Fault Estimation and Accommodation for Networked Control Systems with Transfer Delay

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Abstract In this paper, a method of fault estimation and fault tolerant control for networked control system (NCS) with transfer delay and process noise is presented. First, the networked control system is modeled as a multiple-input-multiple-output (MIMO) discrete-time system with transfer delays, process noise, and model uncertainties. Under this model and under some conditions, a fault estimation method is proposed to estimate the system faults. On the basis of the information on fault estimation and the sliding mode control theory, a fault tolerant controller is designed to recover the system performance. Finally, simulation results are used to verify the efficiency of the method.

Key words Fault estimation, fault tolerant control, networked control systems, transfer delay, model uncertainties, sliding mode control

1 Introduction

Networked control systems (NCSs) are control systems in which controller and plant are connected *via* a communication channel[1]. There are many advantages using NCSs, such as low cost, simple installation procedure and maintenance, increased system agility, and reduced system wiring. Due to these advantages, real-time control networks (such as DeviceNet, Profit-bus, FireWire, and Ethernet) are widely applied in real-time distributed control cases, such as electronics, communications, transportation, aircraft, and automatic manufacturing. However, the network itself is a dynamical system that includes the following problems: data dropout, limited bandwidth, time delay due to data transmission, and information loss due to encoding and quantization[2]. Thus, compared with conventional point-to-point control systems, these problems make the analysis and design of NCSs more complex.

Many results on analysis and controller design of NCSs with network-induced delays have been obtained during the last ten years (*e.g.*[3]$\sim$[6]). But, in these research works, system faults have not been considered. When faults occur in NCSs, the initial control laws always cannot guarantee the stability of the systems. So the study on fault detection (FD) and fault-tolerant control (FTC) of NCSs becomes necessary. As for faults detection of the NCSs, the observer-based methods are widely used (*e.g.* [7] $\sim$ [9]). Meanwhile, some work has been done to deal with the fault tolerant-control in the NCSs (see [10] $\sim$ [12]). Compared with fault detection and isolation (FDI), fault estimation and accommodation are not easy tasks. Until now, some results have been obtained on such issue with applications to the aircraft, power systems, robotics, and process control (*e.g.* [13] $\sim$ [17]).

In this paper, we deal with the fault estimation and accommodation for a kind of networked control systems with transfer delays modeled by discrete-time systems. The rest of this paper is organized as follows. Section 2 describes the model of the networked control system. Section 3 proposes the fault estimation scheme. In Section 4, the fault tolerant controller is designed to make the faulty system state track the desired trajectory. Section 5 presents simulation results of a numerical example, followed by some concluding remarks in Section 6.

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2 Preliminaries

Consider an NCS as shown in Fig. 1. The continuous-time, state-space model of the linear time-invariant plant dynamics can be described by the following standard form

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + B\boldsymbol{u}(t) + E\boldsymbol{f}(t) + \boldsymbol{w}(t) \tag{1}$$

$$\boldsymbol{y}(t) = C\boldsymbol{x}(t) = [0 \;\; I]\boldsymbol{x}(t) \tag{2}$$

where $\boldsymbol{x} \in \mathbf{R}^n$ denotes the state vector, $\boldsymbol{u} \in \mathbf{R}^m$ is the control input vector, $\boldsymbol{y} \in \mathbf{R}^r$ is the measurable output vector, $\boldsymbol{f}(t) \in \mathbf{R}^q$ is the vector function to model the process fault or actuator fault, the process noise $\boldsymbol{w}(t)$ is a bounded zero mean random sequence with known covariance matrix and known bounds. A, B, C are known parameter matrices with proper dimensions.

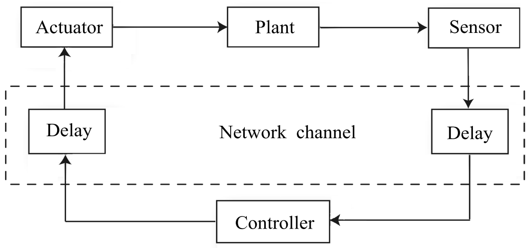


Fig. 1 The block of the networked control system

Remark 1. In the above system, the output matrix C is described as $[0 \;\; I]$. If C is of full row rank, $C = [0 \;\; C_1]$, and C_1 is an $r \times r$ nonsingular matrix, then there exists a similarity transformation $\boldsymbol{x} = \begin{bmatrix} I_{n-r} & 0 \\ 0 & C_1 \end{bmatrix}^{-1} \bar{\boldsymbol{x}}$ that can transform the output equation into the desired form. For this NCS, we give the following assumptions, as in [5].

Assumption 1. The sampling period of the NCS is T, sensor is time-driven, controller is event-driven, and actuator is time-division-driven. We use τ_{sc} and τ_{ca} to represent the sensor-controller and controller-actuator delay, respectively, and they are not larger than the sampling period, *i.e.* $\tau_{sc} < T$, $\tau_{ca} < T$, and $\tau = \tau_{sc} + \tau_{ca} < 2T$.

Assumption 2. In order to prevent the loss of information of the sensor and actuator, buffers that are longer than the biggest delay should be separately set at the sending points of the sensor and actuator.

