Model Transformation and Optimization of the Olympics Scheduling Problem

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Abstract The Olympics scheduling problem is modeled as constraint satisfaction problem, which is transformed into a constrained optimization problem by softening the time constraints of the final matches. A decomposition methodology based on Lagrangian relaxation is presented for the constrained optimization problem. For the dual problem optimization the sub-gradient projection method with variable diameter is studied. The method can converge to the globally optimal solutions and the efficiency is given. Numerical results show that the methods are efficient and the phase transition domain can be recognized by the algorithm

Key words Scheduling, Olympics, Lagrangian relaxation, model transformation

1 Introduction

Since 1980s, modern sports activities have been more and more socialized, specialized, entertainized, commercialized and informatized. As time flies, new technologies play more and more important roles in sports organization. And "High-tech Olympics" has been one of the three themes of "New Beijing, Great Olympics". Research progress on "High-tech Olympics" will benefit the society and economy development.

Olympics organization involves quite many sorts of resources, athletes, referees and gymnasia, as long with various constraints. So the Olympics scheduling is an arduous task, which is one of the important branches of the "High-tech Olympics" research. Sports scheduling research originated in the 1980s. Nemhauser and Trick[1], Henz *et al.*[2], Schaerf[3], Regin[4], McAloon, Tretkoff and Wetzel[5], Schönberger, Mattfeld and Kopfer[6] have explored the scheduling problem. But they focused on the single-event tournament scheduling problem. Research on multi-event and large scaled scheduling research has not been reported. Andreu[7] has studied the DSS for scheduling the 1992 Olympic Games, but the algorithm was not studied.

The Olympics scheduling problem is a timetabling problem. The timetabling problem is a combination problem in nature, and has been proven NP-hard. A wide variety of approaches to timetabling problems have been described in the literature and tested on real data. They can be roughly divided into four types[8]: 1) sequential methods, 2) cluster methods, 3) constraint-based methods, and 4) meta-heuristic methods. Sequential methods and cluster methods present policies that obtain approximate solutions. Constraint-based methods are back-tracking methods in nature. Meta-heuristic methods can provide good solutions, but they are computational consuming.

A novel method might be found by transforming the problem into a new model. In this paper, the time constraints of the final matches are softened, and then the Olympics scheduling problem is transformed into a constrained optimization problem. Although the events are coupled by the field constraints, the constrained optimization problem can be decomposed into single-event subproblems by relaxing the field constraints. Lagrangian relaxation provides a methodology to realize the decomposition. Since 1990s, the authors such as Luh, *et al.* have studied the production scheduling problem with Lagrangian relaxation[9~14]. They relaxed the capacity constraints to decompose the production scheduling problem into job subproblems. The difficulty of Lagrangian relaxation is the optimization of the dual problem. In 1969, Polyak[15] presented a subgradient method to the convex problem, but the convergence of the method depended on the estimated optimal value. In 1996, Kiwiel[16,17] studied the subgradient projection methods for convex optimization, which converged without the estimated optimal value, but the diameter of the variable domain was imported. Kim[18] had presented a variable target value subgradient method in 1991 before Kiwiel, which was nothing but a special case of Kiwiel′s method.

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2 Temporal interval model language

The Olympics system involves time-distribution constraints, field constraints, person constraints and time window constraints. A temporal interval model language is needed as the interface to deal with the constraints.

The matches are scheduled in the periods of the competition days, and the periods are not continuous. So a triple (d, p, τ) is designed for time t, $d = 1, \cdots, n$ denotes the days, $p = 0, 1, \cdots, p_m - 1$ denotes the periods (p_m is the number of the periods in a day), and $\tau = [0, 1, \cdots, \tau_m - 1]$ denotes the time point (τ_m is the number of the time points in a period). In the Olympics scheduling problem, the kth match of event i's jth round is denoted as (i, j, k), and the final match is denoted as $(i, j_{mi}, 1)$. The beginning time and ending time of (i, j, k) is denoted as T_{bijk} and T_{eijk}, the corresponding triples are $(d_{bijk}, p_{bijk}, \tau_{bijk})$ and $(d_{eijk}, p_{eijk}, \tau_{eijk})$. The relations for time intervals are as follows.

