A New Feedback-feedforward Configuration for the Iterative Learning Control of a Class of Discrete-time Systems

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Abstract This paper presents a new feedback-feedforward configuration for the iterative learning control (ILC) design with feedback, which consists of a feedback and a feedforward component. The feedback integral controller stabilizes the system, and takes the dominant role during the operation, and the feedforward ILC compensates for the repeatable nonlinear/unknown time-varying dynamics and disturbances, thereby enhancing the performance achieved by feedback control alone. As the most favorable point of this control strategy, the feedforward ILC and the feedback control can work either independently or jointly without making efforts to reconfigurate or retune the feedforward/feedback gains. With rigorous analysis, the proposed learning control scheme guarantees the asymptotic convergences along the iteration axis.

Key words Iterative learning control, feedforward control, feedback control, nonlinear systems

1 Introduction

Since iterative learning control (ILC) was first proposed by [1] in 1984 for the control of a system that repeats the same task in a finite interval, it has been extensively studied and significant progress has been made in both theory and applications [1~12]. However, although sufficient conditions are given to guarantee the convergence of the learning process, the trajectory error is likely to grow quite significantly before it converges to zero in the process of learning, and the rate of the convergence is often slow. These phenomena are owing to the fact that the control structure is basically an open loop and this control structure alone does not compensate for the output error in each trial. Therefore, the performance in the early stages of learning can be bad for stable plants, and even worse for unstable plants. The use of conventional feedback controllers can help to solve overcome this kind of problem in the transient stages of learning since they can compensate for the control input to reduce the error.

The learning process of making advantage of the current feedback error or the feedback configuration can be found in [8]. Reference [8]proposed a model-based learning scheme for the robot manipulators with feedback controllers, but without giving a rigorous analysis for the convergence of the learning process. [9] suggested an ILC scheme for a class of nonlinear systems with high gain feedback PD controller, which update to the feedforward control input with the feedback controller output. In [7], a discrete ILC was proposed for discrete-time nonlinear time-varying systems with initial state error, input disturbance, and output measurement noise. In [10], a D-type ILC was done in a feedback configuration. The learning process was performed in the feedforward input updated by the previous plant input and the derivative of the previous error. The rapid convergence was shown either by technical proof[7] or by simulation[10] as compared with the traditional feedforward learning.

Some other ILC designs in a feedback configuration could be found in [11,13~14]. However, they still did not show, in a mathematics meaning, what the clear functions of feedforward ILC algorithm and feedback controller in the composite feedforward/feedback configuration are. So far, almost all the descriptions of the functions for the feedforward or feedback component in a combined configuration are based on the qualitative induction rather than the theoretical analysis. This is certainly an important issue both in theory and practical applications.

In this paper, we propose an ILC control scheme for a class of discrete-time nonlinear systems over a finite time interval in a new feedforward-feedback configuration. The function of the feedforward, acting as a compliable component in this configuration, rejecting exogenous disturbances, and compensating for the nonlinear and and time-varying plant, is to meet the high tracking performance requirement. Meanwhile the feedback component serves as the main controller.

2 Problem statement

2.1 Problem formulation

The following discrete-time nonlinear time-varying systems are considered.

$$\boldsymbol{x}_n(k+1) = \boldsymbol{f}(\boldsymbol{x}_n(k), \boldsymbol{y}_n(k), k), \tag{1}$$

$$\boldsymbol{y}_n(k+1) = \boldsymbol{g}(\boldsymbol{x}_n(k), \boldsymbol{y}_n(k), \boldsymbol{u}_n(k), k), \tag{1'}$$

where n and k are the iteration index and discrete-time, respectively. For simplicity in the following discussion, let $\boldsymbol{x}_n(k) \in R^p$, $\boldsymbol{y}_n(k) \in R^p$, and $\boldsymbol{u}_n(k) \in R^p$ for all $k \in [0, K]$ and $n \in [1, \infty)$. $\boldsymbol{u}_n(k)$ is the control variable. $\boldsymbol{f}(\cdot)$ and $g(\cdot)$ are nonlinear functions.

