Robust Adaptive Control of Nonholonomic Systems with Nonlinear Parameterization

WANG Qiang-De[1] WEI Chun-Ling[1, 2]

Abstract A global-adaptive state feedback control strategy is presented for a class of nonholonomic systems in chained form with strong nonlinear drifts and unknown nonlinear parameters. A parameter separation technique is introduced to transform the nonlinear parameterization nonholonomic system into a linear-like parameterized nonholonomic system. Then, the feedback domination design is applied to design a global adaptive stabilization controller and a switching strategy is developed to eliminate the phenomenon of uncontrollability. The proposed controller can guarantee that all the system states globally converge to the origin, while other signals remain bounded. Simulation example demonstrates the effectiveness and the robust features of the proposed controller.

Key words Nonholonomic systems, robust adaptive control, nonlinear drifts, nonlinear parameterization, global-adaptive stabilization

1 Introduction

Over the last few years, the problem of controlling nonholonomic dynamic systems has received considerable attention and become a popular subject in the nonlinear control. There are three main reasons for this. Firstly, the nonholonomic systems are frequently used to describe the practical control systems, such as mobile robots, car-like vehicles, under-actuated satellites and the knife-edge[1∼5], can all be modeled as nonholonomic control system. Secondly, nonholonomic systems represent a special class of inherently nonlinear systems for which the first approximation or feedback linearization method is unfeasible and "pure nonlinear" analysis and synthesis method are needed. Finally, from the technical point of view, the stabilization problems of nonholonomic systems are exceptionally challenging and difficult for the reason that as pointed out by Brockett[6] or Krasnosel'skii-Zabreiko[7], this class of nonholonomic systems can not be asymptotically stabilized by any stationary continuous state feedback, although they may be open loop controllable. This motivates researchers to seek for various approaches, which can be classified into time-varying feedback, discontinuous feedback and the combination of the two. The time-varying feedback approach, which was first proposed in [3], provides smooth/continuous controller and no switching is required[8∼10]. This approach introduces some persistent excitation signals in the control input to guarantee the convergence of the closed-loop signals. However, the convergence rate of this approach is slow. Moreover, linear time-varying system theory[11] and Barbalat′s lemma in [12] are often used to analyze the stability of the closed-loop system. On the other hand, the discontinuous feedback approach uses the discontinuous change of coordinates and a switching control strategy to overcome the difficulty of the loss of controllability[5,13,14]. The advantage is its simplicity and fast transient response and the drawback is that the control input is discontinuous.

However, those aforementioned papers only considered the systems without drifts or with weak nonlinear drifts. A class of nonholonomic systems perturbed by strong nonlinear uncertainties was recently studied in [15∼17]. Discontinuous state and output feedback controllers were designed in [15] and achieved globally exponential stability. However, the paper required that the x_0-subsystem be Lipschitz and there should be no parametric uncertainty. In [16], an input-to-state scaling was introduced to remove the obstacle of zero crossings of u_0 and achieved global stabilization of nonholonomic systems with strong nonlinear drifts and parametric uncertainties. For the same system as in [16], an adaptive state feedback and output feedback control strategies were presented in [17]. In [18], the problem of almost asymptotic stabilization and globally asymptotic regulation for a class of high-order nonholonomic systems in power chained form was solved. However, it should be noticed that all these papers were concerned with systems with linear parameterization. There are very few reports in literature for adaptive control of nonlinearly parameterized nonholonomic systems. However, many practical control systems such as biochemical processes[19] and machines with friction[20], often contain unknown parameters that enter the systems nonlinearly. Indeed, nonlinear parameterizations frequently arise and are inevitable in various realistic dynamic models of practical control problems, as illustrated in [21∼23]. From a theoretical viewpoint, adaptive control of nonlinearly parameterized nonholonomic systems is also interesting, because it represents a new challenge to the theory of nonlinear adaptive control.

Unlike linear parameterization, nonlinear parameterization is exceptionally difficult to estimate. In the recent years, there have been some attempts to deal with this difficult problem. Under a condition that the bound of nonlinear parameters is known, globally adaptive control of a class of nonlinearly parameterized systems was solved by output feedback[21,22]. Recently, adaptive control of nonlinearly parameterized systems has been reported in [24,25] for convex or concave case and in [22] for a general case. More recently, in [26], the globally adaptive control of nonlinearly parameterized systems with uncontrollable linearization has been solved by using parameter separation technique and feedback domination design which were used in many papers[12,21,22,27,28].

