## Switching Logic-based Adaptive **Robust Control of Nonlinearly Parameterized Uncertain Systems**

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In this paper, a switching logic-based adaptive ro-Abstract bust control is proposed for a class of nonlinearly parameterized systems (NPS). Specifically, the controller mainly consists of a robust type term to address the system uncertainty, and a switching logic tuning mechanism to update the involved control gain. The constructed controller achieves a global uniformly ultimate boundedness (GUUB) result for the system errors, and simulation results are included to demonstrate the effectiveness of the control law.

Key words Nonlinear system, switching logic, adaptive robust control, uncertain system

#### 1 Introduction

Over the past decades, adaptive control has attracted much attention of the control field for uncertain nonlinear plants. Typically, an adaptive controller tries to estimate the uncertainty within the system by utilization of on-line gathered information, and thus to obtain knowledge on the system dynamics to the utmost extent<sup>[1]</sup>. In other words, an adaptive control law follows a step-by-step method to obtain a better understanding for the plant, so that a satisfactory control performance is achieved. Different from the more conservative robust control which aims to damp out the system uncertainty by employing a worstcase size dominating term, adaptive control law usually involves much less control energy. Hence, it is often more applicable in reality. In fact, adaptive control has been regarded as one of the most elegant and most important tools for uncertain nonlinear system control. Unfortunately, currently developed adaptive control can only address a class of nonlinear systems whose uncertainty enters the system in a linear way. In other words, the so-called linear parameterization (LP) condition is often required a priori when applying adaptive control for a plant.

Recently, the adaptive control of nonlinearly parameterized systems (NPS) has been a topic of considerable interest in the control field<sup> $[2\sim 8]$ </sup>. For example, Ye *et al.* in [9] designed a global adaptive controller for a class of feedforward nonlinear systems to asymptotically regulate the system errors. In [10], Ge et al. proposed a robust adaptive control approach for a class of time-varying uncertain nonlinear systems which achieves a global uniformly ultimate boundedness (GUUB) stability. In [11], Fang et al. designed an adaptive learning control law to attack a class of nonlinearly parameterized uncertain systems, in which the uncertainty can be separated into an unknown parameter vector multiplying a periodic signal. Unfortunately, for recently developed adaptive controllers targeting NPS, various assumptions have to be imposed to achieve a de-

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sired performance; hence, the control of uncertain NPS is far from being solved and it is actually a very challenging problem for the control field. Recently, some control structures have been reported on addressing uncertain dynamic systems with the control gains tuned via an on-line mechanism. In [12], Ye et al. constructed a switching type adaptive control law to address a class of nonlinearly parameterized system in which some control parameter was tuned via a switching logic. Motivated by these results, in this paper, we propose a switching logic-based adaptive robust control for a class of NPS which enables GUUB tracking result for the system errors. Specifically, we propose a robust type controller whose gain is updated online until the system demonstrates superior performance. The result is based on a stability analysis that involves the application of Lyapunov techniques, and simulation results are included to demonstrate the effectiveness of the designed control law. The contribution of the paper lies in the fact that a switching logic-based adaptive robust control is designed to address the tracking problem of a general NPS. Superior over other controllers with switching logic, the constructed adaptive robust control is continuous and presents no chattering problem which then makes it more applicable in reality. Besides, the contribution of the paper also includes the introduction of the innovative updating mechanism for control parameters based on the performance of the chosen Lyapunov function. This updating mechanism can be fused with other type controllers to achieve desired performance for uncertain complex systems.

#### $\mathbf{2}$ System dynamics

In this paper, we consider the tracking problem for the following nonlinear system

$$\dot{\boldsymbol{x}} = [\boldsymbol{f}(\boldsymbol{x},t) + \Delta \boldsymbol{f}(\boldsymbol{x},\boldsymbol{v},t)] + \boldsymbol{u}$$
(1)

where  $\boldsymbol{x}(t) \in \mathbf{R}^n$  is the system state,  $\boldsymbol{u}(t) \in \mathbf{R}^n$  denotes the control input to the system. The bracketed terms represent the system dynamics where  $f(x, t) \in \mathbf{R}^n$  denotes the so-called nominal dynamics,  $\Delta \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{v}, t) \in \mathbf{R}^n$  is the system uncertainty caused by unmodeled dynamics and unknown disturbance  $\boldsymbol{v} \in \mathbf{R}^m$  exerted on the system. For the system of (1), we assume that if  $\boldsymbol{x}(t) \in \mathcal{L}_{\infty}$ , then  $\boldsymbol{f}(\boldsymbol{x},t) \in \mathcal{L}_{\infty}$ . And for the uncertainty  $\Delta f(\boldsymbol{x}, \boldsymbol{v}, t)$ , there exists a bounding function  $\rho(\boldsymbol{x}, t, \boldsymbol{\theta}) \in \mathbf{R}^1$  in the sense that <sup>[2]</sup>