Assumption 1. Functions $\boldsymbol{f}$ and $\boldsymbol{g}$ are uniformly globally Lipschitz with respect to $\boldsymbol{x}$, $\boldsymbol{y}$ and $\boldsymbol{u}$ for $k \in [0, 1, \ldots, K]$ on a compact set $\Omega \in R^p \times R^p \times [1, K]$ or $\Omega \in R^p \times R^p \times R^p \times [1, K]$, *i.e.*,

$$\begin{aligned}&\|\boldsymbol{f}(\boldsymbol{x}_1(k), \boldsymbol{y}_1(k), k) - \boldsymbol{f}(\boldsymbol{x}_2(k), \boldsymbol{y}_2(k), k)\| \\ &\quad \le k_{fx}\|\boldsymbol{x}_1(k) - \boldsymbol{x}_2(k)\| + k_{fy}\|\boldsymbol{y}_1(k) - \boldsymbol{y}_2(k)\|,\end{aligned} \tag{2}$$

$$\begin{aligned}&\|\boldsymbol{g}(\boldsymbol{x}_1(k), \boldsymbol{y}_1(k), \boldsymbol{u}_1(k), k) - \boldsymbol{g}(\boldsymbol{x}_2(k), \boldsymbol{y}_2(k), \boldsymbol{u}_2(k), k)\| \\ &\quad \le k_{gx}\|\boldsymbol{x}_1(k) - \boldsymbol{x}_2(k)\| + k_{gy}\|\boldsymbol{y}_1(k) - \boldsymbol{y}_2(k)\| \\ &\quad + k_{gu}\|\boldsymbol{u}_1(k) - \boldsymbol{u}_2(k)\|,\end{aligned} \tag{2'}$$

where k_{fx}, k_{fy}, k_{gx}, k_{gy}, k_{gu} are the Lipschitz constants.

Furthermore $g_x = \frac{\partial \boldsymbol{g}((\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}, k)}{\partial \boldsymbol{x}}$, $g_y = \frac{\partial \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}, k)}{\partial \boldsymbol{y}}$, $g_u = \frac{\partial \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}, k)}{\partial \boldsymbol{u}}$ are uniformly bounded for all $(\cdot, \cdot, \cdot, \cdot) \in \Omega$. And there exist constants α_1 and α_2 such that $0 \le \alpha_1 \le g_u \le \alpha_2$.

Assumption 2. The re-initialization condition is satisfied throughout the repeated iterations, *i.e.*, $\boldsymbol{x}_n(0) = \boldsymbol{x}_d(0)$, $\boldsymbol{y}_n(0) = \boldsymbol{y}_d(0)$, $\forall n$.

Assumption 3. There exists a control input $\boldsymbol{u}_d(k)$ that can exactly drive the system output to track the desired trajectory $\boldsymbol{y}_d(k)$ for the systems (1) and (1′) on the finite time interval.

The control objective is to design an iterative learning controller $\boldsymbol{u}_n(k)$ such that the output tracking error between the desired output trajectory $\boldsymbol{y}_d(k)$ and the system output $\boldsymbol{y}_n(k)$ is within an error bound, which can be predetermined.

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2.2 ILC controller add-on to feedback controller

The discrete-time ILC controller is constructed as follows

$$\boldsymbol{u}_n(k) = \boldsymbol{u}_n^f(k) + \boldsymbol{u}_n^b(k), \tag{3}$$

$$\boldsymbol{u}_n^f(k) = \boldsymbol{u}_{n-1}^f(k) + \beta \boldsymbol{e}_{n-1}(k+1), \tag{3$'$}$$

$$\boldsymbol{u}_n^f(0) = \alpha \boldsymbol{e}_n(0) \quad \boldsymbol{u}_n^b(0) \quad \text{given} \quad \text{if} \quad k=0, \tag{3$''$}$$

$$\boldsymbol{u}_n^b(k) = \boldsymbol{u}_n(k-1) + \alpha \boldsymbol{e}_n(k), \quad \text{if} \quad k>0, \tag{3$'''$}$$

where n indicates the iteration number, and β and α are the iterative learning gain matrix and the feedback gain matrix, respectively and $\boldsymbol{e}_n(k+1) = \boldsymbol{y}_d(k+1) - \boldsymbol{y}_n(k+1)$.

3 Convergence Analysis

Two cases are considered in this paragraph. First, we consider the convergence analysis of the pure ILC control for system (1), using the iterative learning controller (3$'$), and then we consider the ILC controller with feedback controller.

