A Dynamic Regulation and Scheduling Scheme for Formation Control

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Abstract This article is concerned with cooperative control problems in formation of mobile robots under the nonholonomic constraints that certain geometrical constraints are imposed on multiple mobile robots throughout their travel. For this purpose, a new method of motion control for formation is presented, which is based on the dynamic regulation and scheduling scheme. It is attractive for its adaptability to the formation structure and desired trajectory. The quality of formation keeping can be evaluated by the instantaneous errors of formation offset and spacing distance. Some kinematics laws are developed to regulate and maintain the formation shape. Simulation results and data analysis show the validity of the proposed approach for a group of robots.

Key words Robot formation, nonholonomic constraints, formation keeping, mobile robots

1 Introduction

Coordination of multiple robots has been considered a promising but complicated research area. Robot formation is one of the fundamental cooperative control problems, which demands that the group of mobile robots establish and maintain some predetermined geometrical shape during moving to their destination. The formation control techniques can be used in many tasks, such as object manipulation, space exploration, and search-rescue[1]. However, the nonholonomic characteristic of mobile robots increases the complexity of formation control. Many strategies have been proposed and tried to resolve the problem. Desai and Ostrowski[2,3] presented a leader-follower approach and derived the kinematics equations of formation to maintain the desired distances and relative orientations among the robots. Lewis[4] and Ren[5] studied the virtual structure approach, which controled the group by a virtual framework without leaders. Balch[6] and Lawton[7] proposed a behavior-based method, and the formation shape was maintained by many cooperative behaviors of the robot team running to the destination. Also, Belta[8] and Fujibayashi[9] studied the kinetic energy shaping method and virtual springs algorithm, respectively. Sanchez[10] used the sliding mode control method in formation, which was the extension of existing trajectory tracking of mobile robots. These formation methods can be classified as mathematic model[2,3,8∼10] and nonmathematic model[4∼7] methods. In addition, most of previous works considered issues in simple Cartesian coordinates only.

In this article, we pursue a new formation approach in which control inputs are easily computable and control laws are decentralized so as to accommodate a flexible and modular dynamic framework. There are three key features in our approach: 1) A special formation coordinate system is constructed, which especially suits the geometric information describing piecewise-smooth motion trajectory and easily copes with the transformation of relative positions. 2) The desired formation shape does not consider the orientation of the reference robot (leader) so as to avoid the motion fluctuations of followers. The orientations of individuals are only used to predict the motion trajectory of formation. 3) By the integration of distributed control laws, the adaptability can be achieved by altering the adjustable formation parameters, so that the formation structure and movement speed are adaptive to the curvature of the reference trajectory and formation errors.

The goal of this article is to present a formation keeping mechanism based on a dynamic regulation and scheduling scheme. Because the formation control model can hardly be accurate under the nonholonomic constraints of the mobile robots, the motion trajectories are approximated as some piecewise arcs, and the formation states of the robots are updated synchronously in instantaneous polar coordinates. The individual trajectory can be calculated independently by each robot, and the group is guaranteed to converge to its desired pose.

2 Dynamic formation framework

Differing from the single robot control, the multirobot formation control should make the robots′ poses not only meet the nonholonomic constraints but also satisfy the holistic-formation shape constraints. The path of the group is planned by a leader, which is a member in the formation in our distributed formation control method. Generally, path planning strategies can be based on probability[11], fuzzy control[12,13], reinforcement learning[14], *etc.* However, they are not the emphases of this paper. We focus on the strategies of maintaining the relative poses among the followers and their leader when the leader moves along its desired trajectory. The trajectory of the group may be offered in advance or be planned real time by the leader. The leader approaches to its desired trajectory with arc trajectory of curvature $k(t)$, so the real trajectory must be piecewise-smooth. Simultaneously, the followers adjust and maintain the formation shape with piecewise-arc trajectory.

Note that, approaching to its trajectory, leader will also consider the movement characters of the whole formation. So the speed of group may be immolated, however, formation accuracy can be improved. And the speed of any robot can be considered invariable in a short time interval because it satisfies the piecewise-constant condition.

In formation control, the relative poses of robots are the most important parameters. To express the relative poses parameters, a formation coordinate system needs to be constructed firstly (see Fig. 1).

Once the linear speed $v_p(t)$ and angular speed $\omega_p(t)$ of leader R_p are determined, the curvature center O of the current instantaneous trajectory can be affirmed. Formation coordinate system is thus created with O as its origin and the direction of polar axis in global coordinates as its direction of polar axis. Note that it is an instantaneous polar coordinate system and may vary with the movement of the group. The instantaneous trajectory of R_p is called

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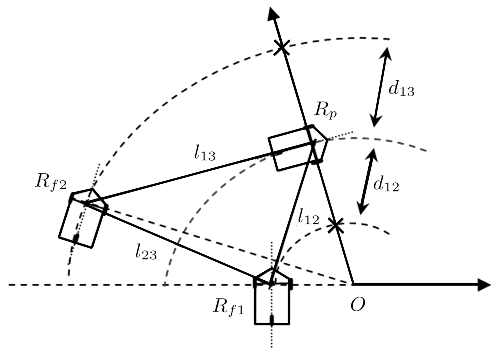

Fig. 1 Triangle formation in formation coordinate system

the reference trajectory, and the direction of trajectory is the reference direction in this formation coordinate system. Also, passing through its current position, each follower has a virtual trajectory that is parallel to the reference trajectory. This virtual trajectory is called its following trajectory.

