Robustly Stable Control of Continuous-time Generalized Predictive Control Combined with QFT Based on GIMC Structure

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Abstract The continuous-time generalized predictive control (CGPC) and the quantitative feedback theory (QFT) are used together to control the plant with high uncertainty. QFT conquers the plant uncertainty and stabilizes the system in the inner loop without affecting the nominal performance based on the generalized internal model control structure. CGPC is used to obtain the necessary control performance in the outer loop. According to several given sufficient conditions, the available tuning parameters of CGPC are selected to make the system robustly stable. Finally, an example is given to show how to use this technique; and it is shown that this combined approach gets better performance than if only one of them is used.

Key words CGPC, QFT, GIMC, small gain theorems, robustly stable

1 Introduction

Long-range predictive control (LRPC) has received a great deal of attention for several decades[1~4]. This is because the strategy that uses the future behavior of the system output makes the LRPC have good robust properties. LRPC including generalized predictive control (GPC) traditionally has been developed in discrete time[5]. There are some problems with purely discrete-time methods, such as numerical sensitivity, sample rate selection, and so on[6]. Although continuous-time generalized predictive control (CGPC) has some properties similar to the discrete-time GPC, it avoids some problems in discrete-time theory and benefits from the advantages of continuous-time approach. Because of the model-process mismatch, CGPC developed for nominal model does not guarantee the stability of the real system. If the plant uncertainty is too high, CGPC based on a nominal model can not stabilize the system.

The quantitative feedback theory (QFT) is very powerful to control uncertain systems[7,8]. However, if some conditions are not quantitative when designing QFT controller, QFT can not assure the control performance and even can make the system unstable. Furthermore, QFT is difficult to use in some uncertain systems because it has to be redesigned if a little change is made in the control performance.

This article proposes a design approach to combine QFT and CGPC. The generalized internal model control GIMC[9] structure is used to overcome the conflict between performance and robustness in the traditional feedback framework. To guarantee the robust stability of the controller, a sufficient condition is given, which can be used to select parameters of CGPC. Finally, simulations are given to show the design process and the effect of the scheme.

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2 Brief review of CGPC

CGPC is based on the following continuous-time linear system model.

$$Y(s) = \frac{B(s)}{A(s)}U(s) + \frac{C(s)}{A(s)}V(s) \tag{1}$$

where $A(s)$, $B(s)$, and $C(s)$ are polynomials in Laplace operators. $Y(s)$, $U(s)$, and $V(s)$ are the system output, control input and disturbance input, respectively.

The minimization of the cost function of CGPC results in (refer to [4])

$$U(s) = g[W(s) - Y(s)] - \frac{G_0}{C}U(s) - \frac{F_0}{C}Y(s) \tag{2}$$

The offset problem can be handled in CGPC by modifying the system model as follows[6].

$$Y(s) = \frac{B(s)}{A(s)}U(s) + \frac{C(s)}{sA(s)}V(s) \tag{3}$$

3 Robustly stable CGPC based on QFT

The basic idea of this approach is that QFT is used to weaken the plant uncertainty and stabilize the system. And CGPC is used to control the system, which has been preparatorily controlled by QFT. Therefore, two closed loops are used in the designing approach. If the additive uncertainty G_a is taken into account and according to (2), then the control structure is shown in Fig. 1, where C_{qft} is the QFT closed loop controller, $G_a = GC_{qft}/(1+GC_{qft}) - G_nC_{qft}(1+G_nC_{qft})$ is the additive uncertainty after the control of C_{qft}, and G_n is the nominal plant for CGPC.

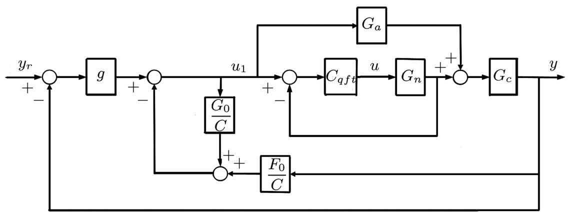

Fig. 1 The structure of CGPC based on QFT with uncertainty

From Fig. 1, the system transfer function is (4) (see next page). If the system transfer function without uncertainty is stable, according to the small gain theorem, the following sufficient condition can guarantee the system stability with uncertainty.

As can be seen from Fig.1 and (5), the CGPC control performance and robust stability not only concern with the nominal model but also are decided directly by the QFT controller, that is, CGPC control signal u_1 is always processed by QFT controller even when there is no uncertainty. The combined method would get better control performance if the robustness and performance controllers were designed separately. GIMC structure provides a good candidate for achieving this objective[9,10]. Assume that K_0 is a stabilizing controller for the nominal plant G_n, and assume G_n and K_0 have the following stable coprime factorizations.