Considering the effect of delay τ and sampling period T, the above plant model is transformed into a stochastic NCS model

$$\boldsymbol{x}(k+1) = A_c\boldsymbol{x}(k) + \sum_{j=0}^{2} B_j\boldsymbol{u}(k-j) + E_c\boldsymbol{f}(k) + \boldsymbol{w}(k) \tag{3}$$

$$\boldsymbol{y}(k) = C\boldsymbol{x}(k) = [0 \quad I]\boldsymbol{x}(k) \tag{4}$$

where $\boldsymbol{x}(k) = \boldsymbol{x}(kT)$, $\boldsymbol{y}(k) = \boldsymbol{y}(kT)$, $\boldsymbol{f}(k) = \boldsymbol{f}(kT)$, $A_c = e^{AT}$.

If $(T - \tau_{sc}) > \tau_{ca}$ then

$$B_0 = \int_{\tau_{sc}+\tau_{ca}}^{T} e^{A(T-t)}\mathrm{d}Bt$$

$$B_1 = \int_{0}^{\tau_{sc}+\tau_{ca}} e^{A(T-t)}\mathrm{d}Bt, \quad B_2 = 0$$

else

$$B_0 = 0, \quad B_1 = \int_{\tau_{sc}+\tau_{ca}-T}^{T} e^{A(T-t)}\mathrm{d}Bt$$

$$B_2 = \int_{0}^{\tau_{sc}+\tau_{ca}-T} e^{A(T-t)}\mathrm{d}Bt$$

$$E_c = \int_0^T e^{A(T-t)}E\mathrm{d}t, \quad \boldsymbol{w}(k) = \int_0^T e^{A(T-t)}\boldsymbol{w}(kT+t)\mathrm{d}t$$

Due to the known covariance matrix and bound of the $\boldsymbol{w}(t)$, as in [18], the bound and covariance matrix of $\boldsymbol{w}(k)$ can be expressed as η_c and $Q_c = Q_c^{\mathrm{T}} > 0$.

In practice, the delays of the NCSs are time-variant. So we use time-stamped[19] and delay window (DW)[18] technique to obtain the delays τ_{sc} and τ_{ca}, respectively. Meanwhile, according to [20], the current delay values of NCSs can be estimated online using Markov process.

Considering the the errors between the real delay values and the estimated ones, the system (3) and (4) can be rewritten as

$$\boldsymbol{x}(k+1) = A_c\boldsymbol{x}(k) + \sum_{j=0}^{2} B_j\boldsymbol{u}(k-j) + E_c\boldsymbol{f}(k) + B_c\boldsymbol{\zeta}(k) + \boldsymbol{w}(k) \tag{5}$$

$$\boldsymbol{y}(k) = C\boldsymbol{x}(k) = [0 \quad I]\boldsymbol{x}(k) \tag{6}$$

where $B_c = \int_0^T e^{A(T-t)}\mathrm{d}Bt$, $\boldsymbol{\zeta}(k)$ is an unknown vector denoting the modeling uncertainties, $B_c\boldsymbol{\zeta}(k)$ is the estimation error.

Remark 2. The transfer delays can be estimated online, and we use them to model the NCSs, thus, the terms B_j $(j = 0, \cdots, 2)$ are known in the above system. From the equations of the delays τ_{sc} and τ_{ca}, we can get $B_c = \sum_{j=0}^{2} B_j$. Thus the estimated errors are in the linear form of the B_c, which can be described as $B_c\boldsymbol{\zeta}(k)$.

3 Fault estimation design

Consider systems (5) and (6). To estimate the fault $\boldsymbol{f}(k)$, the equations are rewritten as

$$\begin{bmatrix} \boldsymbol{x}_1(k+1) \\ \boldsymbol{x}_2(k+1) \\ \boldsymbol{x}_3(k+1) \end{bmatrix} = \begin{bmatrix} A_{c1} \\ A_{c2} \\ A_{c3} \end{bmatrix} \boldsymbol{x}(k) + \sum_{j=0}^{2} \begin{bmatrix} B_{j1} \\ B_{j2} \\ B_{j3} \end{bmatrix} \boldsymbol{u}(k-j) + \begin{bmatrix} E_{c1} \\ E_{c2} \\ E_{c3} \end{bmatrix} \boldsymbol{f}(k) + \begin{bmatrix} B_{c1} \\ B_{c2} \\ B_{c3} \end{bmatrix} \boldsymbol{\zeta}(k) + \begin{bmatrix} \boldsymbol{w}_1(k) \\ \boldsymbol{w}_2(k) \\ \boldsymbol{w}_3(k) \end{bmatrix} \tag{7}$$

$$\boldsymbol{y}(k) = \begin{bmatrix} \boldsymbol{y}_1(k) \\ \boldsymbol{y}_2(k) \end{bmatrix} = \begin{bmatrix} O & I_{r-q} & O \\ O & O & I_q \end{bmatrix} \boldsymbol{x}(k) \tag{8}$$

where $\boldsymbol{x}_1(k) \in \mathbf{R}^{n-r}$, $\boldsymbol{x}_2(k) \in \mathbf{R}^{r-q}$ and $\boldsymbol{x}_3(k) \in \mathbf{R}^{q}$. Therefore, only $\boldsymbol{x}_1(k)$ needs to be estimated in the mean sense.