In this paper, we will investigate the control of nonholonomic systems in a chained form with strong nonlinear drifts and unknown nonlinear parameters. Without imposing any condition on the unknown parameters, combining the parameter separation technique with the feedback domination design, a solution to the problem of global-adaptive control for the uncertain nonholonomic systems is derived. The proposed adaptive control algorithm guarantees that all the states converge to the origin and other variables are bounded.

Received March 7, 2006; in revised form September 14, 2006
Supported by National Natural Science Foundation of P. R. China (60304003, 60574007, 60574080) and Research Startup Foundation of Qufu Normal University
1. College of Electricity Information and Automation, Qufu Normal University, Rizhao 272826, P. R. China 2. Institute of Automation, Southeast University, Nanjing 210096, P. R. China
DOI: 10.1360/aas-007-0399

2 Problem formulation

In this paper, we consider the following uncertain nonholonomic systems with nonlinear parameterization

$$\begin{aligned} \dot{x}_0 &= u_0 + f_0(x_0, \boldsymbol{\theta}) \\ \dot{x}_i &= x_{i+1}u_0 + f_i(x_0, \boldsymbol{x}, \boldsymbol{\theta}), 1 \le i < n \\ \dot{x}_n &= u_1 + f_n(x_0, \boldsymbol{x}, \boldsymbol{\theta}) \end{aligned} \tag{1}$$

where u_0 and u_1 are control inputs, x_0 and $\boldsymbol{x} = (x_1, \cdots, x_n)^{\mathrm{T}}$ are system states, $f_0(x_0, \boldsymbol{\theta})$ and $f_i(x_0, \boldsymbol{x}, \boldsymbol{\theta})$ are continuous functions of their arguments, $\boldsymbol{\theta} \in R^p$ is an unknown constant vector. Functions $f_0(x_0, \boldsymbol{\theta})$ and $f_i(x_0, \boldsymbol{x}, \boldsymbol{\theta})$ denote the possible modeling error and neglected dynamics. Note that function f_i may include uncertain drift terms.

Clearly, when $f_0(x_0, \boldsymbol{\theta}) = 0$ and $f_i(x_0, \boldsymbol{x}, \boldsymbol{\theta}) = 0$ for all $1 \le i \le n$, system (1) becomes a standard chained system which has been extensively studied in the literature. When $f_0(x_0, \boldsymbol{\theta}) = f_0(x_0)^{\mathrm{T}}\boldsymbol{\theta}$ and $f_i(x_0, \boldsymbol{x}, \boldsymbol{\theta}) = f_0(x_0, x_1, \cdots, x_i)^{\mathrm{T}}\boldsymbol{\theta}$, system (1) has been studied by Do in [16] and by Ge in [17].

We impose the following assumption on $f_0(x_0, \boldsymbol{\theta})$ and $f_i(x_0, \boldsymbol{x}, \boldsymbol{\theta})$.

Assumption 1. For $i = 1, \cdots, n$,

$$|f_i(x_0, \boldsymbol{x}, \boldsymbol{\theta})| \le (|x_0| + \cdots + |x_i|) b_i(x_0, x_1, \cdots, x_i, \boldsymbol{\theta}) \tag{2}$$

$$f_0(x_0, \boldsymbol{\theta}) = x_0 b_0(x_0, \boldsymbol{\theta}) \tag{3}$$

where $b_0(x_0, \theta)$ is continuous function and $b_i(x_0, x_1, \cdots, x_i, \boldsymbol{\theta})$ $(i = 1, \cdots, n)$ is nonnegative continuous function.

The above assumption implies that the origin is the equilibrium point of system (1).

The control problem in this paper is stated as follows.

Definition 1. System (1) is said to be adaptive globally asymptotically regulated (AGAR) at the origin by a adaptive state feedback controller of the form

$$\begin{aligned} \dot{\hat{\vartheta}}_0 &= \upsilon_0(x_0, \hat{\vartheta}_0), u_0 = \mu_0(x_0, \hat{\vartheta}_0) \\ \dot{\hat{\vartheta}} &= \upsilon(x_0, \boldsymbol{x}, \hat{\vartheta}_0, \hat{\vartheta}), u_1 = \mu(x_0, \boldsymbol{x}, \hat{\vartheta}_0, \hat{\vartheta}) \end{aligned} \tag{4}$$

if all the solutions of the closed-loop system (1)$\sim$(4) are bounded and well defined over $[0, +\infty)$. Furthermore, $\lim\limits_{t \to +\infty} x_i(t) = 0$ for all $0 \le i \le n$.