1) equal $[t_{11}, t_{12}] = [t_{21}, t_{22}] \Leftrightarrow t_{11} = t_{21} \wedge t_{12} = t_{22}$.
2) before $[t_{11}, t_{12}] < [t_{21}, t_{22}] \Leftrightarrow t_{12} < t_{21}$.
3) after $[t_{11}, t_{12}] > [t_{21}, t_{22}] \Leftrightarrow t_{11} > t_{22}$.
4) repulsive $[t_{11}, t_{12}] \overset{c_1}{\underset{c_2}{\leftrightarrow}} [t_{21}, t_{22}] \Leftrightarrow t_{12} + c_1 < t_{21} \vee t_{11} > t_{22} + c_2$.
5) close $[t_{11}, t_{12}] \overset{c_1}{\underset{c_2}{\approx}} [t_{21}, t_{22}] \Leftrightarrow t_{11} \le t_{22} + c_2 \wedge t_{12} + c_1 \ge t_{21}$.
6) including $[t_{11}, t_{12}] \supset [t_{21}, t_{22}] \Leftrightarrow t_{11} \le t_{21} \wedge t_{12} \ge t_{22}$.
7) during $[t_{11}, t_{12}] \subset [t_{21}, t_{22}] \Leftrightarrow t_{11} \ge t_{21} \wedge t_{12} \le t_{22}$.

The rules above are used to model the Olympics scheduling problem of a schedule system.

3 Modeling and model transforming of the Olympics scheduling problem

In Olympics, the athletes hope to play fairly, the audience want their favorable matches, the sponsors require the gainful advertisement time, and the matches run in some special orders.

For most events, teams or athletes take part in some matches, and the winners are promoted to the next round until the champion is chosen. There are one or more matches in a round, and the matches of the preceding round have to be earlier than those of the successor. These are called order constraints. Some special event has just one round. For some traditional or other reasons, a special round of event A must run before the related round and after the preceding round of event B, for example, swimming man and swimming woman. These are called cross constraints. Some event cannot be much earlier or later than some other event. These are called close constraints. Some event has to be some time earlier or later than some other event. For example, the different groups of the same event had better run one after another, the track events longer than 1000 meters cannot run in a same day. These are called decentralization constraints. All of the above constraints are called time-distribution constraints.

The capacity of the fields is limited, so there cannot be much more matches on the fields at the same time. Some fields may affect each other, and there cannot be matches on the correlative fields at the same time. And these are called field constraints.

In Olympics, some athlete may attend more than one event. So there has to be some time interval between the correlative events, and the athlete cannot run from one field to another in a period of time. These are called person constraints.

The audience and sponsors′ requirement could be expressed by time window constraints.

In fact, the events of the Olympics are coupled by the field constraints. The field uncoupled events could be schedule respectively and independently. Next, a kind of field coupled events represented with the temporal interval model language are analyzed and formulated in Section 2.

First, some symbols are explained. If event i can run on the kind of field l, $S(i,l)=1$, otherwise, $S(i,l)=0$. If match (i,j,k) runs on the mth field at time t, $O[(i,j,k),m,t]=1$, otherwise, $O[(i,j,k),m,t]=0$.

The constraints are formulated as follows:

1) Time-distribution constraints

Order constraints are as follows

$$[T_{bi_1j_1k_1}, T_{ei_1j_1k_1}] < [T_{bi_2j_2k_2}, T_{ei_2j_2k_2}] \tag{1}$$

Cross constraints are as follows

$$[T_{bi_1jk_1}, T_{ei_1jk_1}] < [T_{bi_2jk_2}, T_{ei_2jk_2}] \wedge [T_{bi_2jk_2}, T_{ei_2jk_2}] < [T_{bi_1j+1k_1}, T_{ei_1j+1k_1}] \tag{2}$$

Decentralization constraints are as follows

$$[T_{bi_1j_1k_1}, T_{ei_1j_1k_1}] \overset{c_1}{\underset{c_2}{\leftrightarrow}} [T_{bi_2j_2k_2}, T_{ei_2j_2k_2}] \tag{3}$$

Close constraints are as follows

$$[T_{bi_1j_1k_1}, T_{ei_1j_1k_1}] \overset{c_1}{\underset{c_2}{\approx}} [T_{bi_2j_2k_2}, T_{ei_2j_2k_2}] \tag{4}$$

2) Field constraints

$$\forall l, t, \quad \sum_{i:S(i,l)=1} O[(i,j,k),m,t] \le m_{\max} \tag{5}$$

There cannot be more than $m_{\max}$ matches on the mth field at the same time.