Formation shape can be described by the polar coordinates. For example, in this case, the coordinate matrix of the triangle formation depicted in Fig. 1 can be defined as

$$M = \begin{bmatrix} v_p/\omega_p & \lambda_p - \text{sign}(\omega_p)\pi/2 \\ \rho_{f1} & \theta_{f1} \\ \rho_{f2} & \theta_{f2} \end{bmatrix} \tag{1}$$

where λ_p denotes the orientation of leader, $\rho_{fi}, \theta_{fi} (i = 1, 2)$ are polar coordinates of followers in formation coordinate system. Though the formation shape may be defined by M uniquely, it is not a vivid presentation for the relative poses. To visualize the formation shape, the matrix M can be transformed into two relative-distance matrixes $D_{m\times m} = \{d_{ij}\}_{i,j=1,\cdots,m}$ and $L_{m\times m} = \{l_{ij}\}_{i,j=1,\cdots,m}$. Here, D denotes an offset matrix and L is a spacing matrix. $d_{ij}(i, j = 1, \cdots m)$ is the offset from robot i to the following trajectory of robot j. l_{ij} is the spacing distance from robot i to robot j. Assume that $d_{ij} > 0$ as robot j at the right-side of the reference trajectory of robot i in the reference direction. On the contrary, $d_{ij} < 0$ as robot j at the left-side of that. Thus, different signs of offsets mean the follower′s different-side positions from the reference trajectory.

The headings of robots are given by the orientation vector $\boldsymbol{\lambda} = (\lambda_i)$ $(i = 1, \cdots m)$. The states of formation can thus be represented by offset matrix D , spacing matrix L, and orientation vector $\boldsymbol{\lambda}$. For a special regular triangle formation, its desired offset matrix and spacing matrix can be expressed as

$$Q(D) = \begin{bmatrix} 0 & d_+ & d_- \\ d_- & 0 & 2d_- \\ d_+ & 2d_+ & 0 \end{bmatrix},\ Q(L) = \begin{bmatrix} 0 & l_d & l_d \\ l_d & 0 & l_d^* \\ l_d & l_d^* & 0 \end{bmatrix}$$

where $d_+ = -d_- > 0$ equals the given desired offset from followers to their reference trajectory, l_d denotes the desired spacing distance between any two robots, and $l_d^* = l_d = 2d_+$. Obviously, $Q(D)$ and $Q(L)$ are symmetrical matrixes with main diagonal entries zero.

Especially, for the geometric formation shape, the desired offset d_q and the desired spacing distance l_d should be appropriate. A very small value of l_d may cause collision between robots, and a very big value of d_q may affect the moving speed of the whole group. So, in practical formation control, they have their limits, *viz.* $d_q \in [d_{\min}, d_{\max}]$, $l_d \in [l_{\min}, l_{\max}]$. Furthermore, the offset is relevant to the leader′s moving characters, and the variable desired d is helpful to a more steady and rapid formation. But the offset can be random if the formation goes a straight line. In general, the proper offset should be convenient to the relative observation among individual robots, also the configuration, capability, and task demand of formation should be considered. We may calculate the offset error and spacing distance error as follows.

$$e(d) = |d - d_q|, \quad e(l) = |l - l_d| \tag{2}$$

Once the desired matrixes $Q(D)$ and $Q(L)$ have been determined, the whole formation error $e(F)$ can be evaluated by the geometrical mean value as follows.

$$\frac{1}{2}\sqrt{(\sum_{i>j} |(D - Q(D))_{ij}|)^2 + (\sum_{i>j} |(L - Q(L))_{ij}|)^2} \tag{3}$$

The formation error $e(F)$ should be reduced to its permitted error band, so an adaptive formation control framework should be constructed to adjust the formation network-topological architecture. In the framework, the offset and spacing distance errors are restricted by control law OTR (Offset regulation) and SDR (Spacing distance regulation), respectively.

The dynamic formation framework is shown in Fig. 2. e_1 and e_2 are the given bounds of comparison error $k_l e(l) - k_d e(d)$ and the whole formation error $e(F)$, respectively, where k_l, k_d are comparison coefficients of $e(l)$ and $e(d)$. R is the control object, *viz.* formation system, and S is the states of robots in formation. c_1 denotes that the comparison result is greater than or equal to zero; on the contrary, c_2 denotes that is less than zero.