Assumption 3. Rank$(CE) = q$ and E_{c3} is nonsingular. Define

$$\bar{A}_{c1} \triangleq A_{c1} - E_{c1}E_{c3}^{-1}A_{c3}, \quad \bar{A}_{c2} \triangleq A_{c2} - E_{c2}E_{c3}^{-1}A_{c3} \tag{9}$$

$$\bar{B}_{j1} \triangleq B_{j1} - E_{c1}E_{c3}^{-1}B_{j3}, \quad \bar{B}_{j2} \triangleq B_{j2} - E_{c2}E_{c3}^{-1}B_{j3} \tag{10}$$

$$\bar{B}_{c1} \triangleq B_{c1} - E_{c1}E_{c3}^{-1}B_{c3}, \quad \bar{B}_{c2} \triangleq B_{c2} - E_{c2}E_{c3}^{-1}B_{c3} \tag{11}$$

$$\bar{A}_{c1} \triangleq [\bar{A}_{c11} \quad \bar{A}_{c12} \quad \bar{A}_{c13}], \quad \bar{A}_{c2} \triangleq [\bar{A}_{c21} \quad \bar{A}_{c22} \quad \bar{A}_{c23}] \tag{12}$$

with $\bar{A}_{c11} \in \mathbf{R}^{(n-r)\times(n-r)}$, $\bar{A}_{c12} \in \mathbf{R}^{(n-r)\times(r-q)}$, $\bar{A}_{c13} \in \mathbf{R}^{(n-r)\times q}$, $\bar{A}_{c21} \in \mathbf{R}^{(r-q)\times(n-r)}$, $\bar{A}_{c22} \in \mathbf{R}^{(r-q)\times(r-q)}$, $\bar{A}_{c23} \in \mathbf{R}^{(r-q)\times q}$, $j = 0, 1, 2$.

The following theorem presents a method to estimate the state x_1 of the system described by (7) and (8).

Theorem 1. Suppose that $(\bar{A}_{c11}, \bar{A}_{c21})$ is an observable pair and that Assumption 1 holds. Furthermore, suppose that $E[(\boldsymbol{x}_1(0) - \hat{\boldsymbol{x}}_1(0))(\boldsymbol{x}_1(0) - \hat{\boldsymbol{x}}_1(0))^{\mathrm{T}}] \triangleq P(0)$ is given. Then the state $\boldsymbol{x}_1$ of the discrete time system in (7) and (8) can be estimated with unbiased minimum variance using the following observer

$$\hat{\boldsymbol{\xi}}(k+1) = \begin{bmatrix} \bar{A}_{c11} & \bar{B}_{c1} \\ O & I \end{bmatrix} \hat{\boldsymbol{\xi}}(k) + \begin{bmatrix} \boldsymbol{\rho}(k+1) \\ O \end{bmatrix} + K(k)[\boldsymbol{\lambda}(k+1) - [\bar{A}_{c21} \quad \bar{B}_{c2}]\hat{\boldsymbol{\xi}}(k)] \tag{13}$$

where $\hat{\boldsymbol{\xi}}(k+1)$, $\boldsymbol{\rho}(k+1)$ and $\boldsymbol{\lambda}(k+1)$ are defined as follows

$$\hat{\boldsymbol{\xi}}(k+1) \triangleq \begin{bmatrix} \hat{\boldsymbol{x}}_1(k+1) \\ \hat{\boldsymbol{\zeta}}(k) \end{bmatrix} \tag{14}$$

$$\boldsymbol{\rho}(k+1) \triangleq \bar{A}_{c12}\boldsymbol{y}_1(k) + \bar{A}_{c13}\boldsymbol{y}_2(k) + E_{c1}E_{c3}^{-1}\boldsymbol{y}_2(k+1) + \sum_{j=0}^{2} \bar{B}_{j1}\boldsymbol{u}(k-j) \tag{15}$$

$$\boldsymbol{\lambda}(k+1) \triangleq \boldsymbol{y}_1(k) - E_{c2}E_{c3}^{-1}\boldsymbol{y}_2(k+1) - \bar{A}_{c22}\boldsymbol{y}_1(k) - \bar{A}_{c23}\boldsymbol{y}_1(k) - \sum_{j=0}^{2} \bar{B}_{j2}\boldsymbol{u}(k-j) \tag{16}$$

and $K(k)$ is a Kalman filter gain defined as

$$K(k) = \begin{bmatrix} \bar{A}_{c11} & \bar{B}_{c1} \\ O & I \end{bmatrix} P(k)[\bar{A}_{c21} \quad \bar{B}_{c2}]^{\mathrm{T}} \times \{[\bar{A}_{c21} \quad \bar{B}_{c2}]P(k)[\bar{A}_{c21} \quad \bar{B}_{c2}]^{\mathrm{T}} + \bar{S}\}^{-1} \tag{17}$$

where the error covariance matrix $P(k) \triangleq \mathrm{E}[(\boldsymbol{x}_1(k) - \hat{\boldsymbol{x}}_1(k))(\boldsymbol{x}_1(k) - \hat{\boldsymbol{x}}_1(k))^{\mathrm{T}}]$ is updated by

