tP_tA_t^{\mathrm{T}}+Q-K_{p,t}R_{e,t}K_{p,t}^{\mathrm{T}},P_0=E\boldsymbol{x}_0\boldsymbol{x}_0^{\mathrm{T}}=\pi_0$.

We can summarize the results obtained thus far in the following theorem.

Theorem. Consider the network operating under condition that

$$\begin{cases}\boldsymbol{x}_{t+1}=A_t\boldsymbol{x}_t+B\boldsymbol{u}_{t-\tau}+\boldsymbol{w}_t\\ \boldsymbol{y}_t=C\boldsymbol{x}_t+\boldsymbol{v}_t\qquad t=0,\cdots,N\end{cases}$$

where $\boldsymbol{x}_0$ and the disturbances $\{\boldsymbol{w}_i\}_{i=0}^N$ and $\{\boldsymbol{v}_i\}_{i=0}^N$ are zero-mean random variables with variances given by π_0, (8) and (11), $\boldsymbol{u}_t=\boldsymbol{0},t\in\{-\tau,\cdots,0\}$. Then, the optimal control strategy $\boldsymbol{u}_t=\mathcal{F}_t(\boldsymbol{y}_0,\cdots,\boldsymbol{y}_t)$ that satisfies (12) is given by the law

$$\boldsymbol{u}_t=-K_{t+\tau}^c\hat{\boldsymbol{x}}_{t|t}-\sum_{j=0}^{\tau-1}\Gamma_{t+\tau,j}B\boldsymbol{u}_{t-j-1},\quad t=0,\cdots,N-\tau$$

where K_t^c and $\mathit{\Gamma}_{t,j}$ satisfy (16), $\hat{\boldsymbol{x}}_{t|t}$ are given by the Kalman filter recursions (17).

Remark. The optimal control employs two Riccati recursions: one is P_t and runs forward in time, whereas the other is for P_t^c and runs backward in time. P_t is used to compute the gain matrix $K_{p,t}$, which in turn is used to estimate the state vector from the observations $\boldsymbol{y}_t$. P_t^c is used to compute the gain matrix K_t^c, which is used to determine the optimal control $\boldsymbol{u}_t$. All matrix variables involved are 2×2, and hence, the computational complexity of evaluating the solution is not significant.

4 Simulation results

To illustrate the performance of the proposed algorithm, we simulate a network using the model proposed in [8] for the channel gain from the i-th node to its master node. We take parameters according to the network[8]. The value $c(t)$ is chosen as a random variable between 0 and 0.5. $d(t)$ is 0.05-mean with variance 0.01. Moreover, take $\mu' = 0.8$, $\boldsymbol{R}_t^v = [0.01, 0.01]^{\mathrm{T}}$, and $\alpha = 0.2$. We simulate the proposed algorithm under $B = C = I$, $\hat{\boldsymbol{x}}_{0|-1} = [21.5, 20.5]^{\mathrm{T}}$, and $\boldsymbol{x}_0 = [22, 20]^{\mathrm{T}}$, and we obtain the following results.

The plot in Fig. 1 illustrates the performance of the controller. The plot shows the change of the controller becomes stable with the increase of the number of steps even if there exists time delay in the network. The curves of power control signal $u_p(t)$ and rate control signal $u_f(t)$ are listed with change of steps as follows.

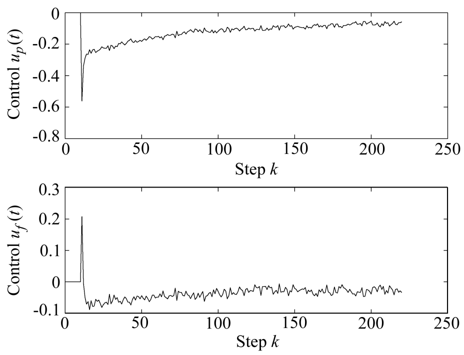

Fig. 1 Evolution of power and rate control curves

The plot in Fig. 2 shows the difference between the actual SNR and the desired SNR. The proposed algorithm reduces the SNR error.