$$\|\Delta \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{v}, t)\| \le \rho(\boldsymbol{x}, t, \boldsymbol{\theta}) \tag{2}$$

where the structure of the function  $\rho(\boldsymbol{x}, t, \boldsymbol{\theta}) \in \mathbf{R}^1$  is known while  $\boldsymbol{\theta} \in \mathbf{R}^p$  denotes unknown parameter vector. Further, it is assumed that the function  $\rho(\boldsymbol{x}, t, \boldsymbol{\theta})$  increases monotonically with respect to each  $\theta_i, i = 1, 2, \cdots, p$ .

It is generally known that if the upper bound of  $\theta_i$  is known a priori, then a robust control can be developed to obtain a GUUB stability result for system (1). If the bounds of  $\theta_i$  is unknown, yet the parameter  $\theta_i$  remains constant and the bounding function  $\rho(\boldsymbol{x}, t, \boldsymbol{\theta})$  satisfies the LP condition, then a robust adaptive control law can be constructed to achieve an asymptotic stability result. Moreover, if  $\rho(\boldsymbol{x}, t, \boldsymbol{\theta})$  can be only fractionally linearly parameterized, then the adaptive robust controller proposed in [2] can be utilized to obtain a practical stability for system (1). In this paper, we remove these assumptions regarding the system dynamics and propose a switching logic-based adaptive robust control to achieve GUUB result for a class of more general NPS.

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**Remark 1.** As can be seen in a nonlinear controller, a feedforward term is usually adopted to damp out (such as in the robust control) or cancel out (such as in the adaptive control) the uncertainty of the dynamics. Unfortunately, such a term is prone to degrade the original ambitious non-linear controller into a simple bang-bang controller due to the fact that this damping or cancelling term usually involves large control input which exceeds the physical limits of the practical controller and thus saturates it into a bang-bang controller. With this in mind, our subsequent analysis aims to design a tight feedforward term to attack the system uncertainty so that the previously stated problem is mostly avoided.

Remark 2. For simplicity, we utilize the first-order system of (1) to demonstrate the process of robust adaptive control construction and stability analysis. However, it should be noted that the proposed design method can be easily extended to address higher-order systems by fusing it with linear filters, provided that the full state information is available for feedback. Specifically, a higher-order system can be transformed into a first-order system by introducing a series of linear filters. Further, based on the property of the linear filters, it is easy to show that the obtained first-order system has the same kind of stability as the previous higher-order system. Based on this fact, if we follow the same process described in the paper to construct an adaptive robust control for the transformed first-order system, then the desired performance can also be achieved for the previous higher-order system.

# 3 Switching logic-based adaptive robust control

#### 3.1 Error system developments

The control objective is to make the system state track some desired trajectory  $\boldsymbol{x}_d(t) \in \mathbf{R}^n$  with the assumption that  $\boldsymbol{x}_d(t), \dot{\boldsymbol{x}}_d(t) \in \mathcal{L}_{\infty}$ . To aid the subsequent controller construction and stability analysis, we define the tracking error signal  $\boldsymbol{e}(t) \in \mathbf{R}^n$  as

$$\boldsymbol{e} = \boldsymbol{x}_d - \boldsymbol{x} \tag{3}$$

To obtain the open-loop dynamics of the error system, we take the time derivative of (3) and then substitute the dynamics of (1) into the resulting expression to obtain

$$\dot{\boldsymbol{e}} = \dot{\boldsymbol{x}}_d - [\boldsymbol{f}(\boldsymbol{x}, t) + \Delta \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{v}, t)] - \boldsymbol{u}$$
(4)

#### 3.2 Adaptive robust control design

For the system parameters defined in (2), we assume that  $\theta_i$  is time-varying in a compact set  $\Omega_i$  and it is bounded in the sense that

$$|\theta_i| \le \bar{\theta}_i, i = 1, 2, \cdots, p \tag{5}$$

where  $\bar{\theta}_i \in \mathbf{R}^1$  represents an unknown positive constant. Based on (5), we define a positive constant  $p^* \in \mathbf{R}^1$  as follows

$$p^* = \max\{\bar{\theta}_i\}, i = 1, 2, \cdots, p$$
 (6)