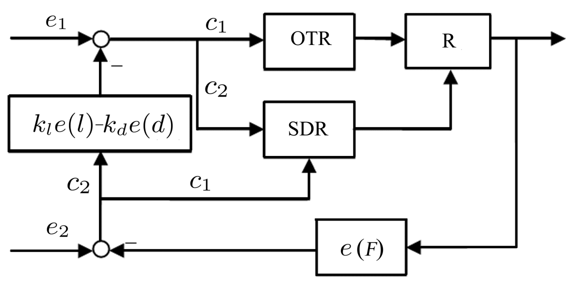

Fig. 2 Abstract view of dynamic formation framework

The formation framework shows the regulating process of formation errors. When formation error $e(F)$ is greater than the given error bound e_2, offset error $e(d)$ and spacing distance error $e(l)$ will be compared, thus, an appropriate formation control laws will be selected between the OTR and SDR in different time-step. Until $e(F)$ is less than e_2, the SDR is used to maintain the formation shape. It allows us to specifically regulate the formation′s shape adaptively to accommodate the nonholonomic constraint of robots.

3 Formation control laws

The formation is constructed by many mobile robots, which will be regarded as an integral body. But any robot must have its own role, either leader or follower, in formation. The decentralized formation control laws described in this article are used by each robot to keep the formation while driving the group to a desired destination. Thus, each robot should consider its own motion constraints and adopt the proper control laws to the role.

The desired trajectory of leader is relevant to the circumstance, and the motion state of leader is determined by the curvature of trajectory. Furthermore, the follower must adapt to the state of its leader in real time and adjust the topological structure of formation. Thus, a formation control scheme can be realized.

3.1 Formation reference adjustment

In general, a single robot in the formation cannot be operated at a random speed. Once the trajectory of leader is calculated either before or online, it can be approximated by many segments of arcs. For a known trajectory, proper velocity of leader is helpful to the stability of formation keeping.

Assuming that the discrete time step is T, the periodic curvature of the desired trajectory of leader may be denoted by a piecewise-constant $k_p(t)$, $(t = T, 2T\cdots)$. Then, the trajectory of the leader is known as $f(\boldsymbol{k}, t) \in SE(2)$, here, $\boldsymbol{k} = \{k_i\}_{i=1,2\cdots n}$. Theoretically, the piecewise-smooth trajectory can approach to any kind of straight path or arc path. The desired linear speed of the leader $\hat{v}_p(t)$ is defined as

$$\hat{v}_p(t) = \frac{2v_{p\max}}{1+e^{|k_p(t)|}} \tag{4}$$

where the numerator $v_{p\max}$ is the feasible maximum linear speed of leader. Also, it can be seen that it is an approximate inversely proportional relationship between $\hat{v}_p(t)$ and $k_p(t)$. In addition, to avoid the jump reaction of speed caused by discontinuity of curvature, the real speed is defined as

$$v_p(t) = v_o + \Delta v_p \alpha_i^2 (4 - 3\alpha_i), \quad \omega_p(t) = k_p(t) v_p(t) \tag{5}$$

where v_o is the original linear speed of the leader, $\Delta v_p = \hat{v}_p - v_o$ and $\alpha_i = m_i/m$ $(m_i = 1, 2, \cdots m)$, is a smoothing constant.

Thus, the leader may track the trajectory with the control input $u_p(v_p(t), \omega_p(t))$. So that it will move quickly at a straight path and slow down during turning. Note that this motion characteristic is just conformable to the general practical case.

For any determinate formation, the valid curvature of the reference trajectory $f(\boldsymbol{k}, t)$ has its maximum $k_{\max}$ in formation coordinate system. That means the reference trajectory whose curvature satisfies $k_p(t) > k_{\max}$ should be synthesized. So a curve fitting method is used to minimize the curvature $k_p(t)$ in this section. The curve fitting process fits equations of approximating curves to the sampling data on $f(\boldsymbol{k}, t)$. The number N of sampling data satisfies

$$N \geq 5l_k / (\pi \frac{1}{n} \sum_{i=1}^{n} 1/k_i) \tag{6}$$

where l_k is the length of $f(\boldsymbol{k}, t)$ to be fitted. Note that under the dynamic formation framework, the type of fitting curves $g(\boldsymbol{k}, t)$ should be circular arc whose curvatures are less than $k_{\max}$. Furthermore, the first and last sampling points must be on the fitting curves to assure the continuity of the valid reference trajectory. Nevertheless, for a given data set, the fitting curves of the given type are generally not unique. Thus, a curve with a minimal deviation from the sampling data is desired. This best-fitting curve can be obtained by the method of the least squares, *i.e.*,

$$\min_{k \in [0, 1/d]} ||g(\boldsymbol{k}, t) - f(\boldsymbol{k}, t)||_2 \tag{7}$$

Assume the trajectory $f(\boldsymbol{k}, t)$ is fitted by h segments of circular arcs, here h is called fitting particle size. Then, $||g(\boldsymbol{k}, t) - f(\boldsymbol{k}, t)||_2$ can be defined as

$$J(\boldsymbol{k}, t) = \sqrt{\sum_{i=1}^{h} \sum_{j=1}^{n} (g(\boldsymbol{k}, t)_{ij} - f(\boldsymbol{k}, t)_{ij})^2} \tag{8}$$

where $J(\boldsymbol{k}, t)$ is the 2-norm of the polar radiuses$'$ distances between the planned trajectory and the fitting trajectory at the same polar angles. So the fitting curvature that minimizes $J(\boldsymbol{k}, t)$ is the desired fitting curvature, which is the valid reference trajectory of leader in the formation coordinate system.