Lemma 1. For any real-value function $\pi(\boldsymbol{x}, \boldsymbol{y}) > 0$,

$$|\boldsymbol{x}\boldsymbol{y}| \le \frac{1}{2}\pi(\boldsymbol{x}, \boldsymbol{y})\boldsymbol{x}^2 + \frac{1}{2}\pi^{-1}(\boldsymbol{x}, \boldsymbol{y})\boldsymbol{y}^2 \tag{5}$$

Lemma 2[26] (Parameter separation technique)**.** For any real-valued continuous function $f(\boldsymbol{x}, \boldsymbol{y})$, where $\boldsymbol{x} \in R^n$ and $\boldsymbol{y} \in R^m$, there are smooth scalar functions $a(\boldsymbol{x}) \ge 0, b(\boldsymbol{y}) \ge 0, c(\boldsymbol{x}) \ge 1$ and $d(\boldsymbol{y}) \ge 1$, such that

$$|f(\boldsymbol{x}, \boldsymbol{y})| \le a(\boldsymbol{x}) + b(\boldsymbol{y}), \quad |f(\boldsymbol{x}, \boldsymbol{y})| \le c(\boldsymbol{x})d(\boldsymbol{y}) \tag{6}$$

Remark 1. By Lemma 2, for $i = 0, \cdots, n$, there exist smooth functions $\gamma_i(x_0, \cdots, x_i) \ge 1$ and $c_i(\boldsymbol{\theta}) \ge 1$ satisfying

$$|b_i(x_0, \cdots, x_i, \boldsymbol{\theta})| \le \gamma_i(x_0, \cdots, x_i)c_i(\boldsymbol{\theta}) \tag{7}$$

Since $\boldsymbol{\theta}$ is a constant vector, $c_i(\boldsymbol{\theta})$ is a constant as well. Let $\vartheta_1 := \sum_{i=0}^n c_i(\boldsymbol{\theta})$ and $\vartheta = \vartheta_1^2$ be two new unknown constants. Then, Assumption 1 implies that

$$|f_0(x_0, \boldsymbol{\theta})| \le |x_0|\,\gamma_0(x_0)\vartheta_1 \tag{8}$$

$$|f_i(x_0, \boldsymbol{x}, \boldsymbol{\theta})| \le (|x_1| + \cdots + |x_i|)\gamma_i(x_0, x_1, \cdots, x_i)\vartheta_1 \tag{9}$$

3 The stabilization of x_0-subsystem

In order to obtain the stabilization of the x_0-subsystem and remove the obstacle of zero crossings of u_0, we design the control input u_0 using feedback domination method such that only when the state x_0 approaches to the origin, u_0 crosses zero. We consider the problem in two cases: $x_0(t_0) \ne 0$ and $x_0(t_0) = 0$. Without loss of generality, we assume that $t_0 = 0$.

Case 1. $x_0(0) \ne 0$.

Select Lyapunov function $V_0(x_0, \tilde{\vartheta}_0) = \frac{1}{2}x_0^2 + \frac{1}{2\Gamma_0}\tilde{\vartheta}_0^2$, where $\tilde{\vartheta}_0 = \vartheta_1 - \hat{\vartheta}_0$, $\hat{\vartheta}_0$ is an estimate of ϑ_1, and $\Gamma_0 > 0$ is a design constant. By Assumption 1 and Remark 1, it is easy to see that

$$\dot{V}_0(x_0, \tilde{\vartheta}_0) \le x_0 u_0 + x_0^2\gamma_0(x_0)(\hat{\vartheta}_0 + \tilde{\vartheta}_0) - \Gamma_0^{-1}\dot{\hat{\vartheta}}_0\tilde{\vartheta}_0 \tag{10}$$

With the choice of the smooth adaptive controller

$$\begin{aligned} u_0(x_0, \hat{\vartheta}_0) &= -x_0\sqrt{k_0^2 + (\gamma_0\hat{\vartheta}_0)^2} - x_0\gamma_0\hat{\vartheta}_0 \\ &= x_0\beta_0(x_0, \hat{\vartheta}_0), \end{aligned} \tag{11}$$

$$\dot{\hat{\vartheta}}_0 = \Gamma_0 x_0^2\gamma_0(x_0) = \Gamma_0\tau_0(x_0) \tag{12}$$

we have

$$\dot{V}_0(x_0, \tilde{\vartheta}_0) \le -x_0^2\sqrt{k_0^2 + (\gamma_0\hat{\vartheta}_0)^2} \le -k_0 x_0^2 \tag{13}$$

where k_0 is a positive constant.

Case 2. $x_0(0) = 0$.