$$P(k+1) = \begin{bmatrix} \bar{A}_{c11} & \bar{B}_{c1} \\ O & I \end{bmatrix} P(k) \begin{bmatrix} \bar{A}_{c11} & \bar{B}_{c1} \\ O & I \end{bmatrix}^{\mathrm{T}} + \begin{bmatrix} \bar{Q} & O \\ O & O \end{bmatrix} - K(k)\{[\bar{A}_{c21} \quad \bar{B}_{c2}]P(k)[\bar{A}_{c21} \quad \bar{B}_{c2}]^{\mathrm{T}} + \bar{S}\}K^{\mathrm{T}}(k) \tag{18}$$

with

$$\bar{S} \triangleq T_1Q_cT_1^{\mathrm{T}}, \qquad \bar{Q} \triangleq T_2Q_cT_2^{\mathrm{T}} \tag{19}$$

$$T_1 \triangleq [I \quad O \quad -E_{c1}E_{c3}^{-1}], \qquad T_2 \triangleq [O \quad I \quad -E_{c2}E_{c3}^{-1}] \tag{20}$$

Proof. By pre-multiplying (7) by

$$\begin{bmatrix} I & O & -E_{c1}E_{c3}^{-1} \\ O & I & -E_{c1}E_{c3}^{-1} \\ O & O & I \end{bmatrix} \tag{21}$$

one can obtain

$$\begin{bmatrix} \boldsymbol{x}_1(k+1) - E_{c1}E_{c3}^{-1}\boldsymbol{y}_2(k-1) \\ \boldsymbol{y}_1(k+1) - E_{c2}E_{c3}^{-1}\boldsymbol{y}_2(k+1) \\ \boldsymbol{y}_2(k+1) \end{bmatrix} =$$

$$\begin{bmatrix} A_{c1} - E_{c1}E_{c3}^{-1}A_{c3} \\ A_{c2} - E_{c2}E_{c3}^{-1}A_{c3} \\ A_{c3} \end{bmatrix}\boldsymbol{x}(k)+$$

$$\sum_{j=0}^{p}\begin{bmatrix} B_{j1} - E_{c1}E_{c3}^{-1}B_{j3} \\ B_{j2} - E_{c2}E_{c3}^{-1}B_{j3} \\ B_{j3} \end{bmatrix}\boldsymbol{u}(k-j)+$$

$$\begin{bmatrix} O \\ O \\ E_{c3} \end{bmatrix}\boldsymbol{f}(k) + \begin{bmatrix} B_{c1} - E_{c1}E_{c3}^{-1}B_{c3} \\ B_{c2} - E_{c2}E_{c3}^{-1}B_{c3} \\ B_{c3} \end{bmatrix}\boldsymbol{\zeta}(k)+$$

$$\begin{bmatrix} \boldsymbol{w}_1(k) - E_{c1}E_{c3}^{-1}\boldsymbol{w}_3(k) \\ \boldsymbol{w}_2(k) - E_{c2}E_{c3}^{-1}\boldsymbol{w}_3(k) \\ \boldsymbol{w}_3(k) \end{bmatrix} \tag{22}$$

Now note that only part of the state of the system, *i.e.* $\boldsymbol{x}_1(k)$, needs to be estimated by the estimator. Using the definitions in (9)$\sim$(12), the first and second block row of (22) can be written as

$$\boldsymbol{x}_1(k+1) = \bar{A}_{c11}\boldsymbol{x}_1(k) + \boldsymbol{\rho}(k+1) + \bar{B}_{c1}\boldsymbol{\zeta}(k) + \bar{\boldsymbol{w}}(k) \tag{23}$$

$$\boldsymbol{\lambda}(k+1) = \bar{A}_{c21}\boldsymbol{x}_1(k) + \bar{B}_{c2}\boldsymbol{\zeta}(k) + \bar{\boldsymbol{v}}(k) \tag{24}$$

where

$$\bar{\boldsymbol{w}}(k) \triangleq \boldsymbol{w}_1(k) - E_{c1}E_{c3}^{-1}\boldsymbol{w}_3(k) \tag{25}$$

$$\bar{\boldsymbol{v}}(k) \triangleq \boldsymbol{w}_2(k) - E_{c2}E_{c3}^{-1}\boldsymbol{w}_3(k) \tag{26}$$

and $\boldsymbol{\rho}(k+1)$, $\boldsymbol{\lambda}(k+1)$ are defined in (15) and (16).

Note that in the dynamical system described by (23) and (24), $\boldsymbol{\lambda}(k+1)$ is completely known. The state of the above system and the unknown vector $\boldsymbol{\zeta}$ can be estimated by using the extended Kalman filter (EKF) approach. First, define a new state vector as in (14)

$$\boldsymbol{\xi}(k) \triangleq \begin{bmatrix} \boldsymbol{x}_1(k) \\ \boldsymbol{\zeta}(k) \end{bmatrix} \tag{27}$$

The augmented system is described as follows

$$\boldsymbol{\xi}(k+1) = \begin{bmatrix} \bar{A}_{c11}\boldsymbol{x}_1(k) + \boldsymbol{\rho}(k+1) + \bar{B}_{c1}\boldsymbol{\zeta}(k) \\ \boldsymbol{\zeta}(k+1) \end{bmatrix} + \begin{bmatrix} \bar{\boldsymbol{w}}(k) \\ O \end{bmatrix} \tag{28}$$

$$\boldsymbol{\lambda}(k+1) = \bar{A}_{c21}\boldsymbol{x}_1(k) + \bar{B}_{c2}\boldsymbol{\zeta}(k) + \bar{\boldsymbol{v}}(k) \tag{29}$$

