Decoupling Control of 5 Degrees of Freedom Bearingless Induction Motors Using α-th Order Inverse System Method

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Abstract A 5-degrees-of-freedom bearingless induction motor is a multi-variable, nonlinear and strong-coupled system. In order to achieve rotor suspension and operation steadily, it is necessary to realize dynamic decoupling control among torque and suspension forces. In the paper, a method based on α-th order inverse system theory is used to study dynamic decoupling control. Firstly, the working principles of a 3-degrees-of-freedom magnetic bearing and a 2-degrees-of-freedom bearingless induction motor are analyzed, the radial-axial force equations of 3-degrees-of-freedom magnetic bearing, the electromagnetic torque equation and radial force equations of the 2-degrees-of-freedom bearingless induction motor are given, and then the state equations of the 5-degrees-of-freedom bearingless induction motor are set up. Secondly, the feasibility of decoupling control based on dynamic inverse theory is discussed in detail, and the state feedback linearization method is used to decouple and linearize the system. Finally, linear control system techniques are applied to these linearization subsystems to synthesize and simulate. The simulation results have shown that this kind of control strategy can realize dynamic decoupling control among torque and suspension forces of the 5-degrees-of-freedom bearingless induction motor, and that the control system has good dynamic and static performance.

Key words Bearingless induction motor, magnetic bearing, inverse system, feedback linearization, decoupling control

1 Introduction

The technologies of bearingless induction motors are a great breakthrough in areas of induction motors and magnetic bearings. They use comparability of structures between magnetic bearings and motor. The windings which produce the suspension forces of the magnetic bearing are put in the stator slot of the induction motor. Compared with traditional motors suspended by magnetic bearings, the bearingless induction motor not only retains the advantages of no lubrication, no friction and no abrasion, but also reduces size and weight of motors, because the suspension force windings no longer occupy added axial room. It changes the structure and drive ways of conventional motors radically and provides technic methods for special electric drives. It has shown great scientific research and application values in fields such as high-speed precision machining, aeronautics and astronautics, energy sources, traffic, life sciences, *etc*[1~3].

The interactional magnetism, which is produced by suspension force windings and armature windings, can let the rotor suspend. There are nonlinear couplings among the torque subsystem, flux linkage subsystem and radial force subsystems. So compared with general motors, the bearingless induction motor is a complicated multi-variable nonlinear and strong-coupled system. In order to make the bearingless induction motor operate steadily, it is necessary to realize nonlinear decoupling control[1,4].

At present, many scholars are carrying out research of the bearingless induction motor. They realized decoupling control among torque force and suspension forces, based on rotor magnetic field oriented or gas magnetic field oriented control method. However, they did not consider time varying characteristic of parameters, which may influence performance of vector decoupling control and is difficult to realize dynamic decoupling control[4]. In the paper, an innovative 5-degrees-of-freedom bearingless induction motor is proposed, which is composed of a 3-degrees-of-freedom axial-radial magnetic bearing and a 2-degrees-of-freedom bearingless induction motor. In order to make the innovative induction motor operate steadily and attain good dynamic and static performance, it is necessary to control the radial suspension forces, axial suspension forces and the torque of the bearingless induction motor independently. A method based on α-th order inverse system theory is used to study the dynamic decoupling control. The feasibility of decoupling control based on dynamic inverse theory for bearingless induction motor is discussed in detail. The state feedback linearization method is used for decoupling control and linearizing the system, and the linear control techniques are applied to linearization subsystems in synthesis and simulation[5~6].