We design the controller $u_0(x_0, \hat{\vartheta}_0)$ as follows

$$u_0 = \begin{cases} x_0\beta_0 + \lambda_0 & \text{when } x_0\beta_0 + \lambda_0 \ge u_0^* \text{ and } t \le t_s \\ x_0\beta_0 & \text{else} \end{cases} \tag{14}$$

where $\lambda_0 > 0$, $t_s > 0$ and $0 < u_0^* < \lambda_0$ are design constants, $\beta_0(x_0, \hat{\vartheta}_0)$ and the update law for $\hat{\vartheta}_0$ are given in (12).

Theorem 1. For any initial conditions $(x_0(0), \hat{\vartheta}_0(0)) \in R^2$, the switching control strategy (11), (12) and (14) guarantees that the solution $x_0(t)$ exists and the parameter estimator $\hat{\vartheta}_0(t)$ is bounded and converges to an invariant set. Furthermore, the control $u_0(x_0, \hat{\vartheta}_0)$ given by (11) or (14) also exists, does not cross zero and satisfies $\lim_{t\to\infty} u_0(x_0(t), \hat{\vartheta}_0(t)) = 0$.

Proof. Consider the Lyapunov function $V_0(x_0, \tilde{\vartheta}_0) = \frac{1}{2}x_0^2 + \frac{1}{2\Gamma_0}\tilde{\vartheta}_0^2$.

For the case of $x_0(0) \ne 0$, from (13), we can see that $\lim_{t\to\infty} x_0(t) = 0$ and $\hat{\vartheta}_0(t)$ is bounded.

For the case of $x_0(0) = 0$, differentiating $V_0(x_0, \tilde{\vartheta}_0)$ along the solution of x_0-subsystem, we can obtain that

$$\dot{V}_0 \le \begin{cases} -k_0 x_0^2 + \lambda_0 x_0 & \text{when } 0 \le t \le t^* \\ -k_0 x_0^2 & \text{else} \end{cases} \tag{15}$$

Clearly, $\lim\limits_{t\to\infty} x_0(t) = 0$ for $t > t^*$. Owing to $-k_0x_0^2 + \lambda_0 x_0 \le -k_0(x_0 - \frac{\lambda_0}{2k_0})^2 + \frac{\lambda_0^2}{4k_0}$, we know that $x_0(t)$ is bounded for $0 \le t \le t^*$.

So, for any initial conditions $(x_0(0), \hat{\vartheta}_0(0)) \in R^2$, $\lim_{t\to\infty} x_0(t) = 0$ and $\hat{\vartheta}_0(t)$ is bounded.

Moreover, we need to show that $x_0(t)$ does not cross zero.

From the selection of t^*, $x_0(t)$ does not cross zero for the case $x_0(0) = 0$ and $0 < t \le t^*$. For the case $x_0(0) \ne 0$, substituting (11) into x_0-subsystem, we have

$$\dot{x}_0 = -x_0(\sqrt{k_0^2 + (\gamma_0\hat{\vartheta}_0)^2} + \gamma_0\hat{\vartheta}_0 - b_0(x_0, \boldsymbol{\theta})) \tag{16}$$

Since $x_0(t)$ and $\hat{\vartheta}_0(t)$ are bounded and $\boldsymbol{\theta}$ is a constant vector, the solution of (16) is

$$x_0(t) = x_0(0)\mathrm{e}^{-\int_0^t \psi(s)\mathrm{d}s} \tag{17}$$

where $\psi(s) = \sqrt{k_0^2 + (\gamma_0(x_0(s))\hat{\vartheta}_0(s))^2} + \gamma_0(x_0(s))\hat{\vartheta}_0(s) - b_0(x_0(s), \boldsymbol{\theta})$.

Clearly, only when $t = \infty$ or at $t = 0$ and $x_0(0) = 0$, $x_0(t) = 0$.

Hence, $u_0(x_0, \hat{\vartheta}_0)$ given by (11) or (14) also exists, does not cross zero and satisfies $\lim_{t\to\infty} u_0(x_0(t), \hat{\vartheta}_0(t)) = 0$. □

4 Input-to-state scaling

The above design can assure that $u_0(t)$ is unequal to zero in finite time scale. However, $u_0(t)$ will converge to zero as t goes to ∞. This phenomenon causes serious trouble in controlling the x-subsystem via the control input u_1, because, in the limit, the x-subsystem is uncontrollable.

To avoid the phenomenon, introduce a state scaling discontinuous transformation as follows

$$z_i = \frac{x_i}{u_0^{n-i}}, \quad i = 1, \cdots, n \tag{18}$$

This discontinuous coordinates transformation was used in [16], and is an improvement of the coordinates transformation used in [13,15,17].

