Variable Structure Model Reference Adaptive Control with Unknown High Frequency Gain Sign

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Abstract A variable structure model reference adaptive control for plants with relative degree greater than one and unknown high frequency gain sign is proposed. A switching scheme is introduced based on a monitoring function designed for the first auxiliary error of the close loop system. It is shown that under the supervision of the monitoring function, the switching stops after at most a finite number of switchings and the tracking error converges to a residual set that can be made arbitrarily small by appropriately choosing some design parameters.

Key words Hybrid control, variable structure control, adaptive control, switching control, high frequency gain

1 Introduction

Backstepping adaptive control (BAC)[1,2] has drawn much attention during the past decade due to its better transient performance than that of the traditional parameter adaptive control schemes[3]. The problem encountered in the BAC to some real systems is that its control law is highly nonlinear and complicated, especially, when the plant relative degree is high. Furthermore, its tracking performance would be deteriorated if input disturbance and unmodelled dynamics exist[4]. On the other hand, some robust model following schemes were also presented during the 1990s, within which variable structure model reference adaptive control (VS-MRAC) was proposed to cope with plant uncertainty and input disturbance with strong stability, disturbance rejection and performance robustness properties[5~7]. However, as with the case of most model following schemes, one of the basic assumptions of the VS-MRAC is that the high frequency gain (HFG) sign is known *a priori*. In [8], a switching method was proposed for the VS-MRAC for plants with relative degree one ($n^*=1$) and without a prior knowledge of HFG sign. This scheme was generalized to the case of $n^*>1$ in [9] but with an adaptive law to estimate the HFG sign, which made the control law design and the stability analysis much complicated. In this paper, we first show that one only needs to design a monitoring function to supervise the behavior of the first auxiliary error of the closed loop system such that the sign switching of control signal can be determined without using any adaptive law. We then show that, under the supervision of the monitoring function, the switching will stop after at most a finite number of switchings, all the closed loop signals are uniformly bounded and the tracking error will converge to a residual set that can be made arbitrarily small by appropriately choosing some design parameters.

2 Problem formulation

Consider the following single input/single output linear time invariant plant

$$y=G_p(s)[u+d]=K_p\frac{A_p(s)}{B_p(s)}[u+d] \tag{1}$$

where y and u are system output and input, respectively, $B_p(s)$ and $A_p(s)$ are monic polynomials of degree n and m, respectively and, d is an input disturbance. With respect to the controlled plant, we make the following assumptions: A1) $G_p(s)$ is of minimum phase. The parameters of $G_p(s)$ are unknown but belong to a known compact set. A2) The order n of $A_p(s)$ is a known constant. A3) The relative degree $n^*>1$. A4) The sign of the high frequency gain $k_p(\neq 0)$ is unknown. A5) The disturbance d satisfies

$$|d(t)|\leq \bar{d}(t), \forall t\geq 0 \tag{2}$$

where $\bar{d}$ is a known, piece-wise continuous and uniformly bounded function.

The reference model is given by

$$y_M=M(s)[r]=\frac{K_M}{B_M(s)}[r] \tag{3}$$

where r is a piecewise continuous and bounded reference input and $B_m(s)$ is a monic and coprime Hurwitz polynomial with $\deg(B_m(s))=n^*$. We define the tracking error as

$$e=y-y_M \tag{4}$$

Much akin to [6], the control signal is defined as

$$u=U^{nom}+\theta_{2n}^{nom}r-u_{vs}=\theta_{\omega}^{nomT}\omega+\theta_{2n}^{nom}r-u_{vs} \tag{5}$$

where u_{vs} is the variable structure control law to be designed,

$$\omega:=[\nu_1^{\mathrm{T}}\ \ y\ \ \nu_2^{\mathrm{T}}]^{\mathrm{T}}\in\Re^{2n-1} \tag{6}$$

in which ν_1 and ν_2 are generated by input/output filters according to

$$\left.\begin{array}{l}\dot{\nu}_1=\varLambda\nu_1+gu,\ \nu_1(0)=0\\ \dot{\nu}_2=\varLambda\nu_2+gy,\ \nu_2(0)=0\end{array}\right\} \tag{7}$$