3.2 Formation state update

In this article, the instantaneous formation coordinates are suitable to depict the motion state with dynamic formation framework. But the formation state in different formation coordinate system should be updated in real-time, which is relevant to the renovation of formation coordinate system. Note that the formation coordinate system depends on not only the current pose of leader but also the magnitude of the control speed $\boldsymbol{u}_p(v_p(t), \omega_p(t))$. However, no matter how $\boldsymbol{u}_p$ changes, the new origin of formation coordinate system will not deviate from current polar radius of leader or its extension line at the switching point of curvature k_p. So the desired piecewise-smooth trajectory is continuous all the while.

Let the discrete-time dynamic model of the formation system be $\dot{\boldsymbol{z}} = f(\boldsymbol{z}, \boldsymbol{u}_f(t))$, where $f(\cdot)$ is nonlinear system function, $\boldsymbol{z}$ is the system state, and $\boldsymbol{u}_f(t) = [v_f(t), \omega_f(t)]^{\mathrm{T}}$ is the system input. The curvature $k_p(t)$ of the leader varies according to the path planning method. Simultaneously, the origin of the formation coordinate system transfers from $\tilde{O}$ to O. Thus, there must exist a nonlinear time-varying coordinates transformation $T : \boldsymbol{z} = T(\tilde{\boldsymbol{z}}, t)$, where $\boldsymbol{z} = (\rho, \theta)$ and $\tilde{\boldsymbol{z}} = (\tilde{\rho}, \tilde{\theta})$ are generalized polar coordinates in the new and original coordinate system, respectively. So, the polar transformation satisfies

$$\left\| O - \tilde{O} \right\| = |\frac{\tilde{k}_p - \kappa k_p}{k_p \tilde{k}_p}| \tag{9}$$

$$\theta_{O\tilde{O}} = \theta_c - \frac{1}{2}\mu(1-\kappa)\pi \tag{10}$$

where κ, μ are two-valued variables and satisfy

$$\kappa = \begin{cases} 1 & \tilde{\omega}_p \omega_p \geq 0 \\ -1 & \tilde{\omega}_p \omega_p < 0 \end{cases}, \quad \mu = \begin{cases} 1 & \omega_p \leq 0 \\ -1 & \omega_p > 0 \end{cases}$$

Formation coordinate system transforms with the movement of the leader, and formation state is updated synchronously. Assume $\tilde{\boldsymbol{s}} = (\tilde{\rho}_p, \tilde{\rho}_d, \tilde{\rho}_f, \tilde{\theta}_p, \tilde{\theta}_d, \tilde{\theta}_f)^{\mathrm{T}}$ is the formation state vector in original coordinate system $\tilde{O}$ and its updated vector is $\boldsymbol{s} = (\rho_p, \rho_d, \rho_f, \theta_p, \theta_d, \theta_f)^{\mathrm{T}}$ in new formation coordinate system O. Here, ρ_p, θ_p belong to the current position $\boldsymbol{z}_p$ of leader, and their updating equations can be summarized as

$$\begin{bmatrix} \rho_p \\ \theta_p \end{bmatrix} = \begin{bmatrix} 1/|k_p| \\ \tilde{\theta}_p - 0.5\mu(1-\kappa)\pi \end{bmatrix} \tag{11}$$

In addition, $\boldsymbol{z}_d(\rho_d, \theta_d)$ is the desired position of follower and the new desired position state may be calculated according to following equation,

$$\begin{bmatrix} \rho_d \\ \theta_d \end{bmatrix} = \begin{bmatrix} \sqrt{\tilde{\rho}_d^2 + \Delta\rho^2 - 2\tilde{\rho}_d \cdot |\Delta\rho| \cos(\tilde{\theta}_p - \tilde{\theta}_d + \mu\sigma\pi)} \\ \theta_p + \mu \arccos((\rho_p^2 + \rho_d^2 - l_d^2)/2\rho_p\rho_d) \end{bmatrix} \tag{12}$$

where $\Delta\rho = \tilde{\rho}_p - \kappa\rho_p$, and σ is a two-value variable and satisfies

$$\sigma = \begin{cases} 1 & \tilde{\omega}_p\omega_p > 0 \cap \rho_p \geq \tilde{\rho}_p \\ 0 & \text{other} \end{cases}$$

Simultaneously, the current position $\boldsymbol{z}_f(\rho_f, \theta_f)$ of follower can be updated as

$$\begin{bmatrix} \rho_f \\ \theta_f \end{bmatrix} = \begin{bmatrix} \sqrt{\tilde{\rho}_f^2 + \Delta\rho^2 - 2\tilde{\rho}_f \cdot |\Delta\rho| \cos(\tilde{\theta}_f - \tilde{\theta}_p - \mu\sigma\pi)} \\ \theta_p - \text{asin}(\tilde{\rho}_f \sin(\tilde{\theta}_f - \tilde{\theta}_p - 0.5(\kappa+1)\pi)/\rho_f) \end{bmatrix} \tag{13}$$