Then based on the assumption that the function  $\rho(\boldsymbol{x}, t, \boldsymbol{\theta})$  increases monotonically with respect to each  $\theta_{i, i} = 1, 2, \cdots, p$ , the following fact can be easily shown <sup>1</sup>

$$\rho(\boldsymbol{x}, t, \boldsymbol{\theta}) = \rho(\boldsymbol{x}, t, \theta_1, ..., \theta_p) \le \rho(\boldsymbol{x}, t, p^*, ..., p^*)$$
(7)

Based on the open loop dynamics of (4) and the subsequent stability analysis, we design the following adaptive robust control law

$$\boldsymbol{u} = \dot{\boldsymbol{x}}_d + k_e \boldsymbol{e} + \frac{\rho^2(\boldsymbol{x}, t, k_p) \boldsymbol{e}}{\rho(\boldsymbol{x}, t, k_p) \|\boldsymbol{e}\| + \varepsilon_\rho} - \boldsymbol{f}(\boldsymbol{x}, t)$$
(8)

where  $k_e \in \mathbf{R}^1$  denotes a positive control gain, while  $\varepsilon_{\rho} \in \mathbf{R}^1$  and  $k_p \in \mathbf{R}^1$  are positive control parameters wherein  $k_p$  is tuned via the subsequently designed mechanism.

After substituting (8) into the system dynamics and cancelling common terms, we obtain the closed-loop dynamics of  $\dot{\boldsymbol{e}}(t)$  as follows

$$\dot{\boldsymbol{e}} = -\Delta \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{v}, t) - k_e \boldsymbol{e} - \frac{\rho^2(\boldsymbol{x}, t, k_p) \boldsymbol{e}}{\rho(\boldsymbol{x}, t, k_p) \|\boldsymbol{e}\| + \varepsilon_{\rho}}$$
(9)

To assist the subsequent control gain mechanism design, we first analyze the system performance by Lyapunov techniques. To do that, we choose a non-negative, scalar function  $V(t) \in \mathbf{R}^1$  as follows

$$V = \frac{1}{2}\boldsymbol{e}^{\mathrm{T}}\boldsymbol{e} \tag{10}$$

After taking the time derivative of (10) and substituting (9) for  $\dot{\boldsymbol{e}}(t)$  into the resulting expression,  $\dot{V}(t)$  can be obtained as follows

$$\dot{V} = \boldsymbol{e}^{\mathrm{T}} \left[ -\Delta \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{v}, t) - k_{e} \boldsymbol{e} - \frac{\rho^{2}(\boldsymbol{x}, t, k_{p}) \boldsymbol{e}}{\rho(\boldsymbol{x}, t, k_{p}) \|\boldsymbol{e}\| + \varepsilon_{\rho}} \right]$$
(11)

We can then apply the property of (2) into (11) to obtain an upper bound of  $\dot{V}(t)$  as follows

$$\dot{V} \leq -k_{e}\boldsymbol{e}^{\mathrm{T}}\boldsymbol{e} + \frac{\rho(\boldsymbol{x}, t, p^{*}) \|\boldsymbol{e}\| \varepsilon_{\rho}}{\rho(\boldsymbol{x}, t, k_{p}) \|\boldsymbol{e}\| + \varepsilon_{\rho}} - \frac{\rho(\boldsymbol{x}, t, k_{p}) \|\boldsymbol{e}\|^{2}}{\rho(\boldsymbol{x}, t, k_{p}) \|\boldsymbol{e}\| + \varepsilon_{\rho}} \left(\rho(\boldsymbol{x}, t, k_{p}) - \rho(\boldsymbol{x}, t, p^{*})\right)$$
(12)

**Remark 3.** Based on the fact that no bound of  $\theta_i$  is known a priori, the positive constant  $p^*$  defined in (6) is subsequently unknown which necessitates the following section of tuning mechanism design for the control gain  $k_p$  introduced in (8).