3) Person constraints

$$[T_{bi_1j_1k_1}, T_{ei_1j_1k_1}] \overset{c_1}{\underset{c_2}{\leftrightarrow}} [T_{bi_2j_2k_2}, T_{ei_2j_2k_2}] \tag{6}$$

Matches (i_1,j_1,k_1) and (i_2,j_2,k_2) involve some common athletes, so match (i_1,j_1,k_1) must be repulsive to match (i_2,j_2,k_2).

$$O[(i,j,k_1),m_1,t_1]=1 \wedge O[(i,j,k_2),m_2,t_2] = 1 \wedge |t_1-t_2| \le \tau \Rightarrow m_1=m_2 \tag{7}$$

Matches (i_1,j_1,k_1) and (i_2,j_2,k_2) involve some common athletes. If they are time-close, they must occupy the same field.

4) Time window constraints

$$[T_{bij_1k}, T_{eij_1k}] \subset [t_1, +\infty] \tag{8}$$

$$[T_{bij_m1}, T_{eij_m1}] \subset [0, t_2] \tag{9}$$

The first round of event i cannot begin earlier than a certain time t_1, and the final match of event i cannot end later than a certain time t_2.

The constraints in the above describe the Olympics scheduling problem. The target is to gain a solution that satisfies all the constraints. This is a constraint satisfaction problem in fact. As mentioned before, constraint-based methods are back-tracking methods in nature. The efficiency of the constraint-based methods depends on the problem′s structure and the back-tracking policy. As the feasible domain gets smaller, the satisfiability would drop abruptly from a certain point, and to find a feasible solution or decide no solution is very difficult near the point[19]. This phenomenon is called phase transition. So we consider model transforming. The feasible domain can be broadened by softening constraints (9), and then we may try some novel methods.

$$\text{Let } J = \sum_i w_i T_i^2 \tag{10}$$

In (10)

$$T_i = \begin{cases} T_{eij_m1} - T_{ei} & T_{eij_m1} > T_{ei} \\ 0 & \text{else} \end{cases} \tag{11}$$

The constraint satisfaction problem can be transformed into a constrained optimization problem as follows:

$$\min_{\text{s.t.}} J$$

(1)~(8) are satisfied.

Constraint (5) can be rewritten as $\boldsymbol{g}(\boldsymbol{x}) \le \boldsymbol{0}$.

4 Solution methodology

4.1 Lagrangian relaxation

Consider the constrained optimization problem. Constraints (1), (2), (3), (4), (6) and (8) are all local constraints, they involve some kind of field. Constraint (7) does not affect the solution. And constraint (5) is the only global constraint. We relax constraint (5) as follows

$$L(\boldsymbol{x},\boldsymbol{\lambda}) = J + \boldsymbol{\lambda}^{\tau}(\sum \boldsymbol{O}[(i,j,k),m,t] - \boldsymbol{m}_{\max}) \tag{12}$$

$\boldsymbol{x}$ is the vector consisting of T_{bijk} and T_{eijk}, $\boldsymbol{\lambda}$ is the vector of Lagrangian multipliers and $\boldsymbol{\lambda} \geq 0$.

The dual problem is

$$q(\boldsymbol{\lambda}) = \min L(\boldsymbol{x},\boldsymbol{\lambda}) \tag{13}$$

s.t. Constraints (1), (2), (3), (4), (6) and (8) are satisfied.

(12) can be rewritten as follows

$$L(\boldsymbol{x},\boldsymbol{\lambda}) = \sum_i L_i - \boldsymbol{\lambda}\boldsymbol{m}_{\max} \tag{14}$$

$$L_i = w_i T_i^2 + \boldsymbol{\lambda}(\sum O[(i,j,k),m,t]) \tag{15}$$

In view of the separability of the original problem, the relaxed problem can be decomposed into I sub-problems as (15).

Due to the duality theorem, $q(\boldsymbol{\lambda}*) = J*$. If the optimal Lagrangian multipliers are known, the original problem can be solved by optimizing the separable sub-problems. And if the approximate optimal Lagrangian multipliers are known, the original problem can be approximately solved. We have a Lagrangian relaxation framework as shown in Fig. 1 to solve the constrained optimization problem.