Under the new $z-$coordinates, the x-subsystem is transformed into

$$\begin{aligned}
&\dot{x}_0 = u_0(x_0, \hat{\vartheta}_0) + f_0\\
&\dot{z}_i = z_{i+1} + \phi_{i1}(x_0, z_i, \hat{\vartheta}_0) + \phi_{i2}(x_0, \bar{\boldsymbol{z}}_i, \hat{\vartheta}_0, \boldsymbol{\theta})\\
&\qquad (1 \le i < n)\\
&\dot{z}_n = u_1 + \phi_{n2}(x_0, \boldsymbol{z}, \boldsymbol{\theta})
\end{aligned} \tag{19}$$

where $\phi_{i1}(x_0, z_i, \hat{\vartheta}_0) = -(n-i)z_i(\frac{\partial u_0}{\partial x_0} + \frac{1}{u_0}\frac{\partial u_0}{\partial \hat{\vartheta}_0}\Gamma_0\tau_0)$, $\phi_{i2}(x_0, \bar{\boldsymbol{z}}_i, \hat{\vartheta}_0, \boldsymbol{\theta}) = \frac{f_i(x_0, x_1, \cdots, x_i, \boldsymbol{\theta})}{u_0^{n-i}} - z_i\frac{n-i}{u_0}\frac{\partial u_0}{\partial x_0}f_0(x_0, \boldsymbol{\theta})$, $(1 \le i < n)$, $\phi_{n2}(x_0, \boldsymbol{z}, \boldsymbol{\theta}) = f_n(x_0, \boldsymbol{x}, \boldsymbol{\theta})$, $\bar{\boldsymbol{z}}_i = [z_1, \cdots, z_i], \boldsymbol{z} = [z_1, \cdots, z_n]$.

Consequently, the following lemma can be established.

Lemma 3. For each $1 \le i \le n$, there exists a smooth function $\omega_i(x_0, \bar{\boldsymbol{z}}_i, \hat{\vartheta}_0) > 0$ such that

$$\left|\phi_{i2}(x_0, \bar{\boldsymbol{z}}_i, \hat{\vartheta}_0, \boldsymbol{\theta})\right| \le (|z_1| + \cdots + |z_i|)\omega_i(x_0, \bar{\boldsymbol{z}}_i, \hat{\vartheta}_0)\vartheta_1 \tag{20}$$

5 Feedback domination design

In this section, we first design the control input u_1 in the case $x_0(0) \ne 0$. Then, the case of $x_0(0) = 0$ will be discussed.

Case 1. $x_0(0) \ne 0$.

Step 1. Consider the Lyapunov function $V_1 = \frac{1}{2}\xi_1^2 + \frac{1}{2\Gamma}\tilde{\vartheta}^2$, where $\xi_1 = z_1$, $\tilde{\vartheta} = \vartheta - \hat{\vartheta}$, $\hat{\vartheta}$ is an estimator of ϑ, and $\Gamma > 0$ is a constant to be designed. From (19) and Lemma 3, we can see that

$$\begin{aligned}
\dot{V}_1 &= \xi_1(z_2 + \phi_{11} + \phi_{12}) - \frac{1}{\Gamma}\tilde{\vartheta}\dot{\hat{\vartheta}}\\
&\le \xi_1(z_2 + \phi_{11}) + |\xi_1| \cdot |\xi_1|\,\omega_1\vartheta - \frac{1}{\Gamma}\tilde{\vartheta}\dot{\hat{\vartheta}}
\end{aligned} \tag{21}$$

Define

$$\begin{aligned}
&\rho_1 = \omega_1, \tau_1 = \Gamma\xi_1^2\rho_1\\
&\alpha_1(x_0, z_1, \hat{\vartheta}_0, \hat{\vartheta}) = -n\xi_1 - \phi_{11} - \xi_1\rho_1\hat{\vartheta}
\end{aligned} \tag{22}$$

Clearly, there is $\alpha_1(x_0, 0, \hat{\vartheta}_0, \hat{\vartheta}) = 0$, then

$$\dot{V}_1 \le -n\xi_1^2 + \xi_1(z_2 - \alpha_1) - (\frac{1}{\Gamma}\tilde{\vartheta} + \eta_1)(\dot{\hat{\vartheta}} - \tau_1) \tag{23}$$

where $\eta_1 = 0$.