To prove the stability of the resulting estimator using the gain in (17) and (18), we denote the estimation error $\boldsymbol{e}(k) \triangleq \boldsymbol{\xi} - \hat{\boldsymbol{\xi}}$, and then model the estimation error dynamics as follows

$$\boldsymbol{e}(k) = \left(\begin{bmatrix} \bar{A}_{c11}\bar{B}_{c1} & \\ O & I \end{bmatrix} - K[\bar{A}_{c21} \quad \bar{B}_{c2}]\right)\boldsymbol{e}(k-1)+ \bar{\boldsymbol{w}}(k-1) - K\bar{\boldsymbol{v}}(k-1) \tag{30}$$

From (25), (26), and (30), as in [21], one can conclude that the estimation error $\boldsymbol{e}_1(k)$ will converge to zero in the mean sense if all the eigenvalues of the matrix $\bar{A}_0 \triangleq \left(\begin{bmatrix} \bar{A}_{c11} & \bar{B}_{c1} \\ O & I \end{bmatrix} - K[\bar{A}_{c21} \quad \bar{B}_{c2}]\right)$ are within the unit circle. □

According to Theorem 1 and (4), the estimation of the state for the linear stochastic system as defined in (3) and (4) is given by

$$\hat{\boldsymbol{x}}(k) = \begin{bmatrix} \hat{\boldsymbol{x}}_1(k) \\ \boldsymbol{y}(k) \end{bmatrix} \tag{31}$$

where $\hat{\boldsymbol{x}}_1(k)$ is given by (13).

From Assumption 1 and (22) or (3), the system fault can be estimated as follows

$$\hat{\boldsymbol{f}}(k-1) = E_{c3}^{-1}[\boldsymbol{y}_2(k) - A_{c3}\hat{\boldsymbol{x}}(k-1) - \sum_{j=0}^{2}B_{j3}\boldsymbol{u}(k-j) - B_{c3}\hat{\boldsymbol{\zeta}}(k-1)] \tag{32}$$

where $\hat{\boldsymbol{x}}(k-1)$ is given by (31).

Remark 3. From (32), it can be seen that the faulty signal at time instant k can be estimated only after the measurements from time instant $(k+1)$ become available. It means that there is a one-step delay in the fault estimation, whose effect on the dynamic response can be neglected for practical application[15]. However, we can avoid such a problem by setting a new vector containing the $\boldsymbol{y}_2(k)$, as in [22].

Remark 4. A good feature of the method proposed in this paper is that it provides information on the shape of the fault, which is not investigated in the existing work on FDD for NCSs.

4 Fault-tolerant controller design

In this section, we consider fault-tolerant controller design for faulty systems to recover the system performance. On the basis of the sliding mode control theorem, we design the control law to achieve the desired performance.

A tracked known time-varying reference $\boldsymbol{r}(k)$ is generated by a specific control purpose under the faulty system. First, we design the controller for the nominal system (*i.e.* $\boldsymbol{f}(k) = \boldsymbol{0}$, $\boldsymbol{w}((k) = \boldsymbol{0})$. Define the sliding surface by the following equations

$$\boldsymbol{S}_c(k) = M\tilde{\boldsymbol{x}}(k), \quad \tilde{\boldsymbol{x}}(k) = \boldsymbol{r}(k) - \boldsymbol{x}(k) \tag{33}$$

where M is the $m \times n$ matrix such that MB_0 is nonsingular. Since the state vectors $\boldsymbol{x}(k)$ and $\boldsymbol{\zeta}(k)$ are unavailable, the estimation values $\hat{\boldsymbol{x}}(k)$ and $\hat{\boldsymbol{\zeta}}(k)$ are substituted for $\boldsymbol{x}(k)$ and $\boldsymbol{\zeta}(k)$. Then, the error equation is rewritten as $\tilde{\boldsymbol{x}}(k) \triangleq \boldsymbol{r}(k) - \hat{\boldsymbol{x}}(k)$.

The discrete-time equivalent control for the nominal system can be obtained by solving the equation $\boldsymbol{S}_c(k+1) = \boldsymbol{0}$

$$\boldsymbol{u}_{eq}(k) = (MB_0)^{-1}MA_c\tilde{\boldsymbol{x}}(k) - (MB_0)^{-1}M\sum_{j=1}^{2}B_j\boldsymbol{u}(k-j) - (MB_0)^{-1}MB_c\hat{\boldsymbol{\zeta}}(k) + (MB_0)^{-1}M[\boldsymbol{r}(k+1) - A_c\boldsymbol{r}(k)] \tag{34}$$

The transient response can be shaped by introducing an additional parameter to the desired sliding function dynamics such that $\boldsymbol{S}_c(k+1) = \beta\boldsymbol{S}_c(k)$. By solving this equation, the control input is obtained as follows

$$\boldsymbol{u}_\beta(k) = \boldsymbol{u}_{eq}(k) - (MB_0)^{-1}M\beta\boldsymbol{S}_c(k), \quad 0 \le \beta < 1 \tag{35}$$