Theorem 1. Under Assumptions 1-3, choose the learning gain matrix β such that $\|1 - \beta g_u\| < 1$, for $\forall g_u \in [\alpha_1, \alpha_2]$, in the learning law (3$'$). Then the output of system (1) controlled by the learning controller (3$'$) will lead to $\lim_{n\to\infty} \|\boldsymbol{u}_n^b(k) - \boldsymbol{u}_d(k)\|_\lambda \le \sigma$, $\lim_{n\to\infty} \|\boldsymbol{y}_n(k) - \boldsymbol{y}_d(k)\|_\lambda \le \sigma$ for some suitably defined constant $\sigma > 0$ that depends on $\|\delta \boldsymbol{x}_n(0)\|$ and $\|\boldsymbol{e}_n(0)\|$. In the sequel, we have $\lim_{n\to\infty} \|\boldsymbol{u}_n^b(k) - \boldsymbol{u}_d(k)\|_\lambda = 0$, $\lim_{n\to\infty} \|\boldsymbol{y}_n(k) - \boldsymbol{y}_d(k)\|_\lambda = 0$, if $\|\delta \boldsymbol{x}_n(0)\| = 0$ and $\|\boldsymbol{e}_n(0)\| = 0$.

Proof. Similar to that of Theorem 2.

Theorem 2. Under Assumptions 1-3, choose the learning gain matrix β such that $\|1 - \beta g_u\| < 1$, for $\forall g_u \in [\alpha_1, \alpha_2]$, in the learning law (3). Then the output of system (1) controlled by the learning controller with a feedback control (3)-(3$'''$) will lead to $\lim_{n\to\infty} \|\boldsymbol{u}_n^b(k) - \boldsymbol{u}_d(k)\|_\lambda \le \sigma$, $\lim_{n\to\infty} \|\boldsymbol{y}_n(k) - \boldsymbol{y}_d(k)\|_\lambda \le \sigma$ for some suitably defined constant $\sigma > 0$ which is a class-K function of $\|\delta \boldsymbol{x}_n(0)\|$, $\|\boldsymbol{e}_n(0)\|$, $\|\delta \boldsymbol{u}_n^b(0)\|$ and M. If they all equal to zero, then $\lim_{n\to\infty} \|\boldsymbol{u}_n^b(k) - \boldsymbol{u}_d(k)\|_\lambda = 0$, $\lim_{n\to\infty} \|\boldsymbol{y}_n(k) - \boldsymbol{y}_d(k)\|_\lambda = 0$.

Proof. Let $\delta \boldsymbol{x}_n(k) = \boldsymbol{x}_d(k) - \boldsymbol{x}_n(k)$, $\delta \boldsymbol{u}_n(k) = \boldsymbol{u}_d(k) - \boldsymbol{u}_n(k)$. Then from (3), we have

$$\boldsymbol{u}_{n+1}^f(k) = \boldsymbol{u}_n^f(k) + \beta \boldsymbol{e}_n(k+1), \tag{4}$$

$$\delta \boldsymbol{u}_n(k) = \boldsymbol{u}_d(k) - \boldsymbol{u}_n^f(k) - \boldsymbol{u}_n^b(k) = \delta \boldsymbol{u}_n^b(k) - \boldsymbol{u}_n^f(k) \tag{4$'$}$$

Using the differential mean value theorem, we have

$$\begin{aligned} &\boldsymbol{e}_n(k+1) = \boldsymbol{y}_d(k+1) - \boldsymbol{y}_n(k+1) = \\ &\quad \boldsymbol{g}(\boldsymbol{x}_d(k), \boldsymbol{y}_d(k), \boldsymbol{u}_d(k), k) - \\ &\quad \boldsymbol{g}(\boldsymbol{x}_n(k), \boldsymbol{y}_n(k), \boldsymbol{u}_n(k), k) = \\ &\quad \boldsymbol{g}_x(\boldsymbol{\xi}_n)\delta \boldsymbol{x}_n(k) + \boldsymbol{g}_y(\boldsymbol{\xi}_n)\boldsymbol{e}_n(k) + \boldsymbol{g}_u(\boldsymbol{\xi}_n)\delta \boldsymbol{u}_n(k) = \\ &\quad \boldsymbol{g}_x(\boldsymbol{\xi}_n)\delta \boldsymbol{x}_n(k) + \boldsymbol{g}_y(\boldsymbol{\xi}_n)\boldsymbol{e}_n(k) + \boldsymbol{g}_u(\boldsymbol{\xi}_n)\delta \boldsymbol{u}_n^b(k) - \\ &\quad \boldsymbol{g}_u(\boldsymbol{\xi}_n)\boldsymbol{u}_n^f(k), \end{aligned} \tag{5}$$