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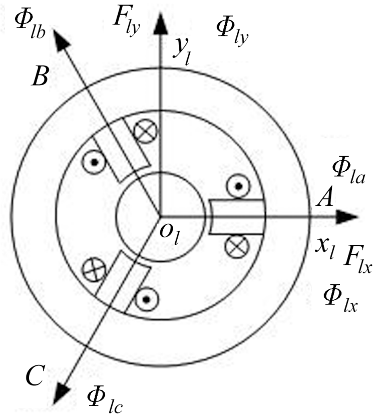
2 Decoupling control of bearingless induction motor

2.1 $\alpha-$th order inverse system method

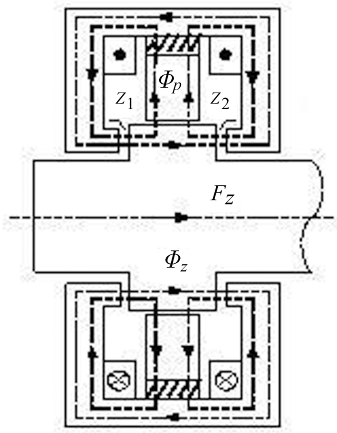
The α-th order inverse system method is to use feedback linearization method to study the system design theory[5]. The basic idea is: firstly, an α-th order inverse system is constructed, which can be realized by feedback linearization method using the inverse model of system object; then the system is transformed to a linear system, namely pseudo-linear system; finally, the linearity system theory is used to synthetize the system.

2.2 Suspension force equations of 3-degrees-of-freedom magnetic bearings

Fig. 1 shows the structure of a 3-degrees-of-freedom radial and axial magnetic bearing[7~9], where Φ_{la}, Φ_{lb} and Φ_{lc} are the magnet fluxes of the windings in A, B and C axes; Φ_{lx} and Φ_{ly} are the equivalent magnet fluxes projected from Φ_{la}, Φ_{lb} and Φ_{lc} to the axes of x_l and y_l; i_{la}, i_{lb} and i_{lc} are the currents of the windings in A, B and C axes; i_{la} and i_{lb} are the currents of equivalent windings in x_l and y_l axes. Because of suction produced by Φ_{la}, Φ_{lb} and Φ_{lc}, the rotor is always on the balance central position. If the rotor is moved from the balance position by the outside interference force, the magnitude and direction of i_{la}, i_{lb} and i_{lc} will be changed accordingly, and the rotor will be returned to the central position.



(a) 3-pole radial magnetic bearing



(b) Magnetic circuit of axial magnetic bearing

Fig. 1 3-degrees-of-freedom radial-axial magnetic bearing

The Maxwell forces $F_{lx}$and F_{ly}, generated by the composite fluxes of Φ_{la}, Φ_{lb} and Φ_{lc}, are projected to the x_l and y_l axes in Fig. 1(a) as follows

$$\begin{bmatrix} F_{lx} \\ F_{ly} \end{bmatrix} = k_{ir} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_{la} \\ i_{lb} \\ i_{lc} \end{bmatrix}$$
$$= k_{ir} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \times$$
$$\begin{bmatrix} i_{lx} \\ i_{ly} \end{bmatrix} = k_{ir} \begin{bmatrix} 3/2 & 0 \\ 0 & 3/2 \end{bmatrix} \begin{bmatrix} i_{lx} \\ i_{ly} \end{bmatrix} \tag{1}$$

where $k_{ir} = \dfrac{\mu_0 \cdot F_m \cdot N_r}{3(\delta_z/2S_z + \delta_r/3S_r)\delta_r}$, k_{ir} is the radial current coefficient; μ_0 is vacuum permeability; δ_r is the length of radial air gap; F_m is magnetic motive force of the permanent magnet; S_z is the length of axial pole area; S_r is radial pole area; N_r is the turns of radial force windings.

In Fig. 1(b), Φ_z is the magnet flux of axial force windings, Φ_p is the magnet flux of permanent magnet. When the rotor is on the balance central position, Φ_p is symmetrical. If the rotor is moved from balance position, the magnitude and direction of i_z will be changed accordingly, and the rotor will be returned to the central position. The rotor acted by the axial force F_z is as follows

$$F_z = k_{iz} i_z + k_z z \tag{2}$$

where k_{iz} is the axial current coefficient, $k_{iz} = \dfrac{\mu_0 \cdot F_m \cdot N_z}{(\delta_z/2S_z + \delta_r/3S_r)\delta_z}$; k_z is axial displacement coefficient, $k_z = -\dfrac{\mu_0 \cdot F_m^2}{2(\delta_z/2S_z + \delta_r/3S_r)^2 \delta_r S_z}$; δ_z is axial air gap; N_z is the turns of axial windings.