$$K_0 = V^{-1}U, \quad G_n = M^{-1}N \tag{6}$$

$$\phi(s) = \frac{C(G_nC_{qft} + G_a + G_aG_nC_{qft})g}{(C + G_0)(1 + G_nC_{qft}) + G_nC_{qft}(F_0 + gC) + G_a(1 + G_nC_{qft})(F_0 + gC)} \tag{4}$$

$$|G_a(j\omega) < \left|\frac{C_{qft}(j\omega)G_n(j\omega)}{1 + C_{qft}(j\omega)G_n(j\omega)} + \frac{C(j\omega) + C_0(j\omega)}{F_0(j\omega) + gC(j\omega)}\right| \quad \text{for} \quad \omega \in [0, \infty) \tag{5}$$

The inner loop in Fig. 2 can be rearranged as Fig. 2 of [10] except some different indications: $U = \tilde{U}$, $V = \tilde{V}$, $G = \tilde{G}$, $M = \tilde{M}$, and $N = \tilde{N}$, where $C_{qft} = (V - QN)^{-1}(U + QM)$ for some Q such that $|V - QN| \neq 0$; the internal stability of the system is not changed. The character of this controller implementation is that the inner loop feedback signal f is always zero if there are no model uncertainties (*i.e.* $G = G_n$), external disturbances or faults. Then, the predictive model for CGPC is $G_nK_0/(1 + G_nK_0)$. The sufficient condition (5) becomes

$$|G_a(j\omega) < \left|\frac{G_n(j\omega)K_0(j\omega)}{1 + G_n(j\omega)K_0(j\omega)} + \frac{C(j\omega) + C_0(j\omega)}{F_0(j\omega) + gC(j\omega)}\right| \quad \text{for} \quad \omega \in [0, \infty) \tag{7}$$

where $G_a = L_{inner} - G_nK_0/(I + G_nK_0), L_{inner} = GV^{-1}Q(N - MG)U/[I + V^{-1}Q(MG - N) + GV^{-1}Q(N - MG)U]$.

As QFT controller is just used to weaken the plant uncertainty and stabilize the system, K_0 need not be complex. If both G_n and $G_n/(1+G_n)$ are stable, K_0 , N , and M can be set to 1, G_n , and 1, respectively.Then, the predictive model for CGPC is $G_n/(1 + G_n)$. The sufficient condition (7) becomes

$$|G_a(j\omega) < \left|\frac{G_n(j\omega)}{1 + G_n(j\omega)} + \frac{C(j\omega) + C_0(j\omega)}{F_0(j\omega) + gC(j\omega)}\right| \quad \text{for} \quad \omega \in [0, \infty) \tag{8}$$

where

$$G_a = \frac{G - G_n}{(G_n + 1)(GC_{qft} + 1)} \tag{9}$$

4 Example study

Consider the following typical two-rank uncertain plant with high parameter uncertainty[11].

$$G_1(s) = \frac{k}{As^2 + Bs + C} \tag{10}$$

with independent uncertainties: $k \in [1, 4], A \in [1, 4], B \in [-2, 2]$, and $C \in [1, 6.25]$.

Horowitz has given a QFT controller[11]. The closed loop controller is

$$C(s) = \frac{9.4 \times 8.5 \times 280^2(s + 14)(4s^2 + 2s + 6.25)}{14s(s + 8.5)(s^2 + 1.2 \times 280s + 280^2)} \tag{11}$$

And the prefilter is

$$F(s) = \frac{21}{(s + 3)(s + 7)} \tag{12}$$

In the following simulation, the parameters are chosen independently as $k = 1, 4, A = 1, 4, B = -2, 2$, and $C = 1, 6.25$. There are several available tuning parameters for CGPC: N_y, N_u, T_1, T_2, λ, $C(s)$, R_n/R_d, and so on. For simple reason, $C(s)$, R_n, and R_d are set to have the following forms: $C(s) = (s/\omega_c + 1)^{na}, R_n = 1$, and $R_d = r_2s + 1$, respectively, unless otherwise specified. It is easy to know that CGPC can not make the plant stable if only CGPC is used. However, it would become possible when the QFT conquered part of the plant uncertainty and stabilized the plant. Here the closed loop controller is chosen as (11). Before the selection of CGPC parameters, the nominal model must be chosen. It can be identified with the output and input data. Here, the nominal model is chosen as

$$G_n(s) = \frac{2.5}{2.5s^2 + s + 3.7} \tag{13}$$

both G_n and $G_n/(1 + G_n)$ are stable, K_0, N, and M can be set to 1, G_n, and 1, respectively.

