Robust H_∞ Networked Control for Uncertain Fuzzy Systems with Time-delay

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Abstract A robust H_∞ networked control method for Takagi-Sugeno (T-S) fuzzy systems with uncertainty and time delay is presented. A state feedback controller is designed *via* the networked control system (NCS) theory. Sufficient condition for robust stability with H_∞ performance is obtained. Network-induced delay in network transmission and packet dropout are analyzed. Simulation result shows the validity of this control scheme.

Key words Robust H_∞ networked control, Takagi-Sugeno fuzzy systems, networked control system theory, network-induced delay

1 Introduction

Fuzzy control is a useful approach to solve the control problems of nonlinear systems[1~5]. Over the past few years, the stability analysis and the synthesis problem of fuzzy systems as an important issue was studied by many researchers. Takagi-Sugeno (T-S) fuzzy system proposed by [6] is widely applied to industrial control because of its simple structure with local dynamics. The typical design approaches are carried out based on fuzzy model *via* the so-called parallel distributed compensation (PDC) method[7]. In recent years, some controller design methods based on linear matrix inequality (LMI) technology have also been used for the stability analysis and the controller design of T-S fuzzy systems.

Recently, many researches about the T-S fuzzy model with time delay term have been presented to deal with the stability and the stabilization problem of the nonlinear system with time-delay[1~5]. The analysis and the synthesis problem for continuous and discrete-time nonlinear systems *via* PDC approach was considered in [1, 2]. Stable fuzzy controller for nonlinear time-delay system was represented by LMI. The main results were based on the state feedback or observer technique. Delay-dependent robust controller was designed *via* state feedback in [3]. On the basis of Lyapunov criterion and Razumikhin theorem, some sufficient conditions were derived, under which the parallel-distributed fuzzy control can stabilize the whole uncertain fuzzy time-delay system asymptotically[4]. Stability analysis was studied for fuzzy control systems with bounded uncertain delays, and design approach based on LMIs was developed[5].

During the past decade, the robust H_∞ control for systems with delay has received considerable attention[8~15]. A novel delay-dependent robust H_∞ control for uncertain systems with a state-delay was proposed based on Lyapunov-Krasovskii functional approach in [10]. Some other results aimed at the interval time-varying delay, input delays, and multiple input-output delays were also given in [8, 12, 13], respectively. Output feedback H_∞ control was applied to communication networks with delays, uncertain stochastic systems with time-varying delays, and uncertain discrete-time-delay fuzzy systems in [9, 14, 15]. Three performance criteria have been established based on quadratic H_2 performance, H_∞, criteria and simultaneous H_2/H_∞ synthesis by [11].

Presently, much attention has been paid to the stability analysis and controller design of networked control systems (NCS) by [16~21]. Both network-induced delay and packet dropout in network transmission have very large influence for stability of the whole system. In [21], a detailed summary was made for review of previous work, and the relationship between the sampling rate and the network-induced delay was captured using a stability region plot. In [19], a model of NCS was provided under consideration of the network-induced delay and the packet dropout in the transmission. Robust controllers for uncertain NCS were also obtained in [20]. How to analyze stability of nonlinear NCS is a challenging and interesting topic. Some results about the stability of nonlinear NCS were obtained in [16~18]. Input-to-state stability (ISS) and input-to-output stability (IOS) were also analyzed in details in [16~17]. However, these methods often require some strict assumptions for system model so they are difficult to practical application.

In this article, we propose a novel control scheme which is the robust H_∞ networked control method for T-S fuzzy system with uncertainty and time delay in network condition. The robust H_∞ performance index of controlled model after considering network action is satisfied. Utilizing fuzzy control method and considering quality of service (QoS) in network system, the corresponding state feedback control law is obtained. We consider the stabilization problem of T-S fuzzy system with uncertainty and time delay in network condition. Further, some sufficient conditions of this control scheme are proposed by solving a set of LMIs. As far as we know, network condition has not been often considered in the usual fuzzy robust H_∞ control method.

2 Problem formulation

In general, a uncertain nonlinear time-delay system can be described by the T-S fuzzy system with uncertainty and time delay, which expresses the nonlinear system as a weighted sum of linear systems. The i-th rule is of the following format:

Rule i :

If $\theta_1(t)$ is $F_{i1}, \cdots$, and $\theta_{\bar n}(t)$ is $F_{i\bar n}$

Then

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= (A_i+\Delta A_i)\boldsymbol{x}(t)+(A_{d_i}+\Delta A_{d_i})\boldsymbol{x}(t-\tau)+\\ &\quad (B_i+\Delta B_i)\boldsymbol{u}(t)+C_i\boldsymbol{w}(t)\\ \boldsymbol{z}(t) &= D_i\boldsymbol{x}(t)+E_i\boldsymbol{u}(t)\\ \boldsymbol{x}(t) &= \boldsymbol{\varphi}(t), -\bar\tau\le t<0, \text{ for } i=1,2,\cdots,r\end{aligned}$$