2.3 Principle of radial force generation of 2-degrees-of-freedom bearingless induction motors

2-pole radial force windings and 4-pole torque windings are wound together in stator slots of bearingless induction motor. The rotation magnetic field and torque are produced by the 4-pole windings. When the rotation magnetic field produced by torque windings and the magnetic field produced by radial force windings satisfy the following three conditions: 1)$P_4 = P_2 \pm 1$, 2) The two magnetic fields have the same rotation direction, 3) The currents which produce the magnetic field have the same frequency, then the interactive magnetic fields will produce radial suspension forces and make the rotor suspend on the balance central position[1~3].

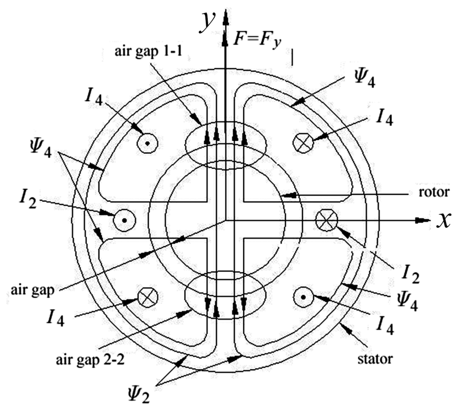


Fig. 2 The principle of producing radial suspension forces of bearingless motors

Fig. 2 shows the working principle of the bearingless induction motor. When 4-pole torque windings and 2-pole radial suspension windings are electrified by I_4 and I_2 as shown, they will generate in the same direction 4-pole torque flux linkage ψ_4 and 2-pole radial flux linkage ψ_2 in air gap 1-1. And the whole flux linkage will be increased to ψ_4+ψ_2, the electromagnetic suction force will be increased. While in air gap 2-2, ψ_4 and ψ_2 have the opposition direction, and the compound flux linkage will be decreased to ψ_4-ψ_2, and electromagnetic suction force will also be decreased. Therefore, the rotor will be effected by electromagnetic compound force in the y positive direction, the rotor will be moved up. If the direction of current in suspension force windings is changed, the radial electromagnetic compound force in y negative direction will be generated. In the same way, electromagnetic compound force will be produced in x direction. So the rotor can be suspended steadily in the balance central position by adjusting the magnitude and direction of the currents in radial suspension force windings.

2.4 Analysis of radial force on 2-degrees-of-freedom bearingless induction motors

The electromagnetic couplings of the bearingless induction motor are very complex, because there are couplings between the 2-pole windings and 4-pole windings, and there are couplings between the windings themselves. In order to analyse easily, 2-phase windings in rotation coordinate, which has been changed from 3-phase windings in static coordinate through $C_{3/2}$ and $C_{r/s}$ transform, is studied. Because 2-phase coordinate of the rotation coordinate are plumb each other, the mutual inductance value among 4-pole windings or 2-pole windings is 0, the self-inductance L_{4s} of torque windings and the self-inductance L_{2s} of radial force windings are constants. The inductance matrix L of the motor can be written as

$$L = \begin{bmatrix} L_{4s} & 0 & -M\alpha & M\beta \\ 0 & L_{4s} & M\beta & M\alpha \\ -M\alpha & M\beta & L_{2s} & 0 \\ M\beta & M\alpha & 0 & L_{2s} \end{bmatrix} \tag{3}$$

where α and β are the rotor radial displacement in the x- and y-directions; M is the mutual inductance coefficient of 4-pole windings and 2-pole windings; the subscript expression s is the component in the stator side.