To compensate for the process noise $\boldsymbol{w}(k)$, an additive term $\boldsymbol{v}(k)$ is added to the control. So we obtain the following equation

$$\boldsymbol{u}_2(k) = \boldsymbol{u}_\beta(k) + \boldsymbol{v}(k) = \boldsymbol{u}_{eq}(k) - (MB_0)^{-1}M\beta\boldsymbol{S}_c(k) + \boldsymbol{v}(k) \tag{36}$$

where $\boldsymbol{v}(k)$ can be chosen as[18]

$$\boldsymbol{v}(k) = (MB_0^{-1})M\boldsymbol{\eta}_c\mathrm{sgn}(\tilde{\boldsymbol{x}}(k)) \tag{37}$$

Now, the sliding surface is designed. A similarity transformation $T_s \in \mathbf{R}^{n\times n}$ such that

$$T_sB_0 = \left[\begin{array}{c} G \\ O \end{array}\right], \quad G \in \mathbf{R}^{m\times m} \tag{38}$$

is introduced. The transformed state variables are defined by $\boldsymbol{\gamma}(k) \triangleq T_s\tilde{\boldsymbol{x}}(k)$, and then the error dynamics is described by

$$\boldsymbol{\gamma}(k+1) = T_s[\boldsymbol{r}(k+1) - A_c\boldsymbol{r}(k)] + F\boldsymbol{\gamma}(k) - \left[\begin{array}{c} G \\ O \end{array}\right]\boldsymbol{u}(k) - T_s\boldsymbol{w}(k) \tag{39}$$

where $F \triangleq T_sA_cT_s^{-1} = \left[\begin{array}{cc} f_{11} & f_{12} \\ f_{21} & f_{22} \end{array}\right]$, $f_{11} \in \mathbf{R}^{m\times m}$, $f_{12} \in \mathbf{R}^{m\times (n-m)}$, $f_{21} \in \mathbf{R}^{(n-m)\times m}$, and $f_{22} \in \mathbf{R}^{(n-m)\times(n-m)}$. Accordingly, sliding surface is designed such that the eigenvalues of $(f_{22} - f_{21}M_1^{-1}M_2)$ are within the unit circle centered at zero to ensure the stability of the system, where $M = [M_1 \quad M_2]$, $M_1 \in \mathbf{R}^{m\times m}$, $M_2 \in \mathbf{R}^{m\times(n-m)}$.

After designing the healthy system controller, we consider the fault-tolerant control for the faulty system. To compensate for the effect of the faults, an additive input term is proposed as follows

$$\boldsymbol{u}_1(k) = -(MB_0)^{-1}ME_c\hat{\boldsymbol{f}}(k) \tag{40}$$

where $\hat{\boldsymbol{f}}(k)$ is given by (32).

The overall fault-tolerant controller design is summarized in the following theorem.

Theorem 2. Considering the system described by (7) and (8), the faulty system states can follow the desired trajectory under the fault-tolerant controller $\boldsymbol{u}_R(k)$ designed as follows

$$\boldsymbol{u}_R(k) = \boldsymbol{u}_1(k) + \boldsymbol{u}_2(k) \tag{41}$$

where $\boldsymbol{u}_1(k)$ and $\boldsymbol{u}_2(k)$ are given by (40) and (36).

Proof. Applying the control (41) to faulty system (7), we get the state dynamic equation

$$\boldsymbol{x}(k+1) = A_c\boldsymbol{x}(k) + \sum_{j=1}^{2}B_j\boldsymbol{u}(k-j) + B_0\boldsymbol{u}_2(k) + B_c\boldsymbol{\zeta}(k) + \boldsymbol{w}(k) + E_c\tilde{\boldsymbol{f}}(k) \tag{42}$$

where $\boldsymbol{u}_2(k)$ is given by (36), $\tilde{\boldsymbol{f}}(k) = \boldsymbol{f}(k) - \hat{\boldsymbol{f}}(k)$.

Note that if $\tilde{\boldsymbol{f}}(k) = \boldsymbol{0}$, the closed-loop system (42) with $\boldsymbol{u}_2$ being given by the healthy control law (36) can track the desired trajectories. Then, we can get

$$\boldsymbol{r}(k+1) - \boldsymbol{x}(k+1) = E_c\tilde{\boldsymbol{f}}(k) = -E_cE_{c3}^{-1}[A_{c3}(\hat{\boldsymbol{x}}(k-1) - \boldsymbol{x}(k-1)) + B_{c3}(\hat{\boldsymbol{\zeta}}(k-1) - \boldsymbol{\zeta}(k-1))] \tag{43}$$

From the above equation, it is clear that the tracking error depends on the state and modeling uncertainty estimation error. Also, we have proved that the state and the modeling uncertainties estimation errors converge to zero in Section 3. Thus the tracking error can converge to zero, and the faulty system states can track the desired trajectory effectively under the fault-tolerant controller given by Theorem 2. □