where $\boldsymbol{\xi}_n = [(\boldsymbol{x}_n(k) + \tau \delta \boldsymbol{x}_n(k))^{\mathrm{T}}, (\boldsymbol{y}_n(k) + \tau \boldsymbol{e}_n(k))^{\mathrm{T}}, (\boldsymbol{u}_n(k) + \tau \delta \boldsymbol{u}_n(k))^{\mathrm{T}}, k]^{\mathrm{T}}$, $\tau \in [0,1]$.

Inserting (5) into (4) gives

$$\begin{aligned} \boldsymbol{u}_{n+1}^f(k) = &(1 - \beta \boldsymbol{g}_u(\boldsymbol{\xi}_n)\boldsymbol{u}_n^f(k) + \beta \boldsymbol{g}_x(\boldsymbol{\xi}_n)\delta \boldsymbol{x}_n(k) + \\ &\beta \boldsymbol{g}_y(\boldsymbol{\xi}_n)\boldsymbol{e}_n(k) + \beta \boldsymbol{g}_u(\boldsymbol{\xi}_n)\delta \boldsymbol{u}_n^b(k). \end{aligned} \tag{6}$$

Taking norm on both sides of (6) yields

$$\begin{aligned} \|\boldsymbol{u}_{n+1}^f(k)\| = &\|(1 - \beta \boldsymbol{g}_u(\boldsymbol{\xi}_n)\|\|\boldsymbol{u}_n^f(k)\| + \sigma_1(\|\delta \boldsymbol{x}_n(k)\| + \\ &\|\boldsymbol{e}_n(k)\| + \|\delta \boldsymbol{u}_n^b(k)\|). \end{aligned} \tag{7}$$

with $\sigma_1 = \sup_{k\in[1,K]}(\|\beta \boldsymbol{g}_x(\boldsymbol{\xi}_n)\|, \|\beta \boldsymbol{g}_y(\boldsymbol{\xi}_n)\|, \|\beta \boldsymbol{g}_u(\boldsymbol{\xi}_n)\|)$.

From (1) and (2), we can get

$$\|\delta \boldsymbol{x}_n(k)\| \le k_{fx}\|\delta \boldsymbol{x}_n(k-1)\| + k_{fy}\|\boldsymbol{e}_n(k-1)\| \tag{8}$$

From (2$'$) and (4$'$), we have

$$\begin{aligned} \|\boldsymbol{e}_n(k)\| \le\ &k_{gx}\|\delta \boldsymbol{x}_n(k-1)\| + k_{gy}\|\boldsymbol{e}_n(k-1)\| + \\ &k_{gu}\|\delta \boldsymbol{u}_n(k-1)\| \le \\ &k_{gx}\|\delta \boldsymbol{x}_n(k-1)\| + k_{gy}\|\boldsymbol{e}_n(k-1)\| + \\ &k_{gu}(\|\delta \boldsymbol{u}_n^b(k-1)\| + \|\boldsymbol{u}_n^f(k-1)\| \end{aligned} \tag{9}$$