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where $i = 1, 2, \cdots, r$ is the number of fuzzy rules, $\boldsymbol{x}(t) \in \mathbf{R}^n$ and $\boldsymbol{z}(t) \in \mathbf{R}^q$ denote the state vector and the measurement output vector, $\boldsymbol{w}(t) \in \mathbf{R}^p$ is the disturbance input vector, $\boldsymbol{u}(t) \in \mathbf{R}^m$ is the control input, A_i and $A_{d_i} \in \mathbf{R}^{n\times n}$ are known system matrices, $B_i \in \mathbf{R}^{n\times m}$ is the input matrix, $C_i \in \mathbf{R}^{n\times p}$ is the disturbance input matrix, $D_i \in \mathbf{R}^{q\times n}$, and $E_i \in \mathbf{R}^{q\times m}$, respectively, of the i-th subsystem. We assume that the admissible uncertainties satisfy $\Delta A_i = M_{1i}F(t)N_{1i}$, $\Delta A_{di} = M_{2i}F(t)N_{2i}$, and $\Delta B_i = M_{3i}F(t)N_{3i}$, where $M_{k_1 i}$ $(k_1 = 1, 2, 3)$, $N_{k_2 i}$ $(k_2 = 1, 2, 3)$, and $F^{\mathrm{T}}(t)$ are real matrices with appropriate dimensions, and satisfy $F^{\mathrm{T}}(t)F(t) \leq I$. τ is the constant bounded time delay in the state and it is assumed to be $0 < \tau \leq \bar{\tau}$, $\theta_1(t)$, $\theta_2(t)$, $\ldots$, $\theta_{\bar{n}}(t)$ are premise variables, F_{ig} is a fuzzy set $(g = 1, 2, \cdots, \bar{n})$. The inferred system is described by

$$\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{r} h_i(\theta(t))[(A_i + \Delta A_i)\boldsymbol{x}(t) + (A_{d_i} + \Delta A_{d_i})\boldsymbol{x}(t-\tau) + (B_i + \Delta B_i)\boldsymbol{u}(t) + C_i\boldsymbol{w}(t)]$$

Considering the network action, the state feedback controller is

$$\boldsymbol{u}(t) = \sum_{i=1}^{r} h_i(\theta(t))K_i\boldsymbol{x}(t_k)$$

The inferred fuzzy system is reconstructed in the following form

$$\begin{aligned}\dot{\boldsymbol{x}}(t) = &\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(t))[(A_i + \Delta A_i)\boldsymbol{x}(t) + (A_{d_i} + \\ &\Delta A_{d_i})\boldsymbol{x}(t-\tau) + (B_i + \Delta B_i)K_j\boldsymbol{x}(t_k) + C_i\boldsymbol{w}(t)], \\ &\text{for } t \in [t_k + \tau_k,\ t_{k+1} + \tau_{k+1})\end{aligned} \tag{1}$$

where t_k is the sampling instant, and $\boldsymbol{x}(t_k)$ is the state vector of plant at the instant t_k, which is a piecewise constant function, by using a zeroth-order-hold (ZOH), τ_k denotes the network-induced delay $k = 0, 1, 2, \cdots, (\tau_0 = 0)$. $t_s = t_{k+1} - t_k$ is the sampling period.

Remark 1. The packet is transmitted at the instant t_k, which contains the measured value of the plant state vector, $\boldsymbol{x}(t_k)$. Note that $\boldsymbol{x}(t_k)$ keeps constant in the interval $t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1})$ until next update. It is assumed that there does not exist controller-to-actuator delay, so $\boldsymbol{u}(t)$ can be sent to the plant as control input immediately. Obviously, while $\tau_0 = 0$,

$$\lim_{N\to\infty} \bigcup_{k=0}^{N} [t_k + \tau_k,\ t_{k+1} + \tau_{k+1}) = [t_0, \infty), t_0 \geq 0$$

3 Robust H_∞ networked control via state feedback

In this section, the robust H_∞ networked control *via* state feedback will be designed according to (1). Before giving the controller design method, we make the following assumptions:

Assumption 1. The sensor is time-driven. The controller and actuator are event-driven. The clocks among them are synchronized.

Assumption 2. The signal transmission is with in a single packet. Also the computational delay is negligible.

Assumption 3. The overall closed-loop system is under zero initial condition.

Assumption 4. We introduce the notion of a maximum allowable transfer interval $\delta > 0$. The maximum allowable transfer interval is a deadline; if a transmission of packet takes place at time t_k and the control signal will reach the plant at the instant $t_k + \tau_k$, then the next control signal must arrive within the time interval $(t_k,\ t_k + \delta]$. It is explicit that the next control signal will arrive at the instant $t_{k+p} + \tau_{k+q}$ if the packet dropout in network transmission is considered. The following condition is assumed

$$t_{k+p} - t_k + \tau_{k+p} \leq \delta,\ k = 0, 1, 2, \cdots,\ p = 1, 2, \cdots, p_{\max} \tag{2}$$

where p, and p_{max} are positive integers, which denote the sampling number and the maximum sampling number within δ, and in fact δ has a upper bound under the condition of guaranteed stability of the closed-loop system. We assume that the upper bound is smaller than $\bar{\tau}$.