where $\varLambda\in\Re^{(n-1)\times(n-1)}$ is a Hurwitz matrix, $g\in\Re^{n-1}$, and $(\varLambda,\ g)$ is a controllable pair, and

$$\left.\begin{array}{l}\theta_{\omega}^{nom}:=[\theta_1^{\mathrm{T}}\ y\ \theta_2^{\mathrm{T}}]^{\mathrm{T}}\in\Re^{2n-1}\\ \theta_{2n}^{nom}\in\Re\end{array}\right\} \tag{8}$$

are the nominal values of θ_{ω}^* and θ_{2n}^*, which, modulo exponentially decaying terms due to initial conditions, satisfy

$$y=G_p(s)[\omega^{\mathrm{T}}\theta_{\omega}^*+\theta_{2n}^*r]=M(s)r=y_M \tag{9}$$

Label K_p^{nom} as the nominal value of K_p and

$$\left.\begin{array}{l}k^{nom}:=K_p^{nom}/K_M\\ \kappa:=(K_p-K_p^{nom})/K_p^{nom}\\ \rho:=1+\kappa\end{array}\right\} \tag{10}$$

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Then, after some algebraic manipulations, the tracking error can be expressed as

$$e = k^{nom} M(s)[-u_{vs} - \bar{U}] + \varepsilon \tag{11}$$

where ε decays exponentially due to nonzero initial conditions and

$$\bar{U} = \rho(\theta_\omega^{*\mathrm{T}}\omega - U^{nom} - W_d(s)[d]) + (1/k^{nom} - \rho\theta_{2n}^{nom})r + \kappa U_{vs} \tag{12}$$

with $W_d(s)$ being a proper and stable transfer function from d to e.

Because $n^* > 1$, define Hurwitz polynomial

$$L(s) = \prod_{i=1}^{n^*-1} L_i(s) = \prod_{i=1}^{n^*-1} (s + \alpha_i), \alpha_i > 0 \tag{13}$$

such that $M(s)L(s)$ is an SPR (Strictly positive real) function. The augmented signal is given by

$$y_a := k^{nom} M(s)L(s)[U_0 - L^{-1}(s)[U_N]] \tag{14}$$

and the auxiliary errors are

$$\left.\begin{aligned} e_0 &:= e - y_a = k^{nom} M(s)L(s)[-U_0 - L^{-1}(s)[\bar{U}]] + \varepsilon \\ e_i &:= F_i^{-1}(s)[U_{i-1}] - L_i^{-1}(s)[U_i],\ i = 1, 2, \cdots, N \end{aligned}\right\} \tag{15}$$

with

$$U_N := u_{vs} \tag{16}$$

and the averaging filters

$$F_i^{-1}(s) := (\tau_i s + 1)^{-1} \tag{17}$$

with τ_i small positive constants. To simplify our analysis, let $\tau_1 = \cdots = \tau_N := \tau$; hence

$$F_1(s) = \cdots = F_N(s) := F(s) \tag{18}$$

Our control objective is to design the control signal u_{vs} which, as shown in (15) and (16), is achieved in a recursive manner, so that the tracking error (11) can converge to a small residual set for plants with $n^* > 1$ and unknown HFG sign.

3 Main results

3.1 Switching signals

Since the sign of K_p is unknown, one must decide what signals should be switched. Obviously, the nominal value of K_p, say, K_p^{nom}, should have the same sign as K_p and therefore its sign should be changed if it is considered to be estimated incorrectly. From (15), U_0 needs the knowledge of the sign of K_p^{nom} due to $k^{nom} := K_p^{nom}/K_M$. Further, whenever the sign of K_p^{nom} changes, θ_ω^{nom} and θ_{2n}^{nom} in (5) should also be changed correspondingly. Define

$$\theta^{nom} := [(\theta_\omega^{nom})^{\mathrm{T}}\ \theta_{2n}^{nom}]^{\mathrm{T}} \tag{19}$$

hence the above analysis shows that only the sign of (U_0, θ^{nom}) needs to be switched. Define