By (9), we get

$$\begin{aligned} &\|\delta \boldsymbol{u}_n^b(k)\| = \|\boldsymbol{u}_d(k) - \boldsymbol{u}_n(k-1) - \alpha \boldsymbol{e}_n(k)\| = \\ &\quad \|\boldsymbol{u}_d(k) - \boldsymbol{u}_n^f(k-1) - \boldsymbol{u}_n^b(k-1) - \alpha \boldsymbol{e}_n(k)\| \le \\ &\quad \|\delta \boldsymbol{u}_n^b(k-1)\| + \|\boldsymbol{u}_n^f(k-1)\| + \\ &\quad \alpha\|\boldsymbol{e}_n(k)\| + \|\triangle \boldsymbol{u}_d(k)\| \le \\ &\quad \|\delta \boldsymbol{u}_n^b(k-1)\| + \|\boldsymbol{u}_n^f(k-1)\| + \\ &\quad \alpha[k_{gx}\|\delta \boldsymbol{x}_n(k-1)\| + k_{gy}\|\boldsymbol{e}_n(k-1)\| + \\ &\quad k_{gu}\|\delta \boldsymbol{u}_n^b(k-1)\| + k_{gu}\|\boldsymbol{u}_n^f(k-1)\|] \\ &\quad + \|\triangle \boldsymbol{u}_d(k)\| = \\ &\quad (1 + \alpha k_{gu})(\|\delta \boldsymbol{u}_n^b(k-1)\| + \|\boldsymbol{u}_n^f(k-1)\|) + \\ &\quad \alpha k_{gx}\|\delta \boldsymbol{x}_n(k-1)\| + \\ &\quad k_{gy}\|\boldsymbol{e}_n(k-1)\| + \|\triangle \boldsymbol{u}_d(k)\|, \end{aligned} \tag{10}$$

where $\triangle \boldsymbol{u}_d(k) = \boldsymbol{u}_d(k) - \boldsymbol{u}_d(k-1)$

Adding (8), (9), and (10), using Assumption 2, gives

$$\begin{aligned} &(\|\delta \boldsymbol{x}_n(k)\| + \|\boldsymbol{e}_n(k)\| + \|\delta \boldsymbol{u}_n^b(k)\|) \le \\ &\quad \sigma_2(\|\delta \boldsymbol{x}_n(k-1)\| + \\ &\quad \|\boldsymbol{e}_n(k-1)\| + \|\delta \boldsymbol{u}_n^b(k-1)\|) + \\ &\quad (k_{gu} + 1 + \alpha k_{gu})\|\boldsymbol{u}_n^f(k-1)\| + \\ &\quad \|\triangle \boldsymbol{u}_d(k)\| \le \\ &\quad \ldots \le \sigma_2^k(\|\delta \boldsymbol{x}_n(0)\| + \|\boldsymbol{e}_n(0)\| + \|\delta \boldsymbol{u}_n^b(0)\|) + \\ &\quad (k_{gu} + 1 + \alpha k_{gu})\sum_{i=0}^{k-1}\sigma_2^{k-i-1}\|\boldsymbol{u}_n^f(i)\| + \\ &\quad \sum_{i=1}^{k}\sigma_2^{k-i}\|\triangle \boldsymbol{u}_d(i)\| \le \\ &\quad \sigma_2^k(\|\delta \boldsymbol{x}_n(0)\| + \|\boldsymbol{e}_n(0)\| + \|\delta \boldsymbol{u}_n^b(0)\|) + \\ &\quad (k_{gu} + 1 + \alpha k_{gu})\sum_{i=0}^{k-1}\sigma_2^{k-i-1}\|\boldsymbol{u}_n^f(i)\| + \\ &\quad M\frac{\sigma_2^K - 1}{\sigma_2 - 1}, \end{aligned} \tag{11}$$

where $\sigma_2 = \sup_{k\in[0,K]}\{(k_{fx} + k_{gx} + \alpha k_{gx}), (k_{fy} + k_{gy} + \alpha k_{gy}), (k_{gu} + 1 + \alpha k_{gu})\}$, and $M = max_{k\in[0,K]}\|\triangle \boldsymbol{u}_d(k)\|$. Without loss of generality, we will assume $\sigma_2 > 1$ for the following discussions.

Multiplying $\sigma_2^{-\lambda k}$ on both sides of (11) on interval $[0, K]$

and then taking supreme norm, we have

$$\begin{aligned}&(\|\delta \boldsymbol{x}_n(k)\|_\lambda+\|\boldsymbol{e}_n(k)\|_\lambda+\|\delta \boldsymbol{u}_n^b(k)\|_\lambda) \leq \\ &\quad(\|\delta \boldsymbol{x}_n(0)\|+\|\boldsymbol{e}_n(0)\|+\|\delta \boldsymbol{u}_n^b(0)\|)+ \\ &\quad(k_{gu}+1+\alpha k_{gu})\|\boldsymbol{u}_n^f(k)\|_\lambda \frac{1-\sigma_2^{-(\lambda-1)K}}{\sigma_2^\lambda-\sigma_2}+ \\ &\quad M\frac{\sigma_2^K-1}{\sigma_2-1}.\end{aligned} \tag{12}$$