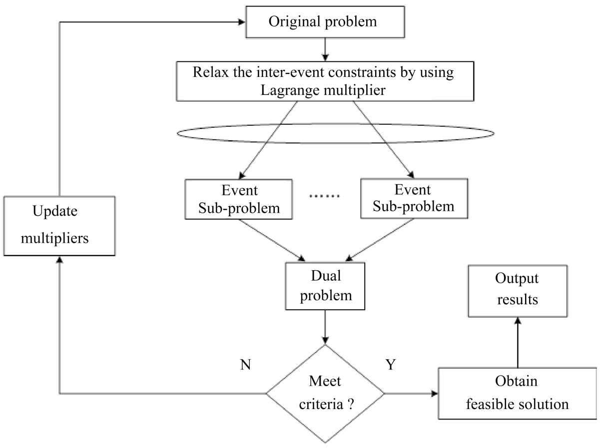

Fig. 1 The Lagrangian relaxation framework

4.2 Lagrangian multipliers updating

The Lagrangian multipliers are the solution of the dual problem, which is proved to be a concave problem. The existing method to solve the dual problem are dependent on variable priori knowledge[15~18]. The subgradient projection method with variable diameter for the dual prolem optimization, which does not dependent upon any priori knownledge, is presented in [20, 21] and modified in the following.

It can be concluded that $\boldsymbol{g}(\boldsymbol{x})$ is the subgradient of $q(\boldsymbol{\lambda})$ by considering that $q(\boldsymbol{\lambda})$ is concave, and for any $\Delta\boldsymbol{\lambda}$, $q(\boldsymbol{\lambda} + \Delta\boldsymbol{\lambda}) \leq q(\boldsymbol{\lambda}) + \langle \mathbf{g}(\boldsymbol{x}^{\boldsymbol{\lambda}}), \Delta\boldsymbol{\lambda}\rangle$.

Let $L(q, q_k^{lev}) = \{\boldsymbol{\lambda} | q(\boldsymbol{\lambda}) \geq q_k^{lev}\}$; then $M^* = L(q, q^*)$ is the set of the optimal solutions. Since $q(\boldsymbol{\lambda})$ is difficult to be expressed directly, we rewrite M^* by the subgradient.

$M^* = \bigcup_{\boldsymbol{\lambda}' \in D} L(\overline{q}(\boldsymbol{\lambda};\boldsymbol{\lambda}'), q^*)$, $\overline{q}(\boldsymbol{\lambda};\boldsymbol{\lambda}') = q(\boldsymbol{\lambda}) + \langle \mathbf{g}(\boldsymbol{x}^{\boldsymbol{\lambda}}), \boldsymbol{\lambda} - \boldsymbol{\lambda}' \rangle$, D is the domain of $\boldsymbol{\lambda}$.

Similarly, the set of the solutions constrained by q_k^{lev} can be rewritten as follows.

$$L(q, q_k^{lev}) = \bigcup\nolimits_{\boldsymbol{\lambda}' \in D} L(\overline{q}(\boldsymbol{\lambda};\boldsymbol{\lambda}'), q_k^{lev})$$

Then let $q^k(\boldsymbol{\lambda}) = \overline{q}(\boldsymbol{\lambda};\boldsymbol{\lambda}_k)$, $H^k = \{\boldsymbol{\lambda} | q^k(\boldsymbol{\lambda}) \geq q_k^{lev}\}$, $\boldsymbol{y}_k = P_{H^k}(\boldsymbol{\lambda}_k)$. $P_{H^k}(\boldsymbol{\lambda}_k)$ denotes the projection of $\boldsymbol{\lambda}_k$ on H^k. $P_+(\bullet)$ is the operator to project on positive semi-space. $d_S(\bullet)$ denotes the distance to space S. With the closed convex set H^k and an admissible stepsize t, the relaxation operator is $R_{H^k,t}(\boldsymbol{\lambda}) = \boldsymbol{\lambda} + t(P_{H^k}(\boldsymbol{\lambda}) - \boldsymbol{\lambda})$ which has the fejér constraction property

$$\left|\boldsymbol{y} - R_{H^k,t}(\boldsymbol{\lambda})\right|^2 \leq |\boldsymbol{y} - \boldsymbol{\lambda}|^2 - t_{\min}(2 - t_{\max}) d_{H^k}^2(\boldsymbol{\lambda}), \forall \boldsymbol{y} \in H^k.$$

Recall that $H^k = \{\boldsymbol{\lambda} | q^k(\boldsymbol{\lambda}) \geq q_k^{lev}\}$, $R_{H^k,t}(\boldsymbol{\lambda}) = \boldsymbol{\lambda} + t(q_k^{lev} - q(\boldsymbol{\lambda}_k))\boldsymbol{g}(\boldsymbol{x}_k)/|\boldsymbol{g}(\boldsymbol{x}_k)|^2$.