As can be seen, the updating equations of formation state are different if the magnitude of $k_p(t)$ alters during the traveling of the robot group, they are shown by the control parameters κ, μ, and σ. Once the formation state $\boldsymbol{s} = (\rho_p, \rho_d, \rho_f, \theta_p, \theta_d, \theta_f)^{\mathrm{T}}$ has been updated, the formation shape regulation may be created to eliminate the error $e(F)$.

3.3 Formation shape regulations

The geometry-based formation control approach is comprehensible, but the main difficulty is to deal with the speed uncertainty of the leader.

The offset of the follower is one of the key factors to show the formation structure in formation coordinates. In general, the initial poses of individual robots are random, and regulating the initial headings firstly is necessary to simplify the offset control rules. In other words, all the initial headings should be turned to the desired formation orientations. Note that the error of offset should be eliminated in a progressive process.

Fig. 3 interprets the quantities of formation shape regulation graphically. $\tilde{O}_p$ denotes the origin of the formation coordinate system, and O_f is the origin of the follower polar coordinates. P denotes the position of the leader, D is the desired following position, and F is that of the follower at the present time. d is the original offset, and d_q is the desired offset. G is the real regulated target of the follower. Offset d can be deduced by

$$d = (\rho_f - \rho_p)\text{sign}(\omega_p) \tag{14}$$

To adjust the offset, the formation control system should satisfy the differential equations

$$\begin{bmatrix} \dot{\omega} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} k_1 & k_2 \end{bmatrix} \begin{bmatrix} 1 \\ -\text{sign}(\omega_p) \end{bmatrix} (d - d_q) \tag{15}$$

where k_1, k_2 are the user-selected controller gains and $0 \leq k_1, k_2 \leq 1$. sign$(\cdot)$ is a symbolic function. d and d_q are the current offset and its desired value, respectively. (15) leads to dynamic equations of the form

$$\dot{\omega} = \omega_f - \omega_p, \quad \dot{h} = h - \tilde{h} \tag{16}$$

where $\dot{\omega}$ is substituted by the difference of the angular speed between follower and leader, and $\dot{h}$ is substituted by the difference of polar radius between coordinate systems $\tilde{O}_p$ and O_f. Thus, the following results can be derived

$$\omega_f = k_1(d - d_q) + \omega_p$$
$$v_f = (-k_2\text{sign}(\omega_p)(d - d_q) + R_n)(k_1(d - d_q) + \omega_p)$$

Note that the followers should not go ahead of their leader in our scheme, so the following inequality constraint must be satisfied:

$$(\theta_p - \hat{\theta}_f)\text{sign}(\omega_p) \geq 0 \tag{17}$$

where $\hat{\theta}_f$ is the polar angle of the real target G in coordinates $\tilde{O}_p$. Finally, the control inputs of the followers can be obtained from the following offset regulation OTR:

$$\begin{bmatrix} \omega_f \\ v_f \end{bmatrix} = \begin{bmatrix} k_1(d - d_q) + \omega_p \\ -k_1k_2\text{sign}(\omega_p)(d - d_q)^2 + k_1(d - d_q)R_n + \\ \omega_pR_n - k_2\text{sign}(\omega_p)(d - d_q)\omega_p \end{bmatrix} \tag{18}$$

where $k_1, k_2 \in [0, 1]$ are control gains, which indicate the rate of convergence in offset regulation.

As can be seen in Fig. 3, spacing distance l between two robot locations P and F is regulated to its desired distance l_q. However, the adjustable process of spacing distance should be progressive in our approach, which can be realized by the following recursion.

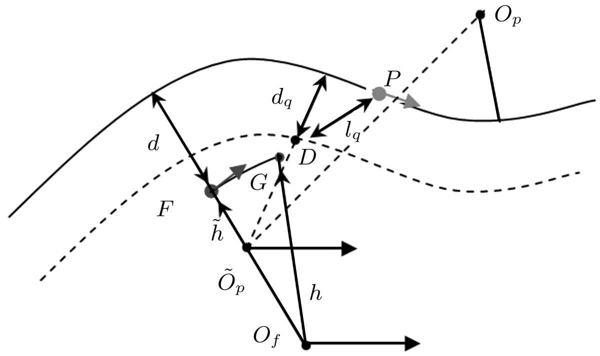

Fig. 3 Graphical depiction of formation shape regulation

$$l_d = (r_1l_d + r_2l_q)/\sum_{i=1}^{2} r_i \tag{19}$$

where r_1 and r_2 are the adjustable spacing distance coefficients. Thus, desired following angle θ_d can be updated by

$$\theta_d = \theta_p + \mu \arccos((\rho_p^2 + \rho_d^2 - l_d^2)/2\rho_p\rho_d) \tag{20}$$