Inserting (12) into (7) gives

$$\begin{aligned}&\|\boldsymbol{u}_{n+1}^f(k)\|_\lambda \leq \|(1-\beta g_u(\boldsymbol{\xi}_n))\|\|\boldsymbol{u}_n^f(k)\|_\lambda+ \\ &\quad\sigma_1(\|\delta \boldsymbol{x}_n(0)\|+\|\boldsymbol{e}_n(0)\|+\|\delta \boldsymbol{u}_n^b(0)\|)+ \\ &\quad\sigma_1(k_{gu}+1+\alpha k_{gu})\|\boldsymbol{u}_n^f(k)\|_\lambda \frac{1-\sigma_2^{-(\lambda-1)K}}{\sigma_2^\lambda-\sigma_2}+ \\ &\quad\sigma_1 M\frac{\sigma_2^K-1}{\sigma_2-1}.\end{aligned} \tag{13}$$

Equation (13) can be rewritten as

$$\begin{aligned}&\|\boldsymbol{u}_{n+1}^f(k)\|_\lambda \leq [\|(1-\beta g_u(\boldsymbol{\xi}_n))\|+ \\ &\quad\sigma_1(k_{gu}+1+\alpha k_{gu})\frac{1-\sigma_2^{-(\lambda-1)K}}{\sigma_2^\lambda-\sigma_2}]\|\boldsymbol{u}_n^f(k)\|_\lambda+ \\ &\quad\sigma_1(\|\delta \boldsymbol{x}_n(0)\|+\|\boldsymbol{e}_n(0)\|+\|\delta \boldsymbol{u}_n^b(0)\|)+ \\ &\quad\sigma_1 M\frac{\sigma_2^K-1}{\sigma_2-1}.\end{aligned} \tag{14}$$

Choosing a sufficiently large constant λ such that the following inequality holds when contractive condition $\|(1-\beta g_u(\boldsymbol{\xi}_n))\|<1$ is satisfied,

$$\begin{aligned}&\|(1-\beta g_u(\boldsymbol{\xi}_n))\|+\sigma_1(k_{gu}+1+\alpha k_{gu})\frac{1-\sigma_2^{-(\lambda-1)K}}{\sigma_2^\lambda-\sigma_2} \\ &\quad\leq \rho<1,\end{aligned} \tag{15}$$

then (14) gives

$$\|\boldsymbol{u}_{n+1}^f(k)\|_\lambda \leq \rho\|\boldsymbol{u}_n^f(k)\|_\lambda+\varepsilon, \tag{16}$$

where $\varepsilon=\sigma_1(\|\delta \boldsymbol{x}_n(0)\|+\|\boldsymbol{e}_n(0)\|+\|\delta \boldsymbol{u}_n^b(0)\|)+\sigma_1 M\frac{\sigma_2^K-1}{\sigma_2-1}$.

Equation (16) means

$$\lim_{n\to\infty}\|\boldsymbol{u}_{n+1}^f(k)\|_\lambda \leq \frac{\varepsilon}{1-\rho} \tag{17}$$

From (12), we can obtain

$$\begin{aligned}&\lim_{n\to\infty}(\|\delta \boldsymbol{x}_n(k)\|_\lambda+\|\boldsymbol{e}_n(k)\|_\lambda+\|\delta \boldsymbol{u}_n^b(k)\|_\lambda) \leq \\ &\quad(\|\delta \boldsymbol{x}_n(0)\|+\|\boldsymbol{e}_n(0)\|+\|\delta \boldsymbol{u}_n^b(0)\|)+ \\ &\quad(k_{gu}+1+\alpha k_{gu})\frac{1-\sigma_2^{-(\lambda-1)K}}{\sigma_2^\lambda-\sigma_2}\lim_{n\to\infty}\|\boldsymbol{u}_n^f(k)\|_\lambda+ \\ &\quad M\frac{\sigma_2^K-1}{\sigma_2-1}.\end{aligned} \tag{18}$$

Then from (17), (18), we can reach the conclusion of this theorem. □

Remark 1. This theorem reveals that the ILC component will play a complementary role in control design, while the feedback component plays the dominant role.