$$\begin{aligned} U_0 &= \begin{cases} U_0^+ = f_0 sgn(e_0), & \text{if } t \in \mathbb{T}^+, \\ U_0^- = -f_0 sgn(e_0), & \text{if } t \in \mathbb{T}^-, \end{cases} \\ \theta^{nom} &= \begin{cases} \theta^{nom+}, & \text{if } t \in \mathbb{T}^+, \\ \theta^{nom-}, & \text{if } t \in \mathbb{T}^-, \end{cases} \end{aligned} \tag{20}$$

where f_0 will be given later, (U_0^+, θ^{nom+}) and (U_0^-, θ^{nom-}) correspond to $K_p > 0$ and $K_p < 0$, respectively, the sets $\mathbb{T}^+$ and $\mathbb{T}^-$ satisfy $\mathbb{T}^+ \cup \mathbb{T}^- = [0, \infty)$ and $\mathbb{T}^+ \cap \mathbb{T}^- = \phi$, and both $\mathbb{T}^+$ and $\mathbb{T}^-$ are the union of the following intervals

$$[t_k,\ t_{k+1}) \cup [t_{k+2},\ t_{k+3}) \cup \cdots \cup [t_j,\ t_{j+1}) \tag{21}$$

where t_k or t_j denotes the switching time of (U_0, θ^{nom}) and will be designed later. We now give the definition for the switching.

Definition 1. For each switching instant t_k $(k \geq 1)$, the switching of (U_0, θ^{nom}) is defined as switched between (U_0^+, θ^{nom+}) and (U_0^-, θ^{nom-}) alternately.

3.2 Monitoring function and switching scheme

We then design a monitoring function to supervise the behavior of the first auxiliary error so that it can decide when (U_0, θ^{nom}) should be switched. To this end, define Lyapunov function

$$V_0 = e_0^2/2 \tag{22}$$

With no loss of generality, let

$$M(s)L(s) = k_M/(s + \lambda), \lambda > 0 \tag{23}$$

Equation (15) can thus be written as

$$\dot{e}_0 = -\lambda e_0 + k^{nom}[-U_0 - L^{-1}(s)[\bar{U}]] + \varepsilon \tag{24}$$

The time derivative of (22) along the solution of (24) yields

$$\dot{V}_0 = -\lambda e_0^2 + k^{nom}[-U_0 - L^{-1}(s)[\bar{U}]]e_0 + \varepsilon e_0 \tag{25}$$

Suppose that after some $\bar{t}_0 \geq 0$, the sign of K_p is correctly estimated and $-U_0 - L^{-1}[\bar{U}]$ in (24) is completely dominated by U_0. Then, we have $k^{nom}[-U_0 - \quad L^{-1}(s)[\bar{U}]]e_0 \leq 0$, which implies that we can write (25) as

$$\dot{V}_0 \leq -2\bar{\lambda}V_0 + \varepsilon^2/2c_\varepsilon \tag{26}$$

where we have applied the triangle inequality $\varepsilon e_0 \leq c_\varepsilon e_0^2/2 + \varepsilon^2/(2c_\varepsilon)$ to the term εe_0 with c_ε a positive constant such that $\lambda - c_\varepsilon/2 := \bar{\lambda} > 0$. The above inequality inspires us to construct the following first-order differential equation

$$\dot{\xi}_0 = -2\bar{\lambda}\xi_0 + \varepsilon^2/2c_\varepsilon, \xi_0(\bar{t}_0) = V_0(\bar{t}_0) \tag{27}$$

Using the Comparison Lemma[10] to (26) and (27), and noting that $\xi_0(\bar{t}_0) = V_0(\bar{t}_0)$, we have $V_0(t) \leq \xi_0(t), \forall t \geq \bar{t}_0$. Since ε is unknown but decays exponentially, there exist positive constants c_δ and δ, such that

$$\varepsilon \leq c_\delta \exp(-\delta t), \forall t \geq 0 \tag{28}$$

Taking (28) into consideration, it can be checked that the solution of (25) satisfies

$$\begin{aligned} &V_0(t) \leq \xi_0(t) \leq \exp[-2\bar{\lambda}(t - \bar{t}_0)]V_0(\bar{t}_0) + c_0\exp(-2\delta t), \\ &\xi_0(\bar{t}_0) = V_0(\bar{t}_0), \forall t \geq \bar{t}_0 \end{aligned} \tag{29}$$