The method is shown as follows.

Algorithm 1. The subgradient projection method with variable diameter

Step 0. Let $k = 0$, $\boldsymbol{\lambda}_0 \geq 0$, ε, δ_0, $d > 0$, $\rho_0 = 0$, $0 < \omega < 1$, $k_d > 1$;

Let $\boldsymbol{x}_0$ be the solution of $q(\boldsymbol{\lambda}_0) = L(\boldsymbol{x}_0, \boldsymbol{\lambda}_0)$, and $\boldsymbol{x}_0^g$ be the corresponding feasible solution

$J_0 = J(\boldsymbol{x}_0^g)$, $q_0 = q(\boldsymbol{\lambda}_0)$, $q_0^{lev} = \omega J_0 + (1-\omega) q_0$, $q_0^{up} = J_0$, $\overline{\boldsymbol{x}} = \boldsymbol{x}_0^g$;

Step 1. If $J_k - q_k \leq \varepsilon$, x is the final solution, and the algorithm ends; Otherwise, continue;

For $0 < t_{\min} \leq t_k \leq t_{\max} < 1 - \delta$, δ is a small positive real number

$$\boldsymbol{\lambda}_{k+1} = P_+(\boldsymbol{\lambda}_k + t_k(q_k^{lev} - q(\boldsymbol{\lambda}_k))\boldsymbol{g}(\boldsymbol{x}_k) \Big/ |\boldsymbol{g}(\boldsymbol{x}_k)|^2);$$

$$\rho_{k+1} = \rho_k + t_k(2 - t_k) d_{Hk}^2(\boldsymbol{\lambda}_k) + d_{RN+}^2(\boldsymbol{\lambda}_k + t_k(P_{Hk}(\boldsymbol{\lambda}_k) - \boldsymbol{\lambda}_k));$$

If $q(\boldsymbol{\lambda}_{k+1}) \geq q_k$, then $q_{k+1} = q(\boldsymbol{\lambda}_{k+1})$; otherwise, $q_{k+1} = q_k$;

$$k = k + 1;$$

Step 2. Let $\boldsymbol{x}_k$ be the solution of $q(\boldsymbol{\lambda}_k) = L(\boldsymbol{x}_k, \boldsymbol{\lambda}_k)$, $\boldsymbol{x}_k^g$ be the corresponding feasible solution;

If $J(\boldsymbol{x}_k^g) \leq J_{k-1}$, then $J_k = J(\boldsymbol{x}_k^g)$, $\overline{\boldsymbol{x}} = \boldsymbol{x}_k^g$; Otherwise, $J_k = J_{k-1}$;

Step 3.1. If $\rho_k > d$, then $q_k^{up} = \min\{q_{k-1}^{lev}, J_k\}$, $\rho_k = 0$;

Step 3.2. If $\rho_k \leq d$, then $q_k^{up} = q_{k-1}^{up}$;

Step 4. $0 \leq \delta_k \leq \delta_{k-1}$, if $q_k > q_k^{lev} - \delta_k$, then $q_k^{up} = J_k$, $d = k_d$, $\rho_k = 0$;

Step 5. $q_k^{lev} = \omega q_k^{up} + (1-\omega) q_k$; go to Step 1;

To demonstrate the convergence of the method, we have 3 lemmas and 1 theorem, where $L_f = \sup\limits_{\boldsymbol{x} \in S} |\boldsymbol{g}(\boldsymbol{x})|$, S is the domain of the primal problem′s variables.

Lemma 1. If $d < |\boldsymbol{\lambda}_k - \boldsymbol{\lambda}^*|^2$, then there exists some k such that $q_k^{lev} \leq q_k + \delta_k$ where δ_k is not smaller than a certain constant.

Lemma 2. If $d \geq |\boldsymbol{\lambda}_k - \boldsymbol{\lambda}^*|^2$, δ is large enough and $\delta_k = 0$ for any k, then $q_k^{up} \geq q^*$.