Inductive step. Suppose there are a set of C^∞ virtual controllers $\alpha_i(x_0, \bar{\boldsymbol{z}}_i, \hat{\vartheta}_0, \hat{\vartheta})$, with $\alpha_i(x_0, 0, \hat{\vartheta}_0, \hat{\vartheta}) = 0$, $1 \le i \le k$, and a Lyapunov function $V_k(x_0, \bar{\boldsymbol{z}}_k, \hat{\vartheta}_0, \hat{\vartheta})$, such that

$$\begin{aligned}
\dot{V}_k \le& -(n-k+1)\sum_{i=1}^k \xi_i^2 - (\frac{1}{\Gamma}\tilde{\vartheta} + \eta_k)(\dot{\hat{\vartheta}} - \tau_k) +\\
&\xi_k(z_{k+1} - \alpha_k)
\end{aligned} \tag{24}$$

where $\xi_i = z_i - \alpha_{i-1}, i = 2, \cdots, k$. Then, in the $(k+1)$th step, we claim that (24) holds as well. The reason of this is as follows. Define $\xi_{k+1} = z_{k+1} - \alpha_k$ and consider the Lyapunov function $V_{k+1} = V_k + \frac{1}{2}\xi_{k+1}^2$. Then, the time derivative of V_{k+1} is

$$\begin{aligned}
\dot{V}_{k+1} \le& -(n-k+1)\sum_{i=1}^k \xi_i^2 - (\tfrac{1}{\Gamma}\tilde{\vartheta} + \eta_k)(\dot{\hat{\vartheta}} - \tau_k) +\\
&\xi_k\xi_{k+1} + \xi_{k+1}[z_{k+2} + \phi_{(k+1)1} + \phi_{(k+1)2} - \tfrac{\partial \alpha_k}{\partial x_0}(u_0 + f_0) -\\
&\sum_{i=1}^k \tfrac{\partial \alpha_k}{\partial z_i}(z_{i+1} + \phi_{i1} + \phi_{i2}) - \tfrac{\partial \alpha_k}{\partial \hat{\vartheta}_0}\Gamma_0\tau_0 - \tfrac{\partial \alpha_k}{\partial \hat{\vartheta}}\dot{\hat{\vartheta}}]
\end{aligned} \tag{25}$$

By Lemma 3 and the fact of $\alpha_i(x_0, 0, \hat{\vartheta}_0, \hat{\vartheta}) = 0$, there are smooth functions $W_{k+1}(\cdot) \ge 0$ and $\bar{W}_{k+1}(\cdot) \ge 0$, such that

$$\begin{aligned}
&\left|\phi_{(k+1)2} - \frac{\partial \alpha_k}{\partial x_0}f_0 - \sum_{i=1}^k \frac{\partial \alpha_k}{\partial z_i}\phi_{i2}\right|\\
&\le (|z_1| + \cdots |z_{k+1}|)\, W_{k+1}(x_0, \bar{\boldsymbol{z}}_{k+1}, \hat{\vartheta}_0, \hat{\vartheta})\vartheta_1\\
&\le (|\xi_1| + \cdots + |\xi_{k+1}|)\, \bar{W}_{k+1}(x_0, \bar{\boldsymbol{z}}_{k+1}, \hat{\vartheta}_0, \hat{\vartheta})\vartheta_1
\end{aligned} \tag{26}$$

Then, from Lemma 2, it is easy to see that there is a smooth function $\rho_{k+1}(\cdot) \ge 0$, such that

$$\begin{aligned}
&|\xi_{k+1}|\left|\phi_{(k+1)2} - \frac{\partial \alpha_k}{\partial x_0}f_0 - \sum_{i=1}^k \frac{\partial \alpha_k}{\partial z_i}\phi_{i2}\right|\\
&\le \sum_{i=1}^k \xi_i^2 + \xi_{k+1}^2\rho_{k+1}(x_0, \bar{\boldsymbol{z}}_{k+1}, \hat{\vartheta}_0, \hat{\vartheta})\vartheta
\end{aligned} \tag{27}$$

So, (25) can be rewritten as follows

$$\begin{aligned}
\dot{V}_{k+1} \le& -(n-k)\sum_{i=1}^k \xi_i^2 - (\frac{1}{\Gamma}\tilde{\vartheta} + \eta_{k+1})(\dot{\hat{\vartheta}} - \tau_{k+1}) +\\
&\xi_{k+1}[z_{k+2} + \bar{\rho}_{k+1} + \xi_{k+1}\rho_{k+1}\hat{\vartheta} - \Gamma\eta_k\xi_{k+1}\rho_{k+1}]
\end{aligned} \tag{28}$$

where $\tau_{k+1} = \tau_k + \Gamma\xi_{k+1}^2\rho_{k+1}$, $\bar{\rho}_{k+1} = \xi_k + \phi_{(k+1)1} - \frac{\partial \alpha_k}{\partial x_0}u_0 - \sum_{i=1}^k \frac{\partial \alpha_k}{\partial z_i}(z_{i+1} + \phi_{i1}) - \frac{\partial \alpha_k}{\partial \hat{\vartheta}_0}\Gamma_0\tau_0 - \frac{\partial \alpha_k}{\partial \hat{\vartheta}}\tau_{k+1}$, $\eta_{k+1} = \eta_k + \xi_{k+1}\frac{\partial \alpha_k}{\partial \hat{\vartheta}}$.