Remark 2. We should notice that the network-induced delay is different to the system delay, because it is time varying and unknown. When the transmission time of a packet exceeds the threshold designed by the common network protocols, the packet is regarded as a data dropout. For example, $t_{k+2} + \tau_{k+2} < t_{k+1} + \tau_{k+1}$ means that the new data packet may reach the plant before the old one. In fact, we first suppose that δ exists. From (2), *i.e.*, $p = 2$, it is required that $t_{k+2} - t_k + \tau_{k+2} \leq \delta$. Thus, the old data packet containing $\boldsymbol{x}(t_{k+1})$ will be discarded. Therefore, when $p > 1$, some packets may be discarded while the whole closed-loop system is still stable under the condition (2). From the point of view of the QoS, the network resource is saved by decreasing the network-induced delay or discarding the old packet, based on δ, which can be realized by a suitable network scheduling method. It is explicit that if $p = 1$, (2) becomes

$$t_{k+1} - t_k + \tau_{k+1} \leq \delta, \quad k = 0, 1, 2, \cdots \tag{3}$$

This means that packet dropout is unallowable in the transmission.

The relationship between δ and the performance of NCS will be analyzed. For given δ, the smaller the sampling period t_s, the higher the allowable packet dropout rate. But the amount of communication required will be more. However, the larger sampling period will lead to the lower allowable packet dropout rate, which may degrade the performance of system.

For simplicity, we assume $p = 1$ in the following discussion.

Lemma 1[22]**.** For any constant symmetric matrix $M \in \mathbf{R}^{m\times n}$, $M = M^{\mathrm{T}} > 0$, scalar $\alpha > 0$, vector function $\boldsymbol{\xi} : [0, \alpha] \to \mathbf{R}^n$, such that the integrations in the following are well defined, then

$$\alpha\int_0^\alpha \boldsymbol{\xi}^{\mathrm{T}}(\beta)M\boldsymbol{\xi}(\beta)\mathrm{d}\beta \geq \left(\int_0^\alpha \boldsymbol{\xi}(\beta)\mathrm{d}\beta\right)^{\mathrm{T}} M \left(\int_0^\alpha \boldsymbol{\xi}(\beta)\mathrm{d}\beta\right)$$

Lemma 2. Let $\mathbb{Q}$ be any $\bar{l} \times n$ matrix, we have for any constant $\epsilon > 0$ and any positive-definite symmetric matrix $\mathbb{T}$ that

$$2\boldsymbol{\zeta}^{\mathrm{T}}\mathbb{Q}\boldsymbol{\eta} \leq \epsilon\boldsymbol{\zeta}^{\mathrm{T}}\mathbb{Q}\mathbb{T}^{-1}\mathbb{Q}^{\mathrm{T}}\boldsymbol{\zeta} + \frac{1}{\epsilon}\boldsymbol{\eta}^{\mathrm{T}}\mathbb{T}\boldsymbol{\eta}$$

for all $\boldsymbol{\zeta} \in \mathbf{R}^{\bar{l}}$, $\boldsymbol{\eta} \in \mathbf{R}^n$ and $\mathbb{T} \in \mathbf{R}^{n\times n}$.

Proof. Similar to the proof in [23], the condition can be obtained. The process is omitted.

Lemma 3[24]**.** For any matrices $\mathbb{D} \in \mathbf{R}^{n\times n_f}$, $\mathbb{E} \in \mathbf{R}^{n_f\times n}$ and $\mathbb{F} \in \mathbf{R}^{n_f\times n_f}$, with $\|\mathbb{F}\| \leq 1$, and scalar $\varepsilon > 0$, the following inequality holds

$$\mathbb{D}\mathbb{F}\mathbb{E} + \mathbb{E}^{\mathrm{T}}\mathbb{F}^{\mathrm{T}}\mathbb{D}^{\mathrm{T}} \leq \varepsilon^{-1}\mathbb{D}\mathbb{D}^{\mathrm{T}} + \varepsilon\mathbb{E}^{\mathrm{T}}\mathbb{E}.$$

Theorem 1. If there exist matrices $P_1 = P_1^{\mathrm{T}} > 0$, $P_2 = P_2^{\mathrm{T}} > 0$, $T = T^{\mathrm{T}} > 0$, matrices K_j $(j = 1, 2, \cdots, r)$, matrices Y_l $(l = 1, 2, 3)$ of appropriate dimensions and constant matrices Y_l $(l = 4, 5, 6)$ of appropriate dimensions such that the following LMIs (4) hold for given scalars $\delta > 0$ and $\varepsilon > 0$, then the closed-loop system (1) with $\boldsymbol{w}(t) \equiv 0$ is asymptotically stable.