Lemma 3. If $d \geq |\boldsymbol{\lambda}_k - \boldsymbol{\lambda}^*|^2$ and $\delta_k = 0$ for any k, then for any $\varepsilon > 0$,

$$k > d(L_f)^2 \big/ (t_{\min}(2 - t_{\max})\omega^2(1-\omega)^2\varepsilon^2) -$$
$$((f(\boldsymbol{x}_0) - q(\boldsymbol{\lambda}_0))^2/\varepsilon)^2 \big/ (2e\ln(\omega)) \Rightarrow q_k^{up} - q_k < \varepsilon.$$

Theorem 1. If there exists some k^*, such that $d \geq |\boldsymbol{\lambda}_k - \boldsymbol{\lambda}^*|^2$ and $\delta_k = 0$ for $k \geq k^*$, then Algorithm 1 converges, and the convergence efficiency is

$$k - k^* > d(L_f)^2/(t_{\min}(2 - t_{\max})\omega^2(1-\omega)^2\varepsilon^2) - ((f(\boldsymbol{x}_0) - q(\boldsymbol{\lambda}_0))^2/\varepsilon)^2/(2e\ln(\omega)) \Rightarrow q_k^{up} - q_k < \varepsilon.$$

The proof of the lemmas and theorem is similar to those in [20, 21]. When some k^* is obtained such that $\delta_k = 0$ for $k \geq k^*$, Algorithm 1 converges to the global optimal solution by the efficiency given in Theorem 1. The efficiency of the whole algorithm also depends on the efficiency to get k^*, which is determined by the original d and sequence δ_k. If δ_k is large before $d \geq |\boldsymbol{\lambda}_k - \boldsymbol{\lambda}^*|^2$ is obtained and descends to 0 sharply after $d \geq |\boldsymbol{\lambda}_k - \boldsymbol{\lambda}^*|^2$ is obtained, then d will get larger rapidly to exceed $|\boldsymbol{\lambda}_k - \boldsymbol{\lambda}^*|^2$. It is better if the original d is more close to $|\boldsymbol{\lambda}_k - \boldsymbol{\lambda}^*|^2$.

Though Algorithm 1 converges without any prior knowledge, it is correspondingly complicated to obtain q_k^{lev}. If the original problem is feasible, it can be concluded that $q(\boldsymbol{\lambda}^*) = J^* = 0$, the algorithm could be reduced by fixing $q_k^{lev} = 0$.

4.3 Sub-problems

Each of the sub-problems involves only one event, so it is correspondingly simple. We solve the sub-problems with enumeration method.

4.4 Construct feasible solutions

Since the field constraints are relaxed, the solutions obtained in the iteration process would violate the field constraints, based on which we must construct feasible solutions. In our work, a heuristic method is developed. The method enumerates all the matches in the order of beginning time, and postpones the current beginning time where the field constraints are violated.

5 Numerical results

The algorithm has been implemented using Matlab and tested on Celeron 4, CPU 1.4 GHz, 256 M SDRAM. The Olympics lasts for 16 days around, but any group of events which are field-coupled one another do not last longer than the track and field, which last for about 10 days. For the tested problem, we deal with the events during 7 days, and each day is partitioned into 3 periods, each period is partitioned into 9 time intervals. Each match lasts for 1$\sim$5 intervals, and all matches spread uniformly on the time axis. All the numbers of intervals and beginning times are generated randomly. The problem involves 20$\sim$90 events, and for a fixed number of events, the problem is generated and computed 25 times.

In the numerical results by Algorithm 1, J_{max} is the maximal J that arises in the optimization process, which denotes the maximal cost of violating the final time constraints. And $|q|_{max}$ is the absolute value of the minimal q, which is the minimal dual value. The difference between J and q is the gap of the original scheduling problem and the dual problem, which is the measurement of the computational error.

If J_{min}/J_{max} is smaller than a certain value, it can be decided that an acceptable solution is obtained, which violates the final time constraints not badly. If some q >0 is got, it can be decided that the original scheduling problem is unsatisfiable, *i.e.* there are no solutions which satisfy all the constraints. And as a practical criterion, we can set a positive value $\underline{q}$ to judge that a scheduling problem is unsatisfiable if $q > \bar{q}$.

By the results, the ratio of the total time consumed and the number of events is nearly a constant under the fixed steps of iteration. J_{min}/J_{max}, $|q|_{min}/|q|_{max}$, the time to get J_{min} and the number of tests to get q >0 increase as the number of events increases. All the possible solutions of the problems can be obtained in no more than 10 mins. The phase transition can be recognized by the number of tests to get q >0 and the time first to get q >0.