Under the dynamic formation framework, the controlled quantities of the leader are piecewise constants. Thus, the desired value of following angle θ_d must satisfy $\hat{\theta}_d(t) =$

$\theta_d(t)+\omega_p(t)\,t$. As a result, the spacing distance regulation SDR can be summarized as

$$\begin{bmatrix} \omega_f \\ v_f \end{bmatrix} = \frac{1}{T} \begin{bmatrix} (\hat{\theta}_d - \lambda_f - \frac{\pi}{2} + \eta\pi) \\ \rho_f \cdot (\hat{\theta}_d - \lambda_f - \frac{\pi}{2} + \eta\pi)\tan(\hat{\theta}_d - \theta_f) \end{bmatrix} \tag{21}$$

where $\eta = 0$ if $\omega_p \leq 0$, and $\eta = -1$ if $\omega_p > 0$. Thus, the states updating and SDR may adjust the formation shape and keep it. In addition, the speed and reachable location of the robot are limited, so we must assure the poses of robot controlled by input $u_f(v_f(t), \omega_f(t))$ satisfy the corresponding nonholonomic constraints synchronously.

State updating (11) and (13) and formation shape regulation (18) and (21) are the main conclusive control laws. Note that the adaptability to any special formation shape can be shown from the flexible coefficients k_i and r_i, and meet the different initial formation errors and the requirement of different control precision.

4 Simulation results

To validate the formation control based on the dynamic formation framework, this section presents the simulation results. Assume that each robot travels under the nonholonomic constraint with a maximum velocity of 1m/s. the distance between two driving wheels is 0.4m, and control period is 1s.

Many experiments have been done in our mobile robots simulation platform. Here, a typical triangle formation manipulation is given. Two robots follow the leader robot in a distributed control model. In initial formation coordinate system, the leader is positioned at the initial location (12, 0.5π, 0), it moves to its destination along the given curvilinear trajectory $g(t) \in SE(2)$. Simultaneously, two followers start off from (7, 0.6π, 0.8π) and (22, 0.55π, 0.5π), respectively. The desired offset between followers and leader trajectory is 2m and -2m, and the desired spacing distances are 4m. Simulation parameters are shown in Table 1.

Table 1 Parameters for Simulation

V	a	T	$l_1q(l_2q)$	k_l	k_d	d_1q	d_2q
1(m/s)	0.4(m)	1(s)	4(m)	2	3	2	-2

Figs. 4∼8 show this maneuver when the leader runs along the curvilinear path. Note that the control parameters k_i and r_i $(i = 1, 2)$ should be selected according to the current formation errors and motion capability of robots.

Figs. 4 and 5 demonstrate the convergent curves of key formation parameters $d_i(i = 1, 2)$ to their desired values. Here, the dotted lines show the varied values when $k_i(i = 1, 2)$ are constant, where the continuous lines show the regulated values when $k_i(i = 1, 2)$ are functions of current error $e(d_i)$. It can be seen that d_1 can be regulated to the approximate value of desired distance with $k_1 = 0.2e^{-0.8e(d_1)}$, $k_2 = 0.6e^{0.7e(d_1)}$ when $t = 48$s that is earlier than the regulation process with $k_1 = 0.02$, $k_2 = 0.6$ in about 16 seconds. Similarly, it is almost the same to the regulation of d_2. Also, the errors of offsets d_1 and d_2 decline about 3m and 7m independently. Thus, if k_1 is a constant, though the offset can converge to its desired value, the converging speed is slow because k_1 must be small enough to adapt to the limitation of robot movement speed and the magnitude of initial formation errors.