5 Simulation

In this section, the proposed approach is applied to actuator fault estimation and fault tolerant control of an aircraft in the vertical plane[24]. The discrete-time system with delay can be described as follows

$$A_c = \left[\begin{array}{cccc} 0.9996 & 0.0003 & 0.0002 & -0.0037 \\ 0.0005 & 0.9900 & -0.0002 & -0.0406 \\ 0.0010 & 0.0037 & 1.0453 & 10.5644 \\ 0.0000 & 0.0000 & 0.0101 & 1.0524 \end{array}\right]$$

$$B_0 = \left[\begin{array}{cc} 0.0000 & -0.0001 \\ -0.0003 & 0.0004 \\ -0.0075 & 0.0049 \\ 0.0000 & -0.0001 \end{array}\right], \quad B_1 = \left[\begin{array}{cc} 0.0044 & 0.0019 \\ 0.0356 & -0.0759 \\ -0.0484 & 0.0405 \\ -0.0003 & 0.0003 \end{array}\right]$$

$$C = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

where the state variable vector $\boldsymbol{x}(k) \in \mathbf{R}^4$ is composed of $x_1 = u$, the longitudinal velocity, $x_2 = \omega$, the vertical velocity, $x_3 = \omega_y$, the rate of pitch, and $x_4 = \theta$, the pitch angle. The components of command vector are u_1, the general cyclic command, and u_2 longitudinal cyclic command.

The covariance matrix for the process noise sequence is $Q = \mathrm{diag}\{0.1^2 \quad 0.1^2 \quad 0.01^2 \quad 0.01^2\}$. The loss of actuator effectiveness is considered, that is

$$E = -B, \quad f(k) = \left[\begin{array}{cc} r_1(k) & 0 \\ 0 & r_2(k) \end{array}\right] \tag{44}$$

with $0 \leq r_i(k) \leq 1$ individually representing the percentage degradations in actuator input channels. It can be verified whether all the assumptions in Theorem 1 are satisfied in this aircraft model. In fact, $\mathrm{rank}(CE) = 2$. In the simulation, the two actuator faults are created as follows

$$r_1(k) = \left\{\begin{array}{ll} 0, & t < 4(\mathrm{sec}) \\ 0.4, & 4 \leq t \leq 10(\mathrm{sec}) \end{array}\right.$$

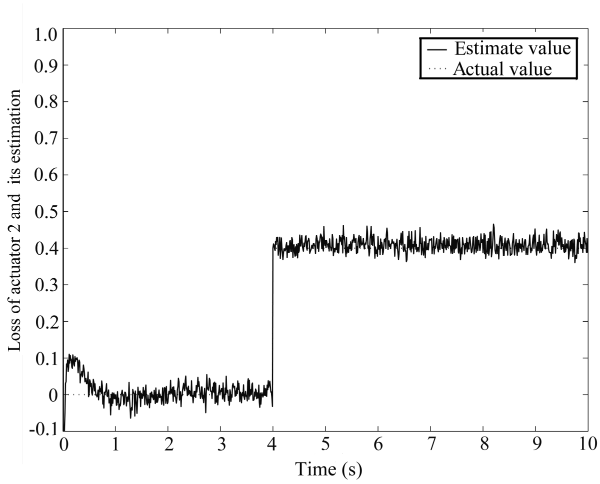
$$r_2(k) = \left\{\begin{array}{ll} 0, & t < 2(\mathrm{sec}) \\ 0.7, & 2 \leq t \leq 10(\mathrm{sec}) \end{array}\right.$$

and the modeling uncertainty is $\zeta(k) = 0.1$.

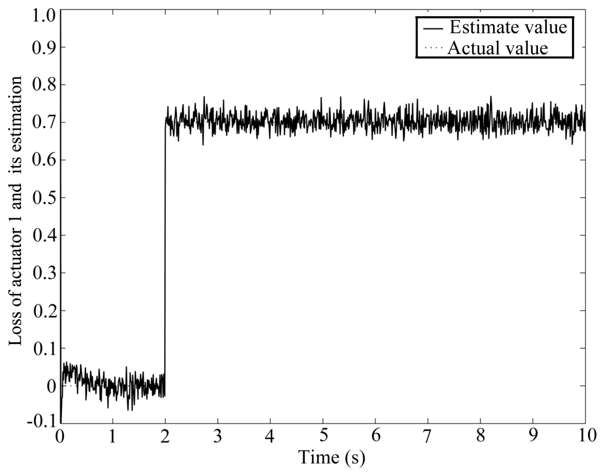
According to Theorem 1, the state estimation is given by (13) and (31), and then the actuator faults are estimated by (32) and compensated by Theorem 2. The sampling period in the simulation is chosen as 0.01s. Fig. 2 shows the actuator fault estimation with satisfactory accuracy. Fig. 3 shows the estimation result of the modeling uncertainty. Fig. 4 depicts the state trajectories of the closed-loop system. It can be seen that the dynamic system states can track the desired trajectory effectively using the proposed fault tolerant controller.