According to the relationship of energy conversion, the magnetic energy stored in the windings can be written as

$$\begin{cases} W_m = \frac{1}{2} I^{\mathrm{T}} L I \\ I = [i_{d4s} \quad i_{q4s} \quad i_{d2s} \quad i_{q2s}]^{\mathrm{T}} \end{cases} \tag{4}$$

where i_{d4s} and i_{q4s} are the 4-pole windings current components in d-q coordinate, respectively; i_{d2s} and i_{q2s} are the 2-pole windings current components in d-q coordinate, respectively.

Neglecting magnetic saturation, the radial forces F_{rx} and F_{ry} in the x- and y-directions can be written as

$$\begin{bmatrix} F_{rx} \\ F_{ry} \end{bmatrix} = \begin{bmatrix} \partial W_m / \partial \alpha \\ \partial W_m / \partial \beta \end{bmatrix} = M \begin{bmatrix} -i_{d4s} & i_{q4s} \\ i_{q4s} & i_{d4s} \end{bmatrix} \begin{bmatrix} i_{d2s} \\ i_{q2s} \end{bmatrix} \tag{5}$$

2.5 Electromagnetic torque of bearingless induction motors

Because the magnetic field produced by radial force windings is much smaller than the magnetic field produced by torque windings, by neglecting magnetic field produced by the radial force windings, the rotor flux linkage satisfies the following equations.

$$\begin{cases} \dot{\psi}_{dr} = -\frac{1}{T_r}\psi_{dr} - \omega_r \psi_{q4s} + \frac{L_{m4r}}{T_r} i_{d4s} \\ \dot{\psi}_{qr} = -\frac{1}{T_r}\psi_{qr} + \omega_r \psi_{d4s} + \frac{L_{m4r}}{T_r} i_{q4s} \end{cases} \tag{6}$$

The torque equation for bearingless induction motor is

$$T_e = p_4 \frac{L_{m4r}}{L_r} (\psi_{dr} i_{q4s} - \psi_{qr} i_{d4s}) \tag{7}$$

where ψ_{dr} and ψ_{qr} are the components of rotor flux linkage in d-q coordinate, respectively; ω_r is the speed of the rotor; ψ_{d4s} and ψ_{q4s} are the components of stator torque flux linkage in d-q coordinate, respectively; T_r is the time constant; p_4 is the pole-pair number of torque windings; L_{m4r} is the mutual inductance between torque windings and rotor.

2.6 State equations of the 5-degrees-of-freedom bearingless induction motor

The subscript "l" denotes the 3-degrees-of-freedom magnetic bearings, the subscript "r" denotes the 2-degrees-of-freedom bearingless motor. After the analysis of forces acting on the rotor, the system motion equations of the 5-degrees-of-freedom bearingless induction motor are as follows[10]

$$\begin{cases} m\ddot{x}_l + F_{lx} = f_{lx} \\ m\ddot{y}_l + F_{ly} = f_{ly} \\ m\ddot{z} + F_z = f_z \\ m\ddot{x}_r + F_{rx} = f_{rx} \\ m\ddot{y}_r + F_{ry} = f_{ry} \\ \frac{J}{p_4}\dot{\omega}_r = T_e - T_L \end{cases} \tag{8}$$

where m is the mass of the rotor; f_{lx}, f_{ly}, f_z, f_{rx} and f_{ry} are the external disturbance forces in the directions of x_l, y_l, z, x_r and y_r axes, respectively; J is the moment of inertia of the rotor; ω_r is the mechanical rotational angular speed of the rotor; T_e and T_L are the electromagnetic torque and the load torque, respectively; F_z is the force in the z-direction.