**Remark 4.** Based on the inequality of (12) and the fact that  $\rho(\boldsymbol{x}, t, k_p)$  increases monotonically with variable  $k_p$ , it is clear that if we choose a control gain large enough ensuring

$$k_p > p^* \tag{13}$$

then  $\dot{V}(t)$  can be upper bounded as

$$\dot{V} \le -k_e \boldsymbol{e}^{\mathrm{T}} \boldsymbol{e} + \varepsilon_{\rho} \tag{14}$$

which can be further rewritten as

$$V \le -2k_e V + \varepsilon_\rho \tag{15}$$

where (10) has been utilized. Based on (10) and (15), it is clear that the tracking error  $\mathbf{e}(t)$  will quickly go to a neighborhood around zero exponentially if the condition (13) is ensured. Unfortunately, as no upper bound of  $\theta_i$  is known, it is usually difficult to select an appropriate gain  $k_p$ . A common way is to pick a large enough  $k_p$  empirically to guarantee the condition of (13) with most confidence. However, this conservative strategy brings some drawbacks such

<sup>&</sup>lt;sup>1</sup>Hereon, for notation simplicity, we will write the function  $\rho(\mathbf{x}, t, p^*, \cdots, p^*)$  as  $\rho(\mathbf{x}, t, p^*)$ .

then increase it gradually based on the performance of the system. This motivates the subsequent tuning mechanism design for the control gain  $k_p$ .

### 3.3 Tuning mechanism design

As stated previously, a strategy is needed to tune the control gain  $k_p$  into a suitable range to achieve satisfactory performance. Besides, it is often a physical requirement that a controller in reality should be designed continuous. Based on this demand, we propose the following algorithm to tune the time derivative of the control gain, denoted as  $k_p$ , so that the control gain  $k_p$  will behave smoothly to help to construct a practical continuous controller.

In the algorithm, a criterion based on the performance of the Lyapunov function (10) is first required to judge whether  $k_p$  has reached an appropriate value. To do so, we solve the differential inequality of (15) to obtain

$$V(t) \le e^{-2k_e(t-t_s)}V(t_s) + \frac{\varepsilon_{\rho}}{2k_e} \left(1 - e^{-2k_e(t-t_s)}\right), \ t \ge t_s$$
(16)

with  $t_s \in \mathbf{R}^1$  represents a certain time from which the condition of (13) holds. Based on the discussion made on Remark 4, it is clear that the inequality of (16) can be utilized as an indication that  $k_p$  has reached a suitable value. According to this observation, we define an auxiliary signal  $\Delta_v(t) \in \mathbf{R}^1$  as follows

$$\Delta_{v}(t) = V(t) - \left[ e^{-2k_{e}(t-t_{s})} V(t_{s}) + \frac{\varepsilon_{\rho}}{2k_{e}} \left( 1 - e^{-2k_{e}(t-t_{s})} \right) \right]$$
(17)

Thus the tuning mechanism can be designed as follows: if  $\Delta_v(t) \leq 0$ , then  $k_p$  is set to zero and  $k_p$  remains unchanged; otherwise,  $k_p$  is set to be a positive value and  $k_p$  increases for some time.

Besides, to make the control gain  $k_p$  reach a suitable value quickly, it is desired to adjust its increasing rate based on the performance of V(t) as well. Motivated by that, we utilize the ratio between  $\Delta_v(t)$  and V(t) as a guide to adjust the increasing rate of  $k_p$ , and we then define the following auxiliary function  $S_v(t) \in \mathbf{R}^1$  to describe the performance of V(t):

$$S_v(t) = \frac{\Delta_v(t)}{V(t) + \varepsilon_v} \tag{18}$$

where  $\varepsilon_v \in \mathbf{R}^1$  denotes a positive constant introduced to avoid the possible singularity of (18) happened at the point of V(t) = 0. After substituting (17) into (18) and performing some mathematical calculation, the function  $S_v(t)$  can be rewritten as follows

$$S_{v}(t) = 1 - \frac{e^{-2k_{e}(t-ts)}V(t_{s}) + \frac{\varepsilon_{\rho}}{2k_{e}}\left(1 - e^{-2k_{e}(t-ts)}\right) + \varepsilon_{v}}{V(t) + \varepsilon_{v}}$$
(10)

Further, it is often beneficial in reality to constrain the increasing rate of  $k_p$  within a reasonable range. This fact

then inspires the introduction of the saturation-like function  $f_{sat}(S_v) \in \mathbf{R}^1$  to maintain the smoothness of the control input into the system:

$$f_{sat}(S_v) = \begin{cases} \alpha_2, & \text{if } S_v > \frac{\alpha_2}{k_v} \\ k_v S_v, & \text{if } \frac{\alpha_1}{k_v} < S_v \le \frac{\alpha_2}{k_v} \\ \alpha_1, & \text{otherwise} \end{cases}$$
(20)

where  $\alpha_1$  and  $\alpha_2 \in \mathbf{R}^+$  denote the lower and upper bounds of the increasing rate, respectively, and  $k_v \in \mathbf{R}^1$  is a weighting positive constant.