$$\begin{aligned} &\Omega_{ii} < 0, \text{ for any } 1 \leq i \leq r \\ &\Omega_{ij} + \Omega_{ji} < 0 \text{ for any } 1 \leq i < j \leq r \end{aligned} \tag{4}$$

where

$$\Omega_{ij} = \begin{bmatrix} \Pi_{1,1} & \Pi_{1,2} & \Pi_{1,3} & \Pi_{1,4} & \Pi_{1,5} & 0 \\ * & \Pi_{2,2} & \Pi_{2,3} & \Pi_{2,4} & \Pi_{2,5} & 0 \\ * & * & \Pi_{3,3} & \Pi_{3,4} & 0 & 0 \\ * & * & * & \Pi_{4,4} & \Pi_{4,5} & \Pi_{4,6} \\ * & * & * & * & \Pi_{5,5} & 0 \\ * & * & * & * & * & \Pi_{6,6} \end{bmatrix}$$

$\Pi_{1,1} = P_2 + Y_1 + Y_1^{\mathrm{T}} - Y_4 A_i - A_i^{\mathrm{T}} Y_4^{\mathrm{T}} + \varepsilon^{-1} Y_4 (M_{1i} M_{1i}^{\mathrm{T}} + M_{2i} M_{2i}^{\mathrm{T}} + M_{3i} M_{3i}^{\mathrm{T}}) Y_4^{\mathrm{T}} + (\varepsilon + 2\varepsilon^{-1}) N_{1i}^{\mathrm{T}} N_{1i}$,
$\Pi_{1,2} = P_1 + Y_3^{\mathrm{T}} + Y_4 - A_i^{\mathrm{T}} Y_6^{\mathrm{T}}$, $\Pi_{1,3} = -Y_4 A_{di}$,
$\Pi_{1,4} = -Y_1 + Y_2^{\mathrm{T}} - Y_4 B_i K_j - A_i^{\mathrm{T}} Y_5^{\mathrm{T}}$, $\Pi_{1,5} = \delta Y_1$,
$\Pi_{2,2} = \delta T + Y_6 + Y_6^{\mathrm{T}} + \varepsilon Y_6 M_{1i} M_{1i}^{\mathrm{T}} Y_6^{\mathrm{T}} + \varepsilon^{-1} Y_6 (M_{2i} M_{2i}^{\mathrm{T}} + M_{3i} M_{3i}^{\mathrm{T}}) Y_6^{\mathrm{T}}$, $\Pi_{2,3} = -Y_6 A_{di}$, $\Pi_{2,4} = -Y_3 + Y_5^{\mathrm{T}} - Y_6 B_i K_j$,
$\Pi_{2,5} = \delta Y_3$, $\Pi_{3,3} = -P_2 + (2\varepsilon + \varepsilon^{-1}) N_{2i}^{\mathrm{T}} N_{2i}$,
$\Pi_{3,4} = -A_{di}^{\mathrm{T}} Y_5^{\mathrm{T}}$,
$\Pi_{4,4} = -Y_2 - Y_2^{\mathrm{T}} - Y_5 B_i K_j - K_j^{\mathrm{T}} B_i^{\mathrm{T}} Y_5^{\mathrm{T}} + \varepsilon Y_5 (M_{1i} M_{1i}^{\mathrm{T}} + M_{2i} M_{2i}^{\mathrm{T}}) Y_5^{\mathrm{T}} + \varepsilon^{-1} Y_5 M_{3i} M_{3i}^{\mathrm{T}} Y_5^{\mathrm{T}}$, $\Pi_{4,5} = \delta Y_2$,
$\Pi_{4,6} = \sqrt{3\varepsilon} K_j^{\mathrm{T}} N_{3i}^{\mathrm{T}}$, $\Pi_{5,5} = -\delta T$, $\Pi_{6,6} = -I$.

Proof. Consider a Lyapunov functional as

$$V(t) = \boldsymbol{x}^{\mathrm{T}}(t) P_1 \boldsymbol{x}(t) + \int_{t-\tau}^{t} \boldsymbol{x}^{\mathrm{T}}(s) P_2 \boldsymbol{x}(s) \mathrm{d}s + \int_{t-\delta}^{t} \int_{s}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(v) T \dot{\boldsymbol{x}}(v) \mathrm{d}v \mathrm{d}s$$

where $P_1 = P_1^{\mathrm{T}} > 0$, $P_2 = P_2^{\mathrm{T}} > 0$, and $T = T^{\mathrm{T}} > 0$. The corresponding time derivative of $V(t)$ for $t \in [t_k + \tau_k,\ t_{k+1} + \tau_{k+1})$ can be given and its formation is omitted.