State variables are chosen as

$$X = [x_1, x_2, \cdots, x_{12}, x_{13}]^{\mathrm{T}} = \\ [x_l, y_l, z, x_r, y_r, \dot{x}_l, \dot{y}_l, \dot{z}, \dot{x}_r, \dot{y}_r, \omega_r, \psi_{dr}, \psi_{qr}]^{\mathrm{T}} \tag{9}$$

Input variables are chosen as

$$U = [u_1, u_2, \cdots, u_6, u_7]^{\mathrm{T}} = [i_{lx}, i_{ly}, i_z, i_{d4s}, i_{q4s}, i_{d2s}, i_{q2s}]^{\mathrm{T}} \tag{10}$$

Output variables are chosen as

$$Y = [y_1, y_2, y_3, y_4, y_5, y_6, y_7]^{\mathrm{T}} = [x_l, y_l, z, x_r, y_r, \omega_r, \psi_r]^{\mathrm{T}} \tag{11}$$

From (1)~(2) and (5) ~ (10), the state equations of the system are written as

$$\begin{cases} \dot{x}_1 = x_6 \\ \dot{x}_2 = x_7 \\ \dot{x}_3 = x_8 \\ \dot{x}_4 = x_9 \\ \dot{x}_5 = x_{10} \\ \dot{x}_6 = \frac{1}{m}(-\frac{3}{2}k_{ir}u_1 + f_{lx}) \\ \dot{x}_7 = \frac{1}{m}(-\frac{3}{2}k_{ir}u_2 + f_{ly}) \\ \dot{x}_8 = \frac{1}{m}(-k_{iz}u_3 - k_z x_3 + f_z) \end{cases}$$

and

$$\begin{cases} \dot{x}_9 = \frac{M}{m}(u_4 u_6 - u_5 u_7) + \frac{1}{m} f_{rx} \\ \dot{x}_{10} = -\frac{M}{m}(u_5 u_6 + u_4 u_7) + \frac{1}{m} f_{ry} \\ \dot{x}_{11} = \frac{P_4^2 L_{m4r}}{J L_r}(x_{12} u_5 - x_{13} u_4) - \frac{P_4}{J} T_L \\ \dot{x}_{12} = -\frac{1}{T_r} x_{12} - x_{11} x_{13} + \frac{L_{m4r}}{T_r} u_4 \\ \dot{x}_{13} = -\frac{1}{T_r} x_{13} + x_{11} x_{12} + \frac{L_{m4r}}{T_r} u_5 \end{cases} \tag{12}$$

Output equations are written as

$$\begin{aligned} Y &= [\ y_1 \quad y_2 \quad y_3 \quad y_4 \quad y_5 \quad y_6 \quad y_7\ ]^{\mathrm{T}} \\ &= [\ x_l \quad y_l \quad z \quad x_r \quad y_r \quad \omega_r \quad \psi_r\ ]^{\mathrm{T}} \\ &= [\ x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_{11} \quad \sqrt{x_{12}^2 + x_{13}^2}\ ]^{\mathrm{T}} \end{aligned} \tag{13}$$

It can be seen from (10) $\sim$ (13) that the state equation of the 5-degrees-of-freedom bearingless induction motor is a 7-input and 7-output nonlinear and strong-coupled MIMO system.

2.7 Analysis of linearization decoupling control based on inverse system theory

From (12) and (13), we can obtain

$$\begin{cases} \dot{y}_1 = \dot{x}_1 = x_6 \\ \ddot{y}_1 = \dot{x}_6 = \frac{1}{m}(-\frac{3}{2}k_{ir}u_1 + f_{lx}) \\ \dot{y}_2 = \dot{x}_2 = x_7 \\ \ddot{y}_2 = \dot{x}_7 = \frac{1}{m}(-\frac{3}{2}k_{ir}u_2 + f_{ly}) \\ \dot{y}_3 = \dot{x}_3 = x_8 \\ \ddot{y}_3 = \dot{x}_8 = \frac{1}{m}(-k_{iz}u_3 - k_z z + f_z) \\ \dot{y}_4 = \dot{x}_4 = x_9 \\ \ddot{y}_4 = \dot{x}_9 = \frac{M}{m}(u_4u_6 - u_5u_7) + \frac{1}{m}f_{rx} \\ \dot{y}_5 = \dot{x}_5 = x_{10} \\ \ddot{y}_5 = \dot{x}_{10} = -\frac{M}{m}(u_5u_6 + u_4u_7) + \frac{1}{m}f_{ry} \\ \dot{y}_6 = \dot{x}_{11} = \frac{P_4^2 L_{m4r}}{JL_r}(x_{12}u_5 - x_{13}u_4) - \frac{P_4}{J}T_L \\ \dot{y}_7 = \dot{\psi}_r = -\frac{1}{T_r}\psi_r + \frac{L_{m4r}}{T_r}\frac{1}{\psi_r}(u_4x_{12} + u_5x_{13}) \end{cases} \tag{14}$$