The tuning process is performed via a step-by-step way until  $k_p$  reaches an appropriate value. The scheme of the tuning algorithm can be summarized as follows. Initialization:

Set  $k_p = k_{p0}$ ,  $t_s = 0$ ,  $k_p = k_{pdot0}$ , where  $k_{p0}$  is usually chosen as a lower bound,  $k_{pdot0}$  is usually set to be relatively large, and  $t_s$  is utilized to memorize the time slot.

Switching logic:

At each time  $t > t_s + \tau$  with  $\tau \in \mathbf{R}^1$  being a positive constant to ensure that each switch of  $k_p$  will last for some time, if

$$\Delta_v(t) > 0 \tag{21}$$

then set

$$k_p = f_{sat}(S_v), \quad t_s = t$$

to increase the control gain  $k_p$ ; otherwise, set

 $\dot{k}_p = 0$ 

and  $k_p$  remains invariable.

**Remark 5.** Based on the function of (20), it is easy to see that when  $\Delta_v(t) > 0$ , the increasing rate of  $k_p$  satisfies the constraint of

$$\alpha_1 \le f_{sat}(S_v) \le \alpha_2 \tag{22}$$

where the bounding constants  $\alpha_1$  and  $\alpha_2$  are chosen according to the system property.

**Remark 6.** Different from the algorithm proposed in [12], the tuning mechanism developed in this paper obtains a continuous control gain  $k_p$  which brings much convenience when combining the mechanism into the corresponding control strategy.

## 4 Stability analysis

**Lemma 1.** The gain tuning mechanism designed previously for system (1) will trigger a finite number of switches (a switch means a change of  $k_p$  from  $f_{sat}(S_v)$  to 0, or vice versa) and the algorithm finally achieves a bounded gain  $k_p$ .

**Proof.** To prove Lemma 1, we first show that  $\dot{k}_p$  can only be switched a finite number of times. Without loss of generality, assume that after the *j*th switch,  $t_s = t_1$  and  $\dot{k}_p$  takes the value of  $f_{sat}(S_v)$  (Note that this is always possible unless no switch happens), and the next switch happens at time  $t_2$  which then sets  $\dot{k}_p$  to 0. Apparently, based on the switching logic, we know that

$$t_2 \ge t_1 + \tau$$

then it is straightforward to show that

$$k_p(t_2) \ge k_p(t_1) + \alpha_1 \tau \tag{23}$$

which demonstrates that over an interval with  $\dot{k}_p = f_{sat}(S_v)$ ,  $k_p$  increases by amount of at least  $\alpha_1 \tau$ . Define

$$N = \left[\frac{p^* - k_{p0}}{\alpha_1 \tau}\right] + 1 \tag{24}$$

where [x] denotes a function to get the maximum integer less than x. Based on this definition, we can then substitute (24) into (23) to show that after N intervals with  $\dot{k}_p = f_{sat}(S_v)$ , denoting the time as  $t_f$ , the control gain  $k_p(t_f)$ can be bounded as

$$k_p(t_f) \ge k_{p0} + N\alpha_1 \tau \ge p^* \tag{25}$$

Therefore, based on the analysis of Remark 4, we know that

$$\dot{V}(t) \leq -2k_e V(t) + \varepsilon_{\rho}, \text{ for } \forall t \geq t_f$$
(26)

where (10) and (14) have been utilized. Further, integrating both sides of (26) yields

$$V(t) \le e^{-2k_e(t-t_f)}V(t_f) + \frac{\varepsilon_{\rho}}{2k_e}(1 - e^{-2k_e(t-t_f)}), \text{ for } \forall t \ge t_f$$

Therefore, after time  $t_f$ , the condition of (21) will never hold, which implies that

$$k_p = 0$$
, for  $\forall t \ge t_f$ .

Hence, during the tuning process, there is at the most N intervals with  $\dot{k}_p = f_{sat}(S_v)$ ; however, as  $\dot{k}_p$  is only switched between 0 and  $f_{sat}(S_v)$ , then there is at the most 2N times of switching involved in the tuning process with the last switch to set  $\dot{k}_p$  as 0. Based on the previous facts, it is then clear that  $k_p$  remains bounded during the tuning process.