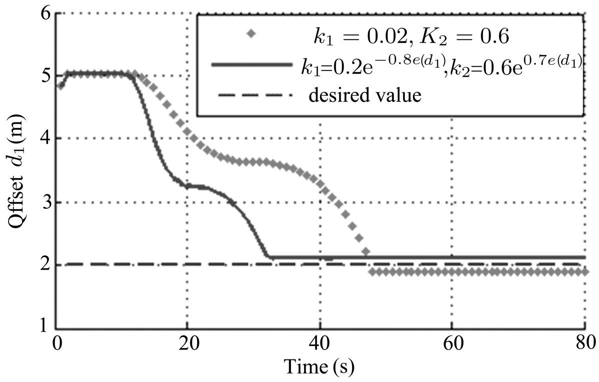

Fig. 4 Comparison of offset d_1 with different $k_i(i = 1, 2)$

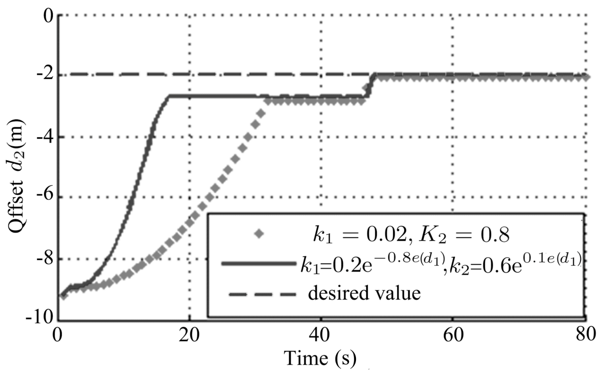

Fig. 5 Comparison of offset d_2 with different $k_i(i = 1, 2)$

However, with the decreasing of the offset error, the small constant k_1 will also limit the regulating speed. In consequence, k_1 should be varied with the offset error $e(d_i)$. k_2 is a direct proportion function of offset $e(d_i)$. Thus, if $e(d_i)$ is increasing, regulation OTR will accelerate the converging with the increasing parameter k_2. However, a very big k_2 is not helpful to the regulation of spacing distance, so a trade-off value should be considered. Generally, k_1 and k_2 satisfy the ranges $k_1 \in [0, 1]$ and $k_2 \in [0, 2]$, and they enlarge or reduce the offset errors. The variations of parameter $k_i(i = 1, 2)$ ensure not only that the formation errors are decreased gradually but also the control inputs are within the practicable ranges of real robots all the time. With the regulation of offset, the spacing errors of robots gradually become the main errors in formation control. Then, the spacing distances l_1 and l_2 should be adjusted to their desired distances. However, the offset may be adjusted in another time according to the dynamic formation framework when the formation error is less than the given limit (see Fig. 5, $t = 48$s). Thus, the formation errors asymptotically converge to zero.

Figs. 6 and 7 show that the regulation speed of spacing distance can be controlled by the given parameters $r_1 : r_2$. The spacing distance l_1 increases slowly at the beginning when the vehicles are controlled by OTR firstly under the condition $k_l e(l) < k_d e(d)$ until the time $t = 40$s ($t = 30$s for l_2). After that, it decreases quickly as controlled by the regulation SDR. Here, the dotted line shows the varied values of spacing distance when $r_1 : r_2 = 8 : 1$, and it is a gentle regulation process and the control speed maximum values are small. While the regulation time can be decreased about 30s when $r_1 : r_2$ is decreased to $1 : 3$. Here, the resultant control inputs can always remain bounded at a reasonable ratio. Thus, spacing distance l_i is regulated by the regulation SDR, and the regulation speed can be varied, *i.e.*, the regulation speed increases with the decreasing of the control ratio $r_1 : r_2$. However, the regulation speed can not be infinity. A reasonable ratio should be selected according to the motion capability of the real vehicles. Consequently, the flexibility of controlled ratio parameters is especially applicable to the formation control of heterogeneous robots.

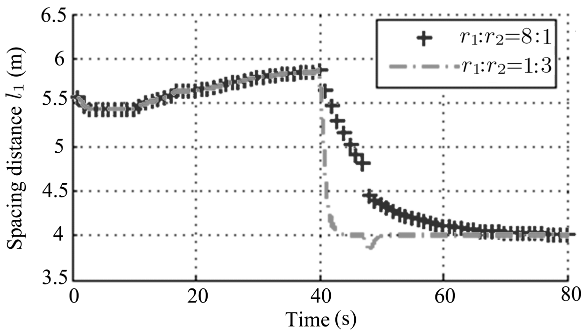

Fig. 6 Comparison of l_1 with different ratio $r_1 : r_2$

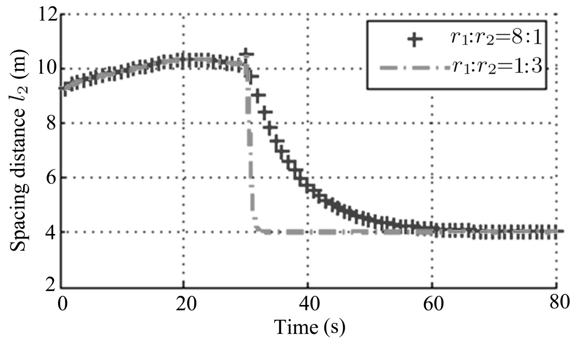

Fig. 7 Comparison of l_2 with different ratio $r_1 : r_2$

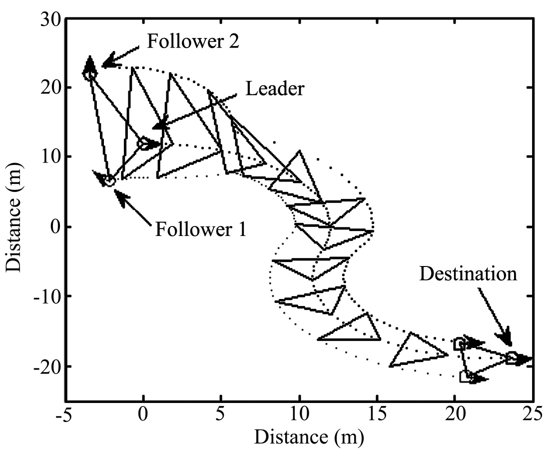

Fig. 8 Path of a typical triangle formation maneuver

With simulation parameters defined in Table 1, initial poses and curvilinear trajectory $g(t)$ mentioned above, the variable gain k_i is the same as the parameter selected in Figs. 4 and 5, and $r_1 : r_2 = 2 : 1$, a visual representation of geometric positions is showed in Fig. 8. The initial and final locations of the mobile robots are depicted by the symbols $\circ$, and the movement trajectories are depicted by a series of points, respectively. In addition, many synchronized positions of the three individual robots are connected each other, they illustrate the size variation of the formation shape during the traveling of the robots.