Then, the virtual controller

$$\begin{aligned}
\alpha_{k+1} =& -(n-k)\xi_{k+1} - \bar{\rho}_{k+1} - \xi_{k+1}\rho_{k+1}\hat{\vartheta} +\\
&\Gamma\eta_k\xi_{k+1}\rho_{k+1}
\end{aligned} \tag{29}$$

renders

$$\dot{V}_{k+1} \leq -(n-k)\sum_{i=1}^{k+1}\xi_i^2 - (\frac{1}{\Gamma}\tilde{\vartheta} + \eta_{k+1})(\dot{\hat{\vartheta}} - \tau_{k+1}) + \xi_{k+1}(z_{k+2} - \alpha_{k+1}) \qquad (30)$$

The aforementioned inductive argument shows that (24) holds for $k = n$. In fact, at the nth step, one can construct a global change of coordinates $(\xi_1, \cdots, \xi_n)$, a positive definite and proper Lyapunov function V_n and a smooth controller $\alpha_n(x_0, \boldsymbol{z}, \hat{\vartheta}_0, \hat{\vartheta})$ of the form (29), such that

$$\dot{V}_n \leq -\sum_{i=1}^{n}\xi_i^2 - (\frac{1}{\Gamma}\tilde{\vartheta} + \eta_n)(\dot{\hat{\vartheta}} - \tau_n) + \xi_n(u_1 - \alpha_n) \qquad (31)$$

Therefore, the smooth adaptive controller

$$\dot{\hat{\vartheta}} = \tau_n = \tau_{n-1} + \Gamma\xi_n^2\rho_n \qquad (32)$$

$$u_1 = \alpha_n = -\xi_n - \bar{\rho}_n - \xi_n\rho_n\hat{\vartheta} + \Gamma\eta_{n-1}\xi_n\rho_n \qquad (33)$$

renders

$$\dot{V}_n \leq -(\xi_1^2 + \cdots + \xi_n^2) \qquad (34)$$

Case 2. $x_0(0) = 0$.

In this case, the control input u_0 is given in (14). During the time period $[0, t^*]$, applying the same design procedure as that in case 1 to the z-subsystem in (19) with u_0 defined in the first equation of (14), we can design $u_1 = u_1^*(x_0, \boldsymbol{z}, \hat{\vartheta}_0, \hat{\vartheta})$ and $\dot{\hat{\vartheta}} = \hat{\vartheta}^*(x_0, \boldsymbol{z}, \hat{\vartheta}_0, \hat{\vartheta})$ to guarantee that z-state can not blow up. At $t = t^*$, since $x_0(t^*) \neq 0$, we switch the control input u_0 and u_1 to (11) and (33), respectively.

Theorem 2. Under Assumption 1, if the above switching control scheme, designed in Sections 3∼5, is applied to system (1), then the closed-loop system is globally asymptotic-regulated at the origin and all the signals are globally bounded.

6 Simulation

To verify our proposed controller, we consider the following system.

$$\dot{x}_0 = u_0 + x_0^{\theta_0}$$
$$\dot{x}_1 = u_0 x_2 + x_1\theta_1^{x_1}$$
$$\dot{x}_2 = u_1 + \ln(1 + (\theta_2 x_2)^2)$$

where $\theta_i, i = 0, 1, 2$ is unknown bounded parameter and satisfies $\theta_0 > 1, \theta_1 > 0$ and $\theta_2 \in R$. The control objective is to design u_0 and u_1 such that $(x_0(t), x_1(t), x_2(t)) \to 0$ as $t \to \infty$ for any initial value.

From Theorems 1 and 2, we can design the adaptive controller as follows:

When $x_0(0) \neq 0$,

$$u_0 = x_0\beta_0$$
$$\dot{\hat{\vartheta}}_0 = \Gamma_0 x_0^2 \mathrm{e}^{\frac{1}{8}\ln^2(1+x_0^2)}$$
$$u_1 = \xi_2 - \phi_{21} - \xi_2\rho_2\hat{\vartheta}$$
$$\dot{\hat{\vartheta}} = \Gamma(\xi_2^2\rho_2 + \xi_1^2\rho_1)$$