The phase transition is shown in Fig.2, where the x-coordinate is the number of events, the y-coordinate is the number of tests not to get q >0 and the average time first to get q >0. From 40 to 70 on the x-coordinate, the number of tests not to get q >0 decreases fast, and the average time first to get q >0 is distinctly longer than that outside the region.

Consider the numerical results by the reduced algorithm. The average J_{min}/J_{max} 's of the variable event number are shown in Fig. 3. And the average time to get J_{min} by Algorithm 1 and the reduced algorithm are shown in Fig. 4. Fig. 3 shows that when the number of events is larger than 50, the average J_{min}/J_{max} increases fast. Comparing to the results by Algorithm 1, phase transition happens around 50 events. So it can be concluded that an original problem is infeasible if J_{min}/J_{max} is larger than a certain value. Fig. 4 shows that when the event number is smaller than 40, the difference between the average times to get J_{min} by Algorithm 1 and the reduced algorithm are not notable, when the event number is larger than 40, the time by Algorithm 1 is remarkably larger than that by the reduced algorithm, and the difference has a maximal value between 60 and 70. Figs 3 and 4 cannot provide enough information to recognize the phase transition.

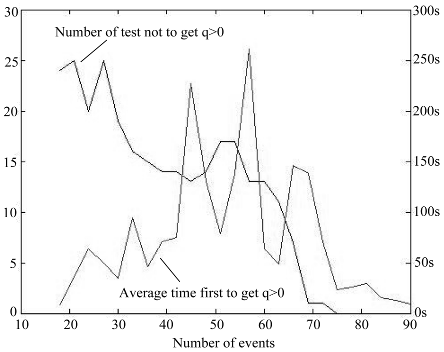

Fig. 2 Phase transition

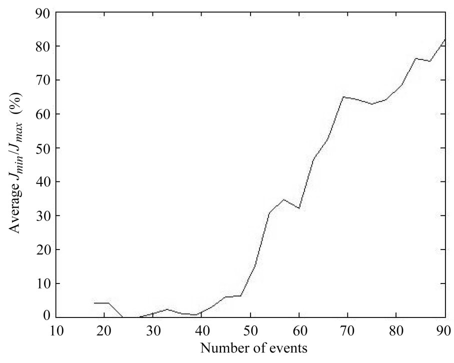

Fig. 3 Average J_{min}/J_{max}

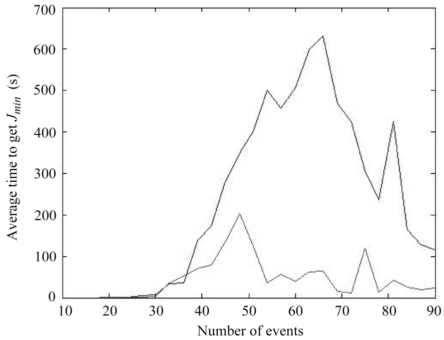

Fig. 4 Average time to get J_{min}

All the above numerical results show that Algorithm 1 and the reduced algorithm can both be applied to the Olympics scheduling problem. Algorithm 1 is more suitable for recognizing the phase transition while the reduced algorithm is less time consuming.

6 Conclusion

In this paper, the Olympics scheduling problem is modeled as a constraint satisfaction problem. By softening the time constraints of the final matches, the constraint satisfaction problem is transformed into a constrained optimization problem. A decomposition methodology based on Lagrangian relaxation is presented for the constrained optimization problem. The dual problem optimization is the key challenge, for which the subgradient projection method with variable diameter is studied. The method can converge to the globally optimal solutions, and the efficiency is given. Numerical results show that the methods are efficient, and the phase transition domain can be recognized by Algorithm 1.

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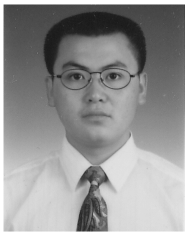

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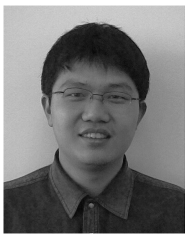

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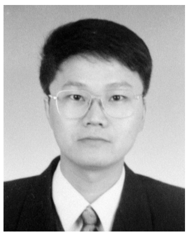

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