From (3) and Lemmas 1-2, we can get for $t \in [t_k + \tau_k,\ t_{k+1} + \tau_{k+1})$

$$\int_{t_k}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s) T \dot{\boldsymbol{x}}(s) \mathrm{d}s \leq \int_{t-\delta}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s) T \dot{\boldsymbol{x}}(s) \mathrm{d}s$$

and

$$\begin{aligned} &-2(\boldsymbol{x}^{\mathrm{T}}(t) Y_1 + \boldsymbol{x}^{\mathrm{T}}(t_k) Y_2 + \dot{\boldsymbol{x}}^{\mathrm{T}}(t) Y_3) \int_{t_k}^{t} \dot{\boldsymbol{x}}(s) \mathrm{d}s \leq \\ &\delta \boldsymbol{\Lambda}^{\mathrm{T}}(t) \bar{Y} T^{-1} \bar{Y}^{\mathrm{T}} \boldsymbol{\Lambda}(t) + \int_{t-\delta}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s) T \dot{\boldsymbol{x}}(s) \mathrm{d}s \end{aligned}$$

where $\bar{Y}^{\mathrm{T}} = [Y_1^{\mathrm{T}}\ \ Y_3^{\mathrm{T}}\ \ 0\ \ Y_2^{\mathrm{T}}]$, $\boldsymbol{\Lambda}^{\mathrm{T}} = [\boldsymbol{x}^{\mathrm{T}}(t)\ \ \dot{\boldsymbol{x}}^{\mathrm{T}}(t)\ \ \boldsymbol{x}^{\mathrm{T}}(t-\tau)\ \ \boldsymbol{x}^{\mathrm{T}}(t_k)]$.

Utilizing Lemma 3 and Schur complement, the conditions in Theorem 1 can be obtained. □

In the following derivation process, we will consider the robust stability of (1) with H_∞ performance index. In order to attenuate the external disturbance of the fuzzy system (1), we introduce H_∞ performance index, *i.e.*,

$$\int_{t_0}^{\infty} \boldsymbol{z}^{\mathrm{T}}(t) \boldsymbol{z}(t) \mathrm{d}t \leq \gamma^2 \int_{t_0}^{\infty} \boldsymbol{w}^{\mathrm{T}}(t) \boldsymbol{w}(t) \mathrm{d}t \tag{5}$$

where $\gamma > 0$ denotes prescribed attenuation level.

Theorem 2. If there exist matrices $\bar{P}_1 = \bar{P}_1^{\mathrm{T}} > 0$, $\bar{P}_2 = \bar{P}_2^{\mathrm{T}} > 0$, $\bar{T} = \bar{T}^{\mathrm{T}} > 0$, matrices $\bar{K}_j$ $(j = 1, 2, \cdots, r)$, matrices $\bar{Y}_l$ $(l = 1, 2, 3)$ of appropriate dimensions and constant matrices $\bar{Y}_l$ $(l = 4, 5, 6)$ of appropriate dimensions such that the following LMIs (6) hold for given scalars $\delta > 0$, $\varepsilon > 0$, and $\gamma > 0$, then the closed-loop system (1) is robustly stable with H_∞ performance index (5).

$$\begin{aligned} &\Omega_{ii}^{'} < 0, \text{ for any } 1 \leq i \leq r \\ &\Omega_{ij}^{'} + \Omega_{ji}^{'} < 0 \text{ for any } 1 \leq i < j \leq r \end{aligned} \tag{6}$$

where

$$\Omega_{ij}^{'} = \begin{bmatrix} \Pi_{1,1}^{'} & \Pi_{1,2}^{'} & \Pi_{1,3}^{'} & \Pi_{1,4}^{'} & \Pi_{1,5}^{'} & \Pi_{1,6}^{'} & 0 & 0 \\ * & \Pi_{2,2}^{'} & \Pi_{2,3}^{'} & \Pi_{2,4}^{'} & \Pi_{2,5}^{'} & \Pi_{2,6}^{'} & 0 & 0 \\ * & * & \Pi_{3,3}^{'} & \Pi_{3,4}^{'} & 0 & 0 & 0 & 0 \\ * & * & * & \Pi_{4,4}^{'} & \Pi_{4,5}^{'} & \Pi_{4,6}^{'} & \Pi_{4,7}^{'} & \Pi_{4,8}^{'} \\ * & * & * & * & \Pi_{5,5}^{'} & 0 & 0 & 0 \\ * & * & * & * & * & \Pi_{6,6}^{'} & 0 & 0 \\ * & * & * & * & * & * & \Pi_{7,7}^{'} & 0 \\ * & * & * & * & * & * & * & \Pi_{8,8}^{'} \end{bmatrix}$$