Consider the case of a line formation approaching a narrow passageway through obstacles. In these experiments, the leader performs an exploratory mission while the formation changes in a decentralized fashion as required by the environment. In the presence of obstacles, the pair-wise robots should switch to appropriate mode to negotiate the obstacles while going to their destination.

Shown in Fig. 9 are the initial and the final configurations of these robots and their paths. In line formation control, the offset d should tend to the spacing distance l. The left scenario is almost the same as the right one, but the passageway width is smaller than the latter. In the left, the formation shape should be changed in order to squeeze through a narrow passage. Here, the formation change is performed by changing only the shape variables and not the planned trajectories. This strategy also shows the flexibility of formation control under our dynamic formation framework. As it can be seen, the group avoids the static obstacle at district a, and the offset reduces to 1m from 5m at district b. Note that the velocity of follower increases suddenly at district b, which is caused by the changing of desired shape variables and the switching of the control regulations, as can be seen in Fig. 10. However, there is no collision among the robots during this process of cooperative collision avoidance. Having crossed the narrow passageway, the follower recovers its original relative position to the leader rapidly.

The right scenario in Fig. 9 shows the case that the formation size is smaller than the gap between obstacles, thus, the formation parameters can be invariable with a rigid formation shape. After the robots avoid the first obstacles at district a, the leader detects the passage and finds that the passage width is more than 5m. So it triggers another process of holistic collision avoidance. This case is different from the left scenario. The pair-wise robots run around the obstacles with the rigid formation shape and arrive at the destination.

Fig. 10 shows that the control regulation switches internally between OTR and SDR depending on the dynamic formation framework of the formation. The upper and below figures are corresponding to the left and right scenarios in Fig. 9, respectively. Firstly, OTR operates until the time t =35s and decreases the big initial errors of offset $e(d)$. SDR is then the next control regulation. However, the upper figure shows that OTR and SDR will operate alternately while the formation shape changes to pass the narrow passage (100s $<t<$ 138s). Finally, the mobile robot group adapts to the variation of the offset and spacing distance between robots and it achieves its task.

The simulation results indicate that the formation shape can be maintained steadily and the convergence of formation error $e(F)$ shows the validity of the proposed formation approach. Especially, our approach periodically considers only short time intervals when computing the next steering command to avoid the enormous complexity of the general motion planning. In practical applications, the control parameters can be selected adaptively.

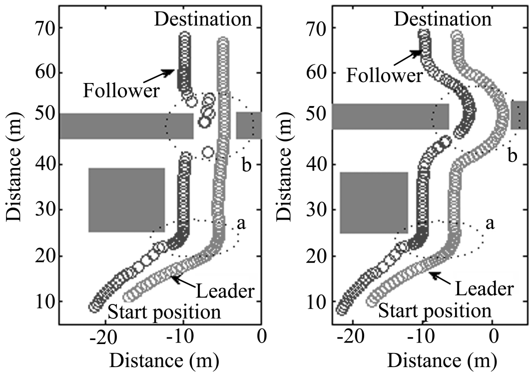

Fig. 9 Path of line formation maneuver

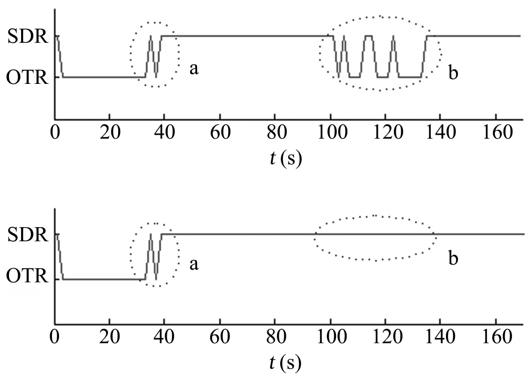

Fig. 10 Switching of control regulation

5 Conclusion

A formation control method based on dynamic formation framework is developed in this article. The main contributions includes formation framework describing, formation error analyzing, states updating and control laws constructed in an instantaneous formation coordinate system. Especially, the speed of the leader is constrained by the curvature of its trajectory, and creating of the formation coordinates is helpful to update the state of the group and construct the formation control laws. Because the formation error can be calculated from the offset and spacing distance matrixes periodically, an adaptive regulating and maintaining mechanism of the formation shape is configurable. Deciding the optimal formation shape for a given environment and implementing coordination tasks with more robots are also important directions in our future work.

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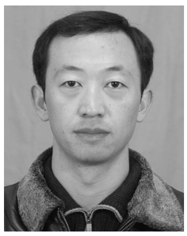

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