where

$$c_0 := c_\delta^2/4c_\varepsilon(\bar{\lambda} - \delta) \tag{30}$$

Here, we let $\delta < \bar{\lambda}$ because a less δ can only make the estimate of ε more conservative. The monitoring function is now constructed based on (29) as [8]

$$\vartheta_k(t) := \exp[-2\bar{\lambda}(t - t_k)]V_0(t_k) + (k + 1)\exp(-2\delta_k t) \tag{31}$$

where the switching time t_k is defined by (21) and $\{\delta_k, 0 < \delta_k < \bar{\lambda}\}$ is any monotonically decreasing sequence satisfying

$$\delta_k \to 0, \text{as}\, k \to \infty \tag{32}$$

which, together with the sequence $\{k+1\}$, implies that there exists a finite k such that both $k+1 > c_0$ and $\exp(-2\delta t) < \exp(-2\delta_k t)$ can be satisfied thereafter. At every switching instant t_k, in view of (31), we have $V_0(t_k) < \vartheta_k(t_k)$ which, considering the absolute continuity of $e_0^{[10]}$, implies that the following definition of the switching time is well-defined

$$t_{k+1} = \begin{cases} \min\{t > t_k : \ V_0(t) = \vartheta_k(t)\}, \\ \quad \text{if the minimum exists}, \quad k = 0, 1, \cdots \\ +\infty, \ \text{otherwise} \end{cases} \tag{33}$$

It is clear that a new switching occurs only when $V_0(t)$ increases and is equal to ϑ_k.

3.3 Main theorem

According to the above analysis, the variable structure control signals of (15) for the case of unknown HFG sign are

$$\left.\begin{array}{l} U_0 \quad \text{given by (20)} \\ U_l = f_l \text{sgn}(e_l), \ l = 1, \cdots, N-1 \\ U_N = f_N \text{sgn}(e_N) \end{array}\right\} \tag{34}$$

where $f_i\,(i = 0, \cdots, N-1)$ and f_N are defined by

$$\left.\begin{array}{l} f_i = \text{BND}\{F_{1,i}^{-1}(s)L_{i+1,N}^{-1}(s)L(s)[\rho(\theta_\omega^* - \theta_\omega^{nom})^{\rm T}\bar{\omega} + \\ \kappa(L^{-1}(s)[u] - \theta_\omega^{nom{\rm T}}\bar{\omega}) + \rho W_d(s)L^{-1}(s)[d]]\} + \Delta_i \\ \\ f_N = \text{BND}\{F_{1,N}^{-1}(s)L(s)[(\theta_\omega^* - \theta_\omega^{nom})^{\rm T}\bar{\omega} + \\ (\theta_{2n}^* - \theta_{2n}^{nom})L^{-1}(s)[r] + W_d(s)L^{-1}(s)[d]]\} + \Delta_N \end{array}\right\} \tag{35}$$

in which BND$\{\ \cdot\ \}$ denotes an upper bound of a signal[1], Δ_i are arbitrarily positive constants, and

$$\left.\begin{array}{l} \bar{\omega} = L^{-1}(s)[\omega] \\ L_{i,j}(s) = \prod_{k=i}^{j} L_k(s) \quad (L_{i,j}(s) = 1\, \text{if}\ j < i) \\ F_{i,j}(s) = \prod_{k=i}^{j} F_k(s) \quad (F_{i,j}(s) = 1\, \text{if}\ j < i) \end{array}\right\} \tag{36}$$

We now give the main results of this paper.

Theorem 1. Suppose the plant to be controlled satisfies assumptions A1)~A5). Let the tracking error and the auxiliary errors be defined by (11) and (15), respectively, and let the corresponding VS control signals U_0 and $U_i (i = 1, 2, \cdots, N)$ be given by (34). Let the monitoring function be defined by (31) and (U_0, θ^{nom}) switch according to Definition 1 with switching time defined by (33). Then,

1) the switching stops after at most a finite number of switchings;

2) the tracing error e converges to a residual set proportional to τ and all the signals of the close-loop system are uniformly bounded.