Remark 2. Note that the learning controller design is independent of the feedback controller. Hence the closed-loop characteristics will not be changed by the addition of the ILC part. Thus, whenever necessary, we can simply switch off either of the control module and the remaining one will still work well.

Remark 3. By Theorem 2, $\|\boldsymbol{u}_n^f(k)\|_\lambda \to 0$ when the convergence is obtained. This implies that the control system will be dominated by the feedback controller, and the ILC feedforward is equivalently off.

Remark 4. The underlying idea of this new feedback/feedforward configuration is to learn and reject the repeatable and non-repeatable uncertainties. Learning mechanism is designed to identify all those repeatable components and leave the remaining unknown iteration-dependent components to the feedback control scheme.

Remark 5. The effectiveness, and the advantages, compared with those of [7], of the proposed iterative learning controller and the ILC add-on to the feedback controller have been verified through intensive simulations. Here the results are omited just due to the limitation of paper length.

4 Conclusion

A discrete iterative learning controller with a new feedforward-feedback configuration in which the iterative learning control is add-on to the feedback controller is proposed for the discrete-time nonlinear time-varying systems with initial state error and initial output error. A systematic approach is developed to analyze the convergence of the learning system. It is shown that the feedforward ILC component add-on to the feedback controller does not change any closed loop characteristics and the feedback controller still play the dominant role in the combined control strategy. Furthermore, it is noted that the learning controller design is completely decoupled from the feedback controller. The feedback controller and ILC can work concurrently as two independent modules without interfering with each others. Whenever necessary, we can simply switch off one control module and the remaining one will still work well. It is a perfect modularized fashion in control system design.

References

1 Arimoto S, Kawamura S, Miyazaki F. Bettering operation of robots by learning. *Journal of Robotic Systems*, 1984, **1**(2): 123~140

2 Hwang D H, Bien Z, Oh S R. Iterative learning control method for discrete-time dynamic systems. *IEE Proceedings Algorithms, Control Systems, Control Theory*, 1991, **138**(2): 139~144

3 Jang, T J, Ahn H S, Choi, C H. Iterative learning control for discrete time nonlinear systems. *International Journal of Systems Science*, 1994, **25**(7), 1179~1189

4 Chen Y C, Wen C Y. *Iterative Learning Control: Convergence Robustness and Applications*. Lecture Notes in Control and Information Sciences, Springer, 1999, **248**

5 Xu J X. Analysis of iterative learning control for a class of nonlinear discrete-time systems. *Automatica*, 1997, **33**(10): 1905~1907

6 Chien C J, Liu, J S. A P-type iterative learning controller for robust output tracking of nonlinear time-varying systems. *International Journal of Control*, 1996, **64** (2): 319~334

7 Chen C J. A discrete iterative learning control for a class of nonlinear time-varying systems. *IEEE Transactions on Automatic Control*, 1998, **43**(5): 748~752

8 Atkeson C G, McIntyre J. Robot trajectory learning through practice. In: Proceedings of 1986 IEEE International Conference on Robot and Automatic Conference, 1986, 1737~1742

9 Kuc T Y, Lee J S, Nam K. An iterative learning control theory for a class of nonlinear dynamic systems. *Automatica*, 1992, **28**(6): 1215~1221

10 Jang T J, Choi C H, Ahn H S. Iterative learning control in feedback systems. *Automatica*, 1995, **31**(2): 243∼248

11 Xu J X, Tan Y. Linear and Nonlinear Iterative Learning Control. Lecture Notes in Control and Information Sciences. Springer, 2003, **291**

12 Xu J X, Cao W J. Learning variable structure control approaches for repeatable tracking control tasks. *Automatica*, 2001, **37**(7): 997∼1006

13 Tan K K, Tang J C. Learning-enhanced PI control of ramvelocity in injection molding machines. *Engineering Applications of Artificial Intelligence*, 2002, **15**(1): 65∼72

14 Hou Z S, Xu J X, Zhong H W. Freewat traffic control using iterative learning control based remp metering and speed signaling. *IEEE Transactions on Vehicular Technology, to be published.*

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