Choose

$$A(U) = \begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \\ \ddot{y}_3 \\ \ddot{y}_4 \\ \ddot{y}_5 \\ \dot{y}_6 \\ \dot{y}_7 \end{bmatrix} = \begin{bmatrix} \dot{x}_6 = \frac{1}{m}(-\frac{3}{2}k_{ir}u_1 + f_{lx}) \\ \dot{x}_7 = \frac{1}{m}(-\frac{3}{2}k_{ir}u_2 + f_{ly}) \\ \dot{x}_8 = \frac{1}{m}(-k_{iz}u_3 - k_z z + f_z) \\ \dot{x}_9 = \frac{M}{m}(u_4u_6 - u_5u_7) + \frac{1}{m}f_{rx} \\ \dot{x}_{10} = -\frac{M}{m}(u_5u_6 + u_4u_7) + \frac{1}{m}f_{ry} \\ \dot{x}_{11} = \frac{P_4^2 L_{m4r}}{JL_r}(x_{12}u_5 - x_{13}u_4) - \frac{P_4}{J}T_L \\ \dot{\psi}_r = -\frac{1}{T_r}\psi_r + \frac{L_{m4r}}{T_r}\frac{1}{\psi_r}(u_4x_{12} + u_5x_{13}) \end{bmatrix} \tag{15}$$

Take the derivatives of $A(U)$: $\frac{\partial}{\partial U}\left[y_1^{(2)}\right]$, $\frac{\partial}{\partial U}\left[y_2^{(2)}\right]$, $\frac{\partial}{\partial U}\left[y_3^{(2)}\right]$, $\frac{\partial}{\partial U}\left[y_4^{(2)}\right]$, $\frac{\partial}{\partial U}\left[y_5^{(2)}\right]$, $\frac{\partial}{\partial U}\left[y_6^{(1)}\right]$, $\frac{\partial}{\partial U}\left[y_7^{(1)}\right]$. So rank $\left[\frac{\partial A}{\partial U}\right]$=7, and the matrix $\frac{\partial A}{\partial U}$ is nonsingular. The relative orders of the system are as follows

$$\alpha = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7) = (2, 2, 2, 2, 2, 1, 1)$$

It is easy to obtain that $\sum\limits_{i=1}^{r} \alpha_i = 12$. And the order of state equation of the system is 13, so the system is invertible. The state feedback linearization method is adopted. Now suppose

$$\begin{cases} u_1 = \phi_1 \\ u_2 = \phi_2 \\ u_3 = \phi_3 \\ u_4u_6 - u_5u_7 = \phi_4 \\ u_5u_6 + u_4u_7 = \phi_5 \\ x_{12}u_5 - x_{13}u_4 = \phi_6 \\ \frac{1}{\psi_r}(u_4x_{12} + u_5x_{13}) = \phi_7 \end{cases} \tag{16}$$

From (16), the formulas of state feedback arithmetic are as follows

$$\begin{cases} u_1 = \phi_1 \\ u_2 = \phi_2 \\ u_3 = \phi_3 \\ u_4 = -\frac{x_{13}}{\psi_r^2}\phi_6 + \frac{x_{12}}{\psi_r}\phi_7 \\ u_5 = \frac{x_{12}}{\psi_r^2}\phi_6 + \frac{x_{13}}{\psi_r}\phi_7 \\ u_6 = \frac{u_4}{u_4^2+u_5^2}\phi_4 + \frac{u_5}{u_4^2+u_5^2}\phi_5 \\ u_7 = -\frac{u_5}{u_4^2+u_5^2}\phi_4 + \frac{u_4}{u_4^2+u_5^2}\phi_5 \end{cases} \tag{17}$$