Proof. 1) The proof is achieved by contradiction. Suppose (U_0, θ^{nom}) switches between (U_0^+, θ^{nom+}) and (U_0^-, θ^{nom-}) alternately without stopping. Then, after a finite number of k switchings, (U_0, θ^{nom}) must have a correct sign, *i.e.*, (U_0, θ^{nom})=(U_0^+, θ^{nom+}) if $K_p > 0$ or (U_0, θ^{nom})=(U_0^-, θ^{nom-}) if $K_p < 0$, while

$$\left.\begin{array}{l} c_0 < (k+1) \\ \exp(-2\delta t) < \exp(-2\delta_k t), \ \forall t \geq t_k \end{array}\right\} \tag{37}$$

[1]The algorithm given by [6] may reduce the conservativeness of conventionally used Euclidean norm when we obtain the BND$\{\ \cdot\ \}$.

where c_0 is defined by (30). Therefore, from (29) (replacing $\bar{t}_0$ with t_k), and taking into account (31) and (37), we have

$$V_0(t) < \vartheta_k(t), \forall t > t_k \tag{38}$$

which, from (33) and (38), implies that no switching will occur again, a contradiction. Hence, only finite switchings are defined.

2) By (38) and (22),

$$|e_0| \leq \text{EXP} \Rightarrow e_0 \to 0, \text{as}\, t \to \infty \tag{39}$$

where EXP generically denotes an exponentially decaying function. Applying U_i given by (34) to (15) and noting that these subsystems have no relationship with the sign of K_p, it can be proved, using the same technique as that in [6], that

$$\left.\begin{array}{l} |e_i| \leq \text{EXP}\ (i = 1, \cdots, N-1) \\ |e_N| \leq \tau C \left\|\bar{\omega}\right\|_\infty + \text{EXP} \end{array}\right\} \tag{40}$$

where $\|x\|_\infty := \sup_{t\geq 0}|x(t)|$, and C generically denotes a positive constant. Using Lemma 1 in Appendix A, we have

$$\|e\|_\infty \leq \tau C(1 + \|\bar{\omega}\|_\infty) + \text{EXP} \tag{41}$$

According to (6) and (36), $\bar{\omega}$ can be rewritten as

$$\begin{array}{l} \bar{\omega} = L^{-1}(s)[((sI - \Lambda)^{-1}gG_p^{-1}(s)[e])^{\rm T}, \ e, \\ \quad ((sI - \Lambda)^{-1}g[e])^{\rm T}]^{\rm T} + L^{-1}(s)[((sI - \Lambda)^{-1}g \\ \quad (G_p^{-1}(s)[y_M] - [d]))^{\rm T}, y_M, \ ((sI - \Lambda)^{-1}g[y_M])^{\rm T}]^{\rm T} \end{array} \tag{42}$$

where $G_p(s)$ is of minimum phase and therefore, $L^{-1}(s)(sI - \Lambda)^{-1}gG_p^{-1}(s)$ is proper and stable. Furthermore, since d is bounded and y_M is the output of reference model, (41) and (42) imply that

$$\|\bar{\omega}\|_\infty \leq C(1 + \|e\|_\infty) \tag{43}$$

Combining (41) with (43) and letting $\tau < C^{-1}$, we have

$$\left.\begin{array}{l} \|\bar{\omega}\|_\infty \leq \dfrac{\tau C + C}{1 - \tau C} \\ \|e\|_\infty \leq \tau C \dfrac{1 + C}{1 - \tau C} + \text{EXP} \end{array}\right\} \tag{44}$$

Hence $\bar{\omega}$, $e \in \mathcal{L}_\infty$, and the tracking error e converges to a residual set that can be made arbitrarily small by choosing τ.