Now, we are able to present the main result.

**Theorem 1.** The proposed adaptive sliding mode control law of (8) ensures GUUB tracking result for the system errors in the sense that

$$\lim_{t \to \infty} \|\boldsymbol{e}(t)\| \le \sqrt{\frac{\varepsilon_{\rho}}{k_e}} \tag{27}$$

**Proof.** Based on the result of Lemma 1,  $\dot{k}_p$  can only be switched a finite number of times and  $\dot{k}_p$  equals 0 after the final switching. Let  $t^*$  be the time when the final switch occurs. Then  $t^* \in \mathcal{L}_{\infty}$ , and according to the switching logic, we must have

$$V(t) \le e^{-2k_e(t-t^*)}V(t^*) + \frac{\varepsilon_{\rho}}{2k_e}(1 - e^{-2k_e(t-t^*)}), \text{ for } \forall t \ge t^*$$
(28)

Otherwise, a further switching would occur which contradicts with the final switching fact. Hence, the result of (27) directly follows from (28). A standard signal chasing argument can then be employed to demonstrate that all the signals during closed-loop operation are bounded.

#### 5 Simulation results

To illustrate the performance of the controller (8), we simulate the following nonlinearly parameterized system<sup>2</sup> via Matlab's Simulink

$$\dot{x} = u + xe^{\theta_1 x^2 + \theta_2 x} \tag{29}$$

where  $\theta(t) = [\theta_1, \theta_2]^{\mathrm{T}}$  denotes varying parameter vector as

$$\theta_1(t) = 1 + 0.5 \sin(t) \theta_2(t) = 2 + \cos(2t)$$
(30)

The desired trajectory was selected in the following manner

$$x_d = 0.3\sin(5t) + 0.5\left(1 - e^{-t}\right) - 1 \tag{31}$$

with  $\dot{x}_d(t)$  calculated as

$$\dot{x}_d = 0.15\cos(5t) + 0.5e^{-t} \tag{32}$$

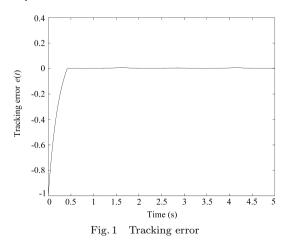
During the simulation, the variable step ode45 algorithm was adopted to solve the involved equation with the maximum step size set to 0.0005, the control and adaptation gains were chosen as follows

$$\alpha_1 = 0.5, \quad \alpha_2 = 5, \quad k_v = 100, \quad \tau = 0.01, \quad k_e = 5.$$
  
 $\epsilon_o = 0.002, \quad \epsilon_v = 0.00001$ 

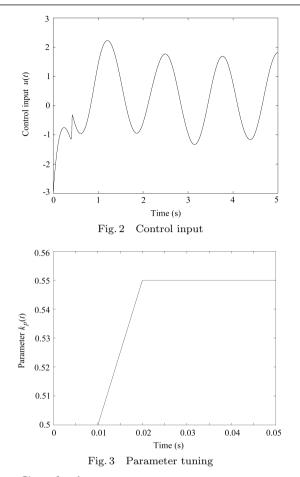
and  $k_p$  and x(t) were initialized as

$$k_{p0} = 0.5, k_{pdot0} = 0, x_0 = 0.$$

The simulation ran for 5 seconds. Fig. 1 illustrates the tracking error of the system state, and Fig. 2 demonstrates the control input while the parameter  $k_p$  tuning process is depicted in Fig. 3. As can be seen from the simulation results, after 2 switches, the system state x tracked well the desired trajectory  $x_d$  in less than 0.5 seconds with the control input u in a very reasonable range. For the parameter  $k_p$ , it converged to 0.55 within 0.02 seconds, thus the curve for the tuning process was only drawn for the time range [0, 0.05].



 $<sup>^2 {\</sup>rm For}$  simplicity, we utilized a SISO system to illustrate the performance of the control system.



## 6 Conclusion

A switching logic-based adaptive robust control is designed in this paper to address a class of NPS. Specifically, we construct a robust type controller whose gain is updated via a switching logic mechanism to achieve satisfactory performance. Based on a stability analysis that involves the application of Lyapunov techniques, it has been shown that the developed control law achieves GUUB tracking result for the system errors. Future work will focus on implementing the proposed control law experimentally. Future work will also target on designing more flexible tuning mechanism to achieve better control results.

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