$\Pi_{1,1}^{'} = \bar{P}_2 + \bar{Y}_1 + \bar{Y}_1^{\mathrm{T}} - \bar{Y}_4 A_i - A_i^{\mathrm{T}} \bar{Y}_4^{\mathrm{T}} + \varepsilon^{-1} \bar{Y}_4 (M_{1i} M_{1i}^{\mathrm{T}} + M_{2i} M_{2i}^{\mathrm{T}} + M_{3i} M_{3i}^{\mathrm{T}}) \bar{Y}_4^{\mathrm{T}} + (\varepsilon + 2\varepsilon^{-1}) N_{1i}^{\mathrm{T}} N_{1i} + D_i^{\mathrm{T}} D_i$,
$\Pi_{1,2}^{'} = \bar{P}_1 + \bar{Y}_3^{\mathrm{T}} + \bar{Y}_4 - A_i^{\mathrm{T}} \bar{Y}_6^{\mathrm{T}}$, $\Pi_{1,3}^{'} = -\bar{Y}_4 A_{di}$,
$\Pi_{1,4}^{'} = -\bar{Y}_1 + \bar{Y}_2^{\mathrm{T}} - \bar{Y}_4 B_i \bar{K}_j - A_i^{\mathrm{T}} \bar{Y}_5^{\mathrm{T}} + D_i^{\mathrm{T}} E_i \bar{K}_j$,
$\Pi_{1,5}^{'} = \delta \bar{Y}_1$, $\Pi_{1,6}^{'} = -\bar{Y}_4 C_i$, $\Pi_{2,2}^{'} = \delta \bar{T} + \bar{Y}_6 + \bar{Y}_6^{\mathrm{T}} + \varepsilon \bar{Y}_6 M_{1i} M_{1i}^{\mathrm{T}} \bar{Y}_6^{\mathrm{T}} + \varepsilon^{-1} \bar{Y}_6 (M_{2i} M_{2i}^{\mathrm{T}} + M_{3i} M_{3i}^{\mathrm{T}}) \bar{Y}_6^{\mathrm{T}}$,
$\Pi_{2,3}^{'} = -\bar{Y}_6 A_{di}$, $\Pi_{2,4}^{'} = -\bar{Y}_3 + \bar{Y}_5^{\mathrm{T}} - \bar{Y}_6 B_i \bar{K}_j$, $\Pi_{2,5}^{'} = \delta \bar{Y}_3$,
$\Pi_{2,6}^{'} = -\bar{Y}_6 C_i$, $\Pi_{3,3}^{'} = -\bar{P}_2 + (2\varepsilon + \varepsilon^{-1}) N_{2i}^{\mathrm{T}} N_{2i}$,
$\Pi_{3,4}^{'} = -A_{di}^{\mathrm{T}} \bar{Y}_5^{\mathrm{T}}$,
$\Pi_{4,4}^{'} = -\bar{Y}_2 - \bar{Y}_2^{\mathrm{T}} - \bar{Y}_5 B_i \bar{K}_j - \bar{K}_j^{\mathrm{T}} B_i^{\mathrm{T}} \bar{Y}_5^{\mathrm{T}} + \varepsilon \bar{Y}_5 (M_{1i} M_{1i}^{\mathrm{T}} + M_{2i} M_{2i}^{\mathrm{T}}) \bar{Y}_5^{\mathrm{T}} + \varepsilon^{-1} \bar{Y}_5 M_{3i} M_{3i}^{\mathrm{T}} \bar{Y}_5^{\mathrm{T}}$, $\Pi_{4,5}^{'} = \delta \bar{Y}_2$,
$\Pi_{4,6}^{'} = -\bar{Y}_5 C_i$, $\Pi_{4,7}^{'} = \bar{K}_j^{\mathrm{T}} E_i^{\mathrm{T}}$, $\Pi_{4,8}^{'} = \sqrt{3\varepsilon} \bar{K}_j^{\mathrm{T}} N_{3i}^{\mathrm{T}}$,
$\Pi_{5,5}^{'} = -\delta \bar{T}$, $\Pi_{6,6}^{'} = -\gamma^2 I$, $\Pi_{7,7}^{'} = -I$, $\Pi_{8,8}^{'} = -I$.

Proof. Consider a Lyapunov functional as

$$V(t) = \boldsymbol{x}^{\mathrm{T}}(t) \bar{P}_1 \boldsymbol{x}(t) + \int_{t-\tau}^{t} \boldsymbol{x}^{\mathrm{T}}(s) \bar{P}_2 \boldsymbol{x}(s) \mathrm{d}s + \int_{t-\delta}^{t} \int_{s}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(v) \bar{T} \dot{\boldsymbol{x}}(v) \mathrm{d}v \mathrm{d}s$$

where $\bar{P}_1 = \bar{P}_1^{\mathrm{T}} > 0$, $\bar{P}_2 = \bar{P}_2^{\mathrm{T}} > 0$, and $\bar{T} = \bar{T}^{\mathrm{T}} > 0$. Similar to the proof in Theorem 1, under Assumption 3, the conditions in Theorem 2 can be obtained. The process is omitted.