To prove that all the signals of the close-loop system are uniformly bounded, we need to prove that $\omega \in \mathcal{L}_\infty$. Note that (44) implies that $e \in \mathcal{L}_\infty$ and therefore y as well as $\nu_2 \in \mathcal{L}_\infty$ since $y = y_M + e$. Hence, from (6), it suffices to prove that $\nu_1 \in \mathcal{L}_\infty$. From (7), we have

$$|\dot{\nu}_1| = |\Lambda\nu_1 + gu| \leq \left|\Lambda\nu_1 + g(\theta_\omega^{nom{\rm T}}\omega + \theta_{2n}^{nom}r - u_{vs})\right| \tag{45}$$

where from (34) and (35),

$$|(u_{vs})_t| = |(U_N)_t| \leq C(1 + \|\bar{\omega}_t\|_\infty) \tag{46}$$

Taking (44) and (46) into consideration, it follows that

$$|\dot{\nu}_1(t)| \leq C + C\left\|(\nu_1)_t\right\| + C\left\|\omega_t\right\|_\infty \leq C(1 + \|(\nu_1)_t\|) \tag{47}$$

which implies that ν_1 is a regular signal(see page 70 in [11]). Using Corollary 3.6.3(see page 140 in [11]), $\bar{\omega} \in \mathcal{L}_\infty$(see (A.5) of Appendix A) implies that ν_1, $\omega \in \mathcal{L}_\infty$. □

In particular, we have

Corollary 1. if $\varepsilon = 0$, then at most one switching is needed.

Proof. The proof is the same as that given by [8] and is therefore omitted. □

4 Simulation results

The plant to be controlled was chosen as the following relative degree two plant

$$G_p(s) = -4/(s^2 - 0.6s - 2),\ x(0) = [0.5, 0.5]^{\mathrm{T}} \tag{48}$$

where x is the state of a controllable canonical form of $G_p(s)$. The reference model was chosen as

$$M(s) = 2/(s+2)^2 \tag{49}$$

The design parameters were chosen as follows: $\Lambda = -2$ and $g = 1$ for the input/output filters; $L(s) = s + 2$ in (13); $K_p^{nom+} = 2$ and $K_p^{nom-} = 2$; hence $\theta^{nom+} = [-2, -2, 2, 1]^{\mathrm{T}}$ and $\theta^{nom-} = [-2, 2, -2, -1]^{\mathrm{T}}$, respectively. In the simulation, we chose the upper bound of $(\theta^{nom})^* - \theta^{nom}$ to be $[5, 5, 10, 10]^{\mathrm{T}}$. We chose $\tau_0 = \tau_1 = 0.05$ for the averaging filters; $\Delta_0 = \Delta_1 = 0.1$ in (35); $\bar{\lambda} = 1.8$ in (26). The reference signal was a square wave with amplitude 1 and frequency 2 rad/sec. The disturbance $d = \sin(0.5t)$. The signals U_0, U_1 and the final control signal u were design according to (34), (35) and (5), respectively. The monitor function ϑ_k was chosen according to (31) with $\delta_k = 1/(k+1)$. In the simulation, we set $K_p = K_p^{nom+}$ at $t = 0$; that is, an incorrect estimate of the sign of K_p was applied at the beginning of the simulation. Figs. 1∼3 show that the unique switching occurred at about $t = 0.1$s, and henceforth the system output y tracked the model output y_M perfectly. Note that to avoid chattering, we replaced $\mathrm{sgn}(x)$ in (34) with $x/(|x| + 0.001)$ in the simulation.