6 Conclusion

In this paper, a novel method of fault estimation and fault tolerant control is proposed for networked control systems, which are modeled as discrete-time systems with delay, noise, and uncertainties. An extended Kalman filter is presented to estimate the unavailable state, from which



(a) Actuator 1



(b) Actuator 2

Fig. 2 Estimation of actuator faults

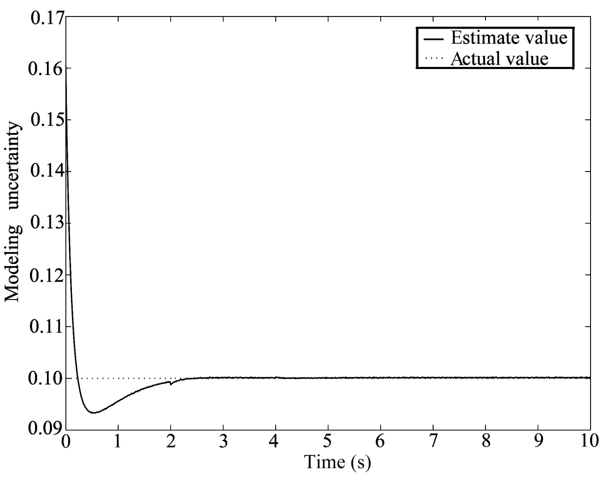
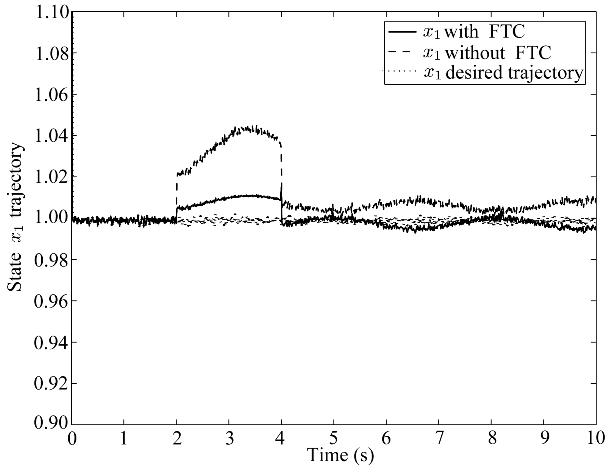
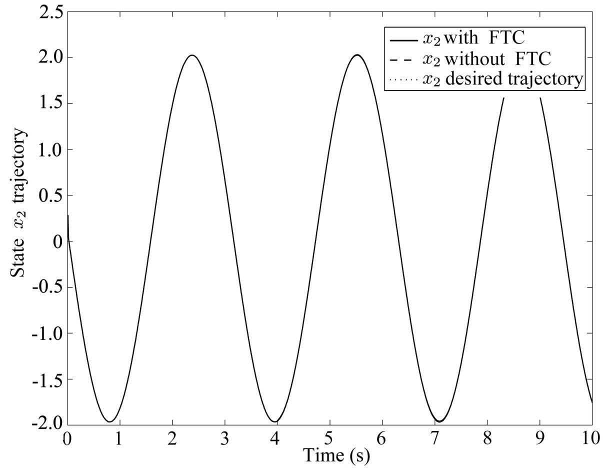


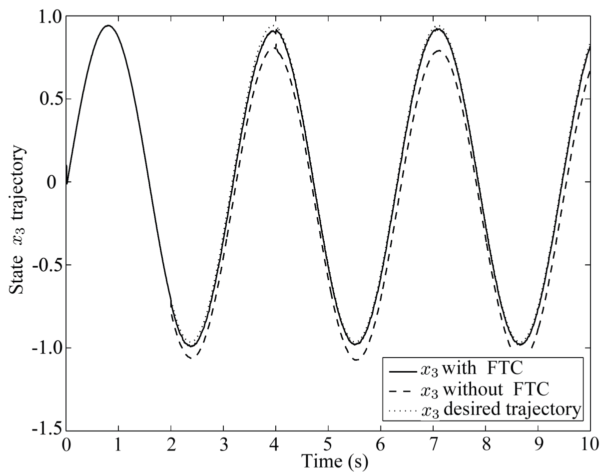
Fig. 3 Estimation of the modeling uncertainty



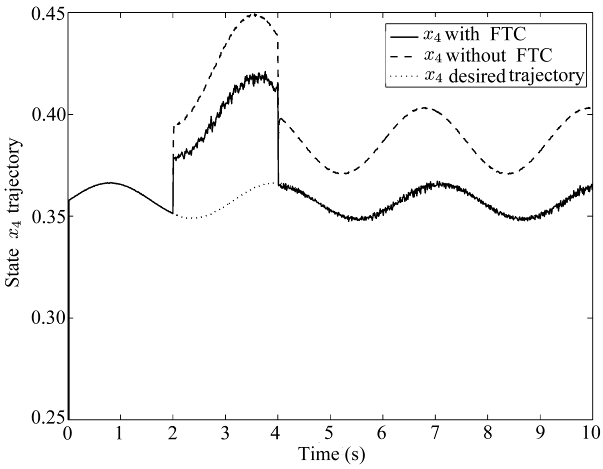
(a) State x_1



(b) State x_2



(c) State x_3



(d) State x_4

Fig. 4 State trajectories

the fault estimation can be obtained. According to the sliding mode control theory, a fault tolerant controller is designed to achieve the state tracking. Besides networks inducing delays, data drop could be considered. FDI for networked control systems then becomes more complex and practical. As a result, extension of the proposed method in this paper to such systems will be investigated in our future work.

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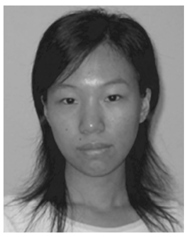

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