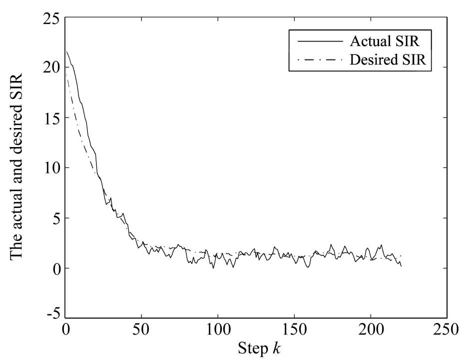

Fig. 2 Comparison of the actual SNR and the desired SNR

5 Conclusions

We have studied the control problem for the systems with the loop delay in wireless networks. A measurement-feedback power and rate control approach with input delay is presented. An explicit power and rate controller is found according to the past and current observations of the actual and the desired SNR levels by using the separation principle and solving two Riccati equations, where one is backward equation for stochastic linear quadratic regulation and the other is the standard filtering Riccati equation. The optimal feedback control law presented in this article is analytical, instead of the linear quadratic regulation (LQR) solution obtained by linear matrix inequality (LMI) methods with state-delayed models in [13]. Simulation results show the better performance of the proposed algorithm.

References

1 Aein J M. Power balancing in systems employing frequency reuse. *COMSAT Technical Review*, 1973, **3**(2): 277~299

2 Zander J. Distributed cochannel interference control in cellular radio system. *IEEE Transactions on Vehicular Technology*, 1992, **41**(3): 305~311

3 Sung C W, Wong W S. A distributed fixed-step power control algorithm with quantization and active link quality protection. *IEEE Transactions on Vehicular Technology*, 1999, **48**(2): 553~562

4 Zhang H S, Chen C S, Wong W S. Distributed power control for time varying systems: performance and convergence analysis. *IEEE Transactions on Vehicular Technology*, 2005, **54**(5): 1896~1904

5 Shoarinejad K, Speyer J L, Pottie G J. A distributed schem for integrated predictive channel and power allocation in cellular radio networks. In: Proceedings of IEEE Global Telecommunications Conferences. IEEE, 2001. **6**: 3623~3627

6 Zhang H S, Ge W Y, Wong, W S, Caines P E. A stochastic approximation approach to the robust power control problem. In: Proceedings of IEEE Conference on Decision and Control. IEEE, 2003. **3**: 2601~2605

7 Ulukus S S, Yates R D. Stochastic power control for cellular radio systems. *IEEE Transactions on Communications*, 1998, **46**(6): 784~798

8 Subramanian A, Sayed A H. Joint rate and power control algorithms for wireless networks. *IEEE Transactions on Signal Processing*, 2005, **53**(11): 4204~4214

9 Subramanian A, Khajehnouri N, Sayed A H. A minimum variance power control algorithm for wireless networks. In: Proceedings of IEEE Vehicular Technology Conference. IEEE, 2003. 1468~1472

10 Gunnarsson F, Gustafsson F, Blom J. Dynamical effects of time delays and time delay compensation in power controlled DS-CDMA. *IEEE Journal on Selected Areas in Communications*, 2001, **19**(1): 141~151

11 Gunnarsson F, Gustafsson F. Time delay compensation in power controlled cellular radio systems. *IEEE Comunications Letters*, 2001, **5**(7): 295~297

12 Rintamaki M, Koivo H, Hartimo I. Adaptive closed-loop power control algorithms for CDMA cellular communication systems. *IEEE Transactions on Vehicular Technology*, 2004, **53**(6): 1756~1768

13 Subramanian A, Sayed A H. A robust power and rate control method for state-delayed wireless networks. *Automatica*, 2005, **41**(11): 1917~1924

14 Kelly F P. Models for a self managed internet. *Philosophical Transactions of the Royal Society A*, 2000, **358**(1773): 2335~2348

KONG Shu-Lan Associate professor at School of Mathematics Science, Qufu Normal University. Her research interest covers optimal estimation and control and wireless communication. Corresponding author of this paper.
E-mail: shulkong@gmail.com

ZHANG Huan-Shui Professor at School of Control Science and Engineering, Shandong University. His research interest covers optimal estimation, robust filtering and control, time delay systems, and wireless communication.
E-mail: huanshuizhang@hotmail.com

ZHANG Zhao-Sheng Associate professor at School of Mathematics Science, Qufu Normal University. His research interest is combinatorial optimization.
E-mail: zzsksl@sina.com

ZHANG Cheng-Hui Professor at School of Control Science and Engineering, Shandong University. His research interest covers optimal control and adaptive control.
E-mail: zchui@sdu.edu.cn