For the sake of minimizing the upper bound of δ under some restrictions, we can use the modified generalized eigenvalue minimization problem (GEVP) technique[25] to solve the suboptimal problem.

Theorem 3. There exists an upper bound of $\delta_{max} = 1/\varsigma$ such that for any $0 < \delta < \delta_{max}$, the closed-loop system (1) can be robustly stabilized with H_∞ performance index

(5), if the following GEVP problem is feasible for matrices $\check{Y} = \check{Y}^{\mathrm{T}} > 0$, $\bar{P}_1 = \bar{P}_1^{\mathrm{T}} > 0$, $\bar{P}_2 = \bar{P}_2^{\mathrm{T}} > 0$, $\bar{T} = \bar{T}^{\mathrm{T}} > 0$, matrices $\bar{K}_j$ $(j = 1, 2, \cdots, r)$, matrices $\bar{Y}_l$ $(l = 1, 2, 3)$ of appropriate dimensions and the given constant matrices $\bar{Y}_l$ $(l = 4, 5, 6)$ of appropriate dimensions, scalars $\varepsilon > 0$ and $\gamma > 0$:

$$\text{Minimize} \quad \varsigma = \frac{1}{\delta} > 0$$
$$\text{s.t.} \begin{cases} \tilde{X} < \begin{bmatrix} \check{Y} & 0 \\ 0 & 0 \end{bmatrix} \\ \check{Y} < \varsigma \tilde{Z} \end{cases} \tag{7}$$

for $1 \le i \le j \le r$, where

$$\tilde{X} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \bar{Y}_1 \\ * & \bar{T} & 0 & 0 & 0 & 0 & 0 & \bar{Y}_3 \\ * & * & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 & 0 & \bar{Y}_2 \\ * & * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & * & -\bar{T} \end{bmatrix},$$

$$\tilde{Z} = \begin{bmatrix} \tilde{\Pi}_{1,1} & \tilde{\Pi}_{1,2} & \tilde{\Pi}_{1,3} & \tilde{\Pi}_{1,4} & 0 & \tilde{\Pi}_{1,6} & 0 \\ * & \tilde{\Pi}_{2,2} & \tilde{\Pi}_{2,3} & \tilde{\Pi}_{2,4} & 0 & \tilde{\Pi}_{2,6} & 0 \\ * & * & \tilde{\Pi}_{3,3} & \tilde{\Pi}_{3,4} & 0 & 0 & 0 \\ * & * & * & \tilde{\Pi}_{4,4} & \tilde{\Pi}_{4,5} & \tilde{\Pi}_{4,6} & \tilde{\Pi}_{4,7} \\ * & * & * & * & \tilde{\Pi}_{5,5} & 0 & 0 \\ * & * & * & * & * & \tilde{\Pi}_{6,6} & 0 \\ * & * & * & * & * & * & \tilde{\Pi}_{7,7} \end{bmatrix}$$

$\tilde{\Pi}_{1,1} = \Pi'_{1,1}$, $\tilde{\Pi}_{1,2} = \Pi'_{1,2}$, $\tilde{\Pi}_{1,3} = \Pi'_{1,3}$, $\tilde{\Pi}_{1,4} = \Pi'_{1,4}$, $\tilde{\Pi}_{1,6} = \Pi'_{1,6}$, $\tilde{\Pi}_{2,2} = \bar{Y}_6 + \bar{Y}_6^{\mathrm{T}} + \varepsilon \bar{Y}_6 M_{1i} M_{1i}^{\mathrm{T}} \bar{Y}_6^{\mathrm{T}} + \varepsilon^{-1} \bar{Y}_6 (M_{2i} M_{2i}^{\mathrm{T}} + M_{3i} M_{3i}^{\mathrm{T}}) \bar{Y}_6^{\mathrm{T}}$, $\tilde{\Pi}_{2,3} = \Pi'_{2,3}$, $\tilde{\Pi}_{2,4} = \Pi'_{2,4}$, $\tilde{\Pi}_{2,6} = \Pi'_{2,6}$, $\tilde{\Pi}_{3,3} = \Pi'_{3,3}$, $\tilde{\Pi}_{3,4} = \Pi'_{3,4}$, $\tilde{\Pi}_{4,4} = \Pi'_{4,4}$, $\tilde{\Pi}_{4,5} = \Pi'_{4,8}$, $\tilde{\Pi}_{4,6} = \Pi'_{4,6}$, $\tilde{\Pi}_{4,7} = \Pi'_{4,7}$, $\tilde{\Pi}_{4,8} = \Pi'_{4,8}$, $\tilde{\Pi}_{5,5} = \Pi'_{8,8}$, $\tilde{\Pi}_{6,6} = \Pi'_{6,6}$, $\tilde{\Pi}_{7,7} = \Pi'_{7,7}$.