To get the model-plant mismatch (MPM), many optimal methods can be used to seek the maximal magnitude of G_a at different frequency according to (9). After the unmodelled dynamic bound is found, the CGPC parameters can be chosen to find out whether they meet (8). If $N_y = 8$, $N_u = 2$, $T_1 = 0$, $T_2 = 0.4$, $\lambda = 0.001$, $r_2 = 0.8$, the stability bound is shown in Fig. 2 with different ω_c. As can be seen from Fig. 2, $\omega_c = 8$ makes the system unstable and $\omega_c = 0.8, 2$, and 4 can guarantee the robust stability of CGPC. As can be seen from Fig. 2, the parameters of $N_y = 8$, $N_u = 2$, $T_1 = 0$, $T_2 = 0.4$, $\lambda = 0.001$, $r_2 = 0.8$, and $\omega_c \in [0.8, 4]$ can guarantee the stability of the system. If $N_y = 8$, $N_u = 2$, $T_1 = 0$, $T_2 = 0.4$, $\lambda = 0.001$, the stability bound is shown in Fig. 3 with different $r_2 = 0.8, 5$, and $R_d = 1$. As can be seen from Fig. 3, the larger of r_2 in some range such as [0.8, 5], the more robustly stability of

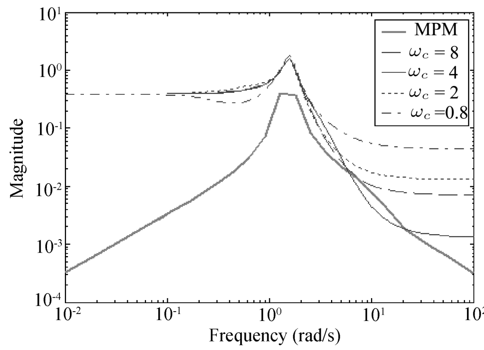

Fig. 2 Variation of the stability bound with ω_c

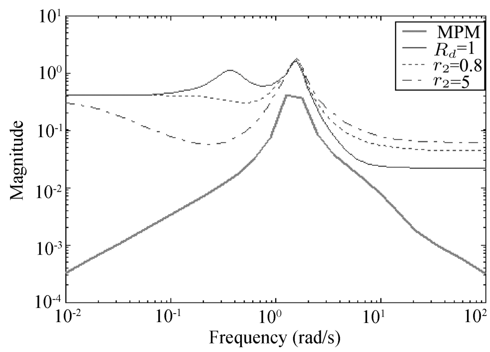

Fig. 3 Variation of the stability bound with R_n/R_d

the controller. Because r_2 reflects the bandwidth of reference model, it can be used to change the system tuning time. The larger of r_2, the longer of the tuning time.

If the plant is the nominal model, the control signal is shown in Fig.4, where (11) and (12) are used for only QFT controller, and (11) and CGPC, whose parameters are chosen as $N_y = 8$, $N_u = 2$, $T_1 = 0$, $T_2 = 0.4$, $\lambda = 0.001$, $\omega_c = 0.8$, and $r_2 = 0.8$, are used for the combined method. If the nominal model parameters are chosen as $k = 1$, $A = 4$, $B = -2$, and $C = 1$, the control signal is shown in Fig.5. As can be seen from Figs. 4 and 5, the control signal fluctuates too violently to be used in controlling the plan if only QFT controller is used. However, the control signal fluctuates very little because the objective of CGPC is to drive the predictive future output as close as possible to the future set-point subject to the input constraints.

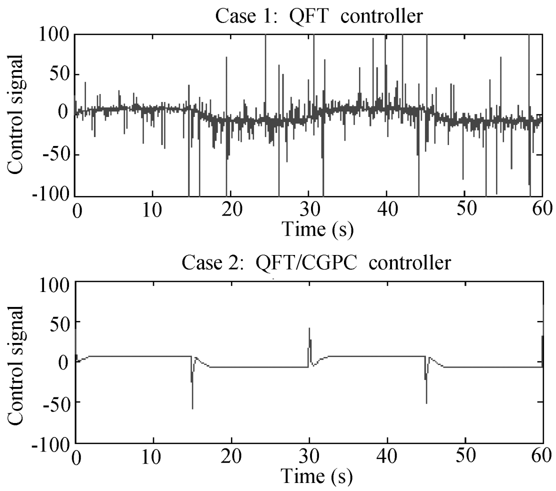

Fig. 4 Controller signal u for plant in nominal state

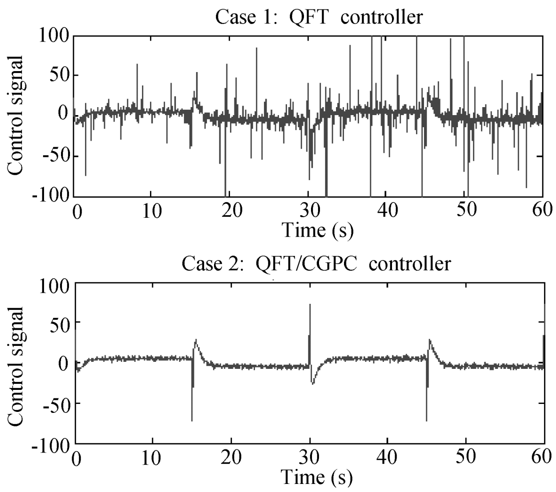

Fig. 5 Controller signal u for plant in boundary state

5 Conclusion

Based on GIMC structure, the combined CGPC/QFT approach is provided for high uncertain plant. This method has the advantages of both CGPC and QFT and makes the controller more flexible. The combined method greatly improves the system performance especially the nominal model.

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