When $x_0(0) = 0$,

$$u_0 = \begin{cases} x_0\beta_0 + \lambda_0, & \text{if } x_0\beta_0(x_0, \hat{\vartheta}_0) + \lambda_0 \geq u_0^* \text{ and } t \leq t_s \\ x_0\beta_0, & \text{else} \end{cases}$$

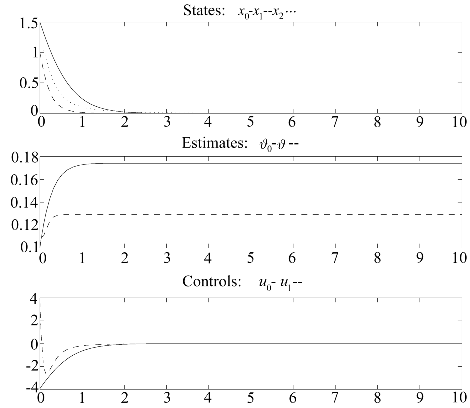

Fig. 1 Simulation results with $(x_0(0), x_1(0), x_2(0)) = (1.5, 1, 1)$

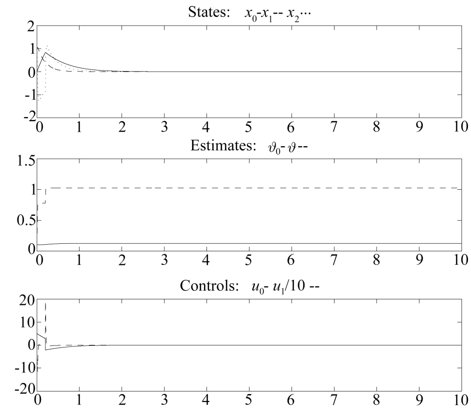

Fig. 2 Simulation results with $(x_0(0), x_1(0), x_2(0)) = (0, 1, 1)$

where $\beta_0 = (-\mathrm{e}^{\frac{1}{8}\ln(1+x_0^2)}\hat{\vartheta}_0 - \sqrt{k_0^2 + (\mathrm{e}^{\frac{1}{8}\ln(1+x_0^2)}\hat{\vartheta}_0)^2}$, $\xi_1 = z_1, \xi_2 = z_2 - \alpha_1$, $z_1 = \frac{x_1}{u_0}, z_2 = x_2$, $\alpha_1 = -2\xi_1 + \xi_1\phi_{11} - \xi_1\rho_1\hat{\vartheta}$, $\phi_{11}, \phi_{21}, \rho_1, \rho_2$ are known functions.

The unknown system parameters θ_0, θ_1 and θ_2 are assumed to be 1.5, 0.5 and 0.5, respectively. In simulation, the design parameters are chosen as $k_0 = 2.5, \lambda_0 = 5, u_0^* = 3$, $t_s = 0.2$, $\Gamma_0 = \Gamma = 0.1$. The simulation results for the case of initial conditions $(x_0(0), x_1(0), x_2(0)) = (1.5, 1, 1)$, $(\hat{\vartheta}_0(0), \hat{\vartheta}(0)) = (0.1, 0.1)$ are shown in Fig.1 while the results for $(x_0(0), x_1(0), x_2(0)) = (0, 1, 1)$, $(\hat{\vartheta}_0(0), \hat{\vartheta}(0)) = (0.1, 0.1)$ are in Fig.2. From the figures, it is clear to see that all of the states asymptotically converge to the origin, the controls are bounded and converge to zero and the parameters estimates are bounded.

7 Conclusion

By using a parameter separation technique, an input-to-state scaling and a feedback domination design, a globally adaptive state feedback controller is designed for a class of uncertain nonholonomic system in chained form with strong nonlinear drifts and nonlinear parameters. When the initial value $x_0(0)$ is unequal to zero, the control law is smooth. When the initial value $x_0(0)$ is equal to zero, a switching control strategy is proposed. The system states have been proved to globally converge to the origin and the parameters estimators are bounded. Simulation result has shown the effectiveness and feasibility of the proposed control strategy.

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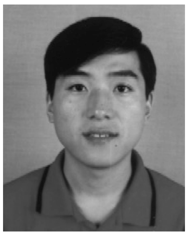

WANG Qiang-De Received his Ph. D. degree in control theory and control engineering from Northeastern University. Now he is an associate professor in Qufu Normal University. His main research interest covers robust control and adaptive control of nonlinear systems. Corresponding author of this paper.
E-mail: wqdwchl@sohu.com

WEI Chun-Ling Ph. D. candidate at the Institute of Automation, Southeast University. She received her B. S. degree from Qufu Normal University in 1998. Her main research interest covers robust covers and adaptive control of nonlinear systems.
E-mail: weichunling@eyou.com