After adopting the state feedback matrix described as (17), the system will be turned into linear system expressed in (18) without couplings. Substituting $f_{lx} = k_s \cdot x_l$, $f_{ly} = k_s \cdot y_l$, $f_z = k_s \cdot z$, $f_{rx} = k_s \cdot x_r$ and $f_{ry} = k_s \cdot y_r$ into (14), and combining with (13) and (15), the system will be transformed into linear system as follows

$$\begin{cases} \ddot{x}_l = -\frac{3}{2m}k_{ir}\phi_1 + \frac{k_s}{m}x_l \\ \ddot{y}_l = -\frac{3}{2m}k_{ir}\phi_2 + \frac{k_s}{m}y_l \\ \ddot{z} = -\frac{1}{m}k_{iz}\phi_3 - \frac{1}{m}k_z x_3 + \frac{k_s}{m}z \\ \ddot{x}_r = \frac{M}{m}\phi_4 + \frac{k_s}{m}x_r \\ \ddot{y}_r = -\frac{M}{m}\phi_5 + \frac{k_s}{m}y_r \\ \dot{\omega}_r = \frac{P_4^2 L_{m4r}}{JL_r}\phi_6 - \frac{P_4}{J}T_L \\ \dot{\psi}_r = -\frac{1}{T_r}\psi_r + \frac{L_{m4r}}{T_r}\phi_7 \end{cases} \tag{18}$$

3 Synthetizing system

3.1 Synthetizing position of rotor system

The normalized linear system described in (18) can be synthesized using the linear system theory. The former five rows of (18) are the x_l, y_l, z, x_r and y_r displacement subsystems of the 5-degrees-of-freedom bearingless induction motor which belongs to the second-order integral system.

For example, the transfer function of the displacement system of the rotor in the x_r-direction is as follows

$$G_k(s) = x(s)/\phi_4(s) = M/m \cdot s^2 \tag{19}$$

The characteristic equation of the system is as follows

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \tag{20}$$

The parameters $\omega_n$and ξ are chosenas ω_n=800 rad/s, $\xi=\sqrt{2}/2$, and the transfer function of state feedback is as follows

$$a_0 s + a_1 = 2\xi\omega_n m/M \cdot s + \omega_n^2 m/M \tag{21}$$

The closed loop transfer function of the system can be obtained as follows

$$G(s) = \frac{6.4 \times 10^5}{s^2 + 1132s + 6.4 \times 10^5} \tag{22}$$

The system overshoot σ is 4.3%, the adjusting time t_s is 7.06 ms.

3.2 Synthetizing speed system

The sixth row of (18) is the subsystem of the speed ω_r, which belongs to the first-order integral system. The transfer function of the speed subsystem can be chosen as

$$G_k(s) = \frac{\omega_r(s)}{\phi_6(s)} = \frac{P_4^2 L_{m4r}}{JL_r} \cdot \frac{1}{s} \tag{23}$$

The speed adjuster can be chosen as PI adjuster. The transfer function of the system is

$$G_c(S) = \frac{k_1(\tau s + 1)}{\tau s} \tag{24}$$

According to requirement of the design adjuster theory, G_c(s) can be chosen as follows

$$G_c(s) = \frac{2JL_r(\tau s + 1)}{P_4^2 L_{m4r}\tau^2 s} \tag{25}$$

The closed loop transfer function of the rotate speed system is

$$\Phi(s) = \frac{2\tau^{-2}(\tau s + 1)}{s^2 + 2\tau^{-1}s + 2\tau^{-2}} \tag{26}$$