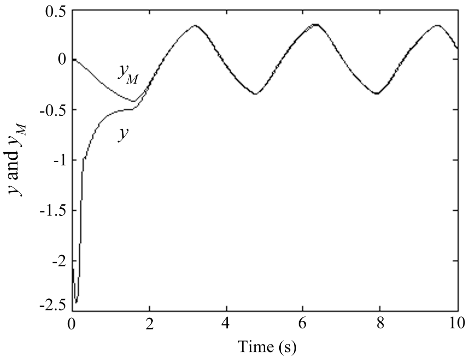

Fig. 1 y and y_M

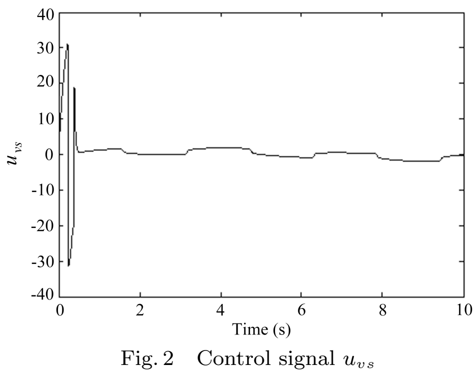

Fig. 2 Control signal u_{vs}

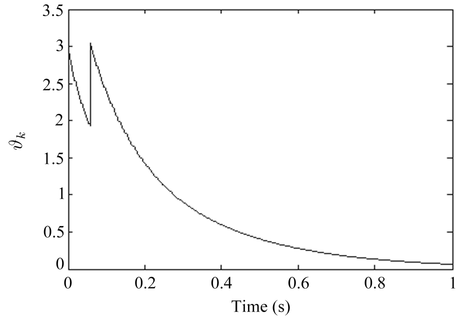

Fig. 3 Monitor function ϑ_k

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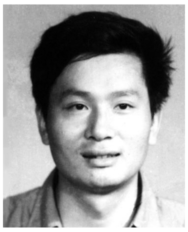

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Appendix A

Lemma 1. If (39) and (40) hold, the tracking error can be expressed as $\|e\|_\infty \le \tau C(1+\|\bar{\omega}\|_\infty)+\mathrm{EXP}$; that is, (41) is satisfied.

Proof. It can be verified from (11), (14) and (15) that

$$e = e_0 + \sum_{i=1}^{N} k^{nom} M(s)L(s)[L_{1,i-1}^{-1}(s)(F_{1,(N-i)}^{-1}(s)[e_i])] - k^{nom} M(s)L(s)[(F_{1,N}^{-1}(s)-1)[U_0]] \tag{A1}$$

where $L_{1,i-1}^{-1}(s)$, $F_{1,(N-i)}^{-1}$ and $F_{1,N}^{-1}(s)$ are defined by (36). Note that the term $[(F_{1,N}^{-1}(s)-1)[U_0]]$ in (A1) can be expressed as

$$[(F_{1,N}^{-1}(s)-1)[U_0]] = -\tau s \sum_{i=1}^{N} \frac{1}{(\tau s+1)^i}[U_0] \tag{A2}$$

Further, since $sM(s)L(s)$ is proper, we can rewrite it as

$$sM(s)L(s) = C + G_{ML}(s) \tag{A3}$$

Therefore, from (A1) and in view of (A2) and (A3), it follows that

$$k^{nom} M(s)L(s)[(F_{1,N}^{-1}(s)-1)[U_0]] = -\tau k^{nom} C[\sum_{i=1}^{N} \frac{1}{(\tau s+1)^i}[U_0]] - \tau k^{nom} G_{ML}(s)[\sum_{i=1}^{N} \frac{1}{(\tau s+1)^i}[U_0]] \tag{A4}$$

Letting $i=0$ in (35) and noting an upper bound of the disturbance is known, we have

$$\|U_0\|_\infty \le C + C\|\bar{\omega}\|_\infty + C\left\|L^{-1}(s)[u]\right\|_\infty \tag{A5}$$

Here, to obtain (A5), we have used the inequality $\|z\|_\infty \le C(1+\|x\|_\infty)$, where $z = H(s)[x]$ with $H(s)$ as a proper and stable transfer function. Noting (16), we can obtain the following equation from: (5)

$$L^{-1}(s)[u] = \theta_\omega^{nom\mathrm{T}}\bar{\omega} + \theta_{2n}^{nom} L^{-1}(s)[r] - L^{-1}(s)[U_N] \tag{A6}$$

Then, taking into account (46), we have

$$\left\|L^{-1}(s)[u]\right\|_\infty \le C(1+\|\bar{\omega}\|_\infty) \tag{A7}$$

which, together with (A5), implies that

$$\|U_0\|_\infty \le C(1+\|\bar{\omega}\|_\infty) \tag{A8}$$

In view of (A4), inequality (A8) means that

$$\left\|k^{nom} M(s)L(s)[(F_{1,N}^{-1}(s)-1)[U_0]]\right\|_\infty \le \tau C(1+\|\bar{\omega}\|_\infty) \tag{A9}$$

Considering (39), (40), (A1) and (A9), it is easy to verify that (41) holds. □