Proof. Given Ω'_{ij}, we exchange column 5 with column 8, and row 5 with row 8. Similar to the proof in [25], the process is omitted.

Remark 3. It is very explicit that the above result may be only suboptimal because some matrices and parameters must be given in advance. In fact, we can also introduce the following design process to obtain another suboptimal value about δ.

Design procedure.

Step 1. Select membership functions and construct fuzzy plant rules.

Step 2. Choose an appropriate attenuation level $\gamma > 0$, a scalar $\varepsilon > 0$ and randomly choose the constant matrices $\bar{Y}_l$ $(l = 4, 5, 6)$ of appropriate dimensions.

Step 3. Choose an initial $\delta > 0$ according to current network demand.

Step 4. Solve the LMIs in (6) to obtain K_j $(j = 1, 2, \cdots, r)$, $\bar{P}_1$, $\bar{P}_2$, $\bar{T}$, and $\bar{Y}_l$ $(l = 1, 2, 3)$ of appropriate dimensions. If the solutions of LMIs do not exist, we repeat Step 2.

Step 5. Increase δ and repeat Step 4 until the cycle indices arrive the setting value.

Step 6. Construct the fuzzy robust H_∞ networked controller.

4 Simulation result

In this section, an example is presented to show the validity of our control scheme. We use the above design method to design a robust H_∞ networked controller for the following nonlinear systems.

Example. Consider the following nonlinear system proposed in [26]; the structures and parameters for this fuzzy system are omitted.

We select

$$\bar{Y}_4 = \begin{bmatrix} -12.3 & -4.3 \\ -3.8 & -6.3 \end{bmatrix}, \bar{Y}_5 = \begin{bmatrix} -3.4 & -4.3 \\ -3.0 & -3.6 \end{bmatrix},$$
$$\bar{Y}_6 = \begin{bmatrix} -4.7 & -3.7 \\ -3.3 & -3.7 \end{bmatrix}, \varepsilon = 1, \gamma = 0.65, \delta = 0.15$$

Applying Theorem 2, the feasible solutions to (6) are given as follows.

$$\bar{P}_1 = \begin{bmatrix} 14.4220 & 10.2578 \\ 10.2578 & 13.7708 \end{bmatrix}, \bar{P}_2 = \begin{bmatrix} 6.2221 & 2.6455 \\ 2.6455 & 1.9581 \end{bmatrix}$$
$$\bar{T} = \begin{bmatrix} 14.6049 & 7.7547 \\ 7.7547 & 10.5054 \end{bmatrix}, \bar{Y}_1 = \begin{bmatrix} -96.7872 & -51.4147 \\ -51.3546 & -69.5131 \end{bmatrix}$$
$$\bar{Y}_2 = \begin{bmatrix} 96.7588 & 51.3676 \\ 51.3474 & 69.4784 \end{bmatrix}, \bar{Y}_3 = \begin{bmatrix} -0.0337 & -0.0271 \\ -0.0336 & -0.0319 \end{bmatrix}$$
$$K_1 = \begin{bmatrix} -1.9558 & -0.6443 \end{bmatrix}, K_2 = \begin{bmatrix} -3.4021 & -0.0378 \end{bmatrix}$$

Next, under the same initial value $\boldsymbol{\varphi}(t) = (0.5 \ \ -1)^{\mathrm{T}}$ for $t \in [-0.5, \ \ 0]$, we show the results with the different network condition. The figures are ignored because of the length restriction.

Case 1. Sampling period $t_s = 0.05$, network-induced delay $\tau_D \le 0.03$, and system state delay time $\tau = 0.5$ are given according to system demand.

Case 2. Sampling period $t_s = 0.1$, network-induced delay $\tau_D \le 0.01$, and system state delay time $\tau = 1$ are given according to system demand.

Case 3. Sampling period $t_s = 0.1$, network-induced delay $\tau_D \le 0.01$, and system state delay time $\tau = 3$ are given according to system demand.

Case 4. Sampling period $t_s \le 0.1$ (random change in this scope), network-induced delay $\tau_D \le 0.01$, and system state delay time $\tau = 3$ are given according to system demand.

It is very explicit that the uncertain fuzzy system is still robustly stable with H_∞ performance after considering different network action.

5 Conclusion

In this study, a robust H_∞ networked control method for T-S fuzzy systems with time delay has been proposed. NCS theory is used to design system controller. Both network-induced delay and packet dropout are considered in an uniform framework. During the sampling interval, the plant is controlled based on the robust H_∞ state feedback control law. Simulation result shows the validity of the presented control scheme. Of course, some special properties of NCS can not be reflected as the system with time-delay, and some random modeling method should be applied to the design. These issues will be researched in our future work.

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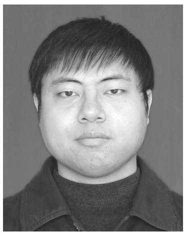

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