4 Simulation of control system

The control strategy can be verified by computer simulation using the parameters of the designed prototype machine. The parameters of the system are as follows: The stator inductance L_s is 16.31×10^{-2} H; the rotor inductance L_r is 16.778×10^{-2} H; the mutual inductance between stator and rotor L_{m4r} is 15.856×10^{-2} H; the mutual inductance coefficient between stator torque winding and radial force winding M is 78.2 H/m; the rotor resistance r is 11.48 Ω; the time constant of the rotor T_r is 1.46×10^{-2} s; the mass of rotor m is 2.85 kg; the moment of inertia J is 0.00769 kg·m^2; the pole-pair number of torque windings P_4 is 2; the pole-pair number of suspension force windings P_2 is 3. So we can obtain the followings

1) The state feedback parameters of x_r position system are

$$a_0 = 2\xi\omega_n m/M = 41.23, \ a_1 = \omega_n^2 m/M = 23324.81$$

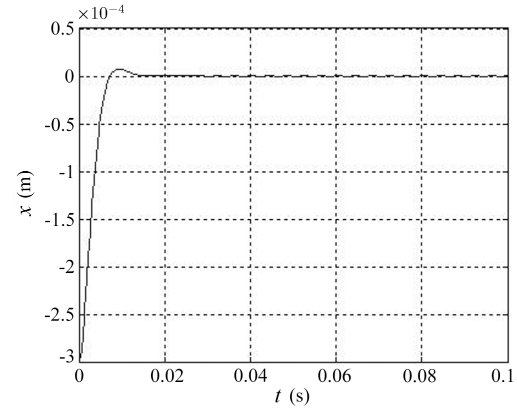
2) The adjust parameter of torque system

From (24) and (25),

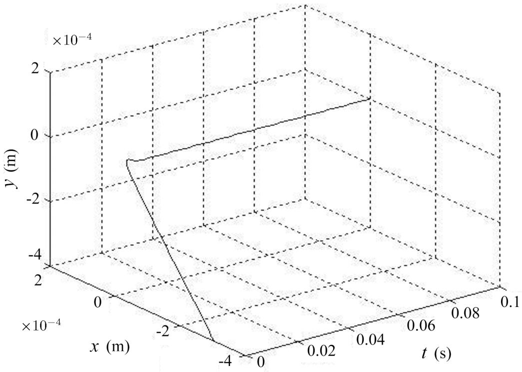
$$k_1 = \frac{2JL_r}{P_z^2 L_{m4r}\tau} = 0.041, \text{ where } \tau \text{ is } 0.1.$$

So from (26), we can obtain

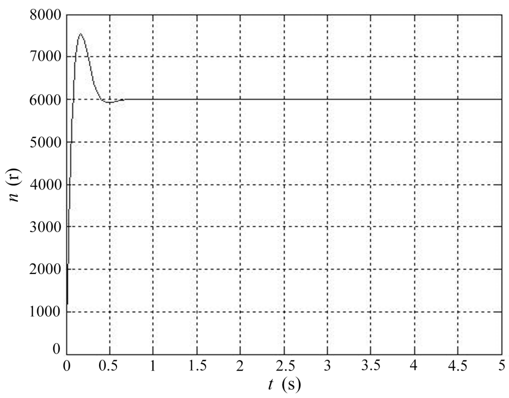
$$\Phi(s) = \frac{2\tau^{-2}(\tau s + 1)}{s^2 + 2\tau^{-1}s + 2\tau^{-2}} = \frac{20s + 200}{s^2 + 20s + 200}$$



(a) Start up displacement subsystem curve in the x-direction



(b) The trajectory of the mass center of the rotor



(c) Performance curve of speed of the bearingless induction motor

Fig. 3 Simulation results

4.1 Process of rotor rising

When the initialization of x is –0.3 mm, the displacement curve starting up in x-direction is shown in Fig. 3(a). The simulation results have shown that the steady-state error of system approaches to 0, the overshoot of system is very small and adjusting time is approach to 0.01s. When the initialization of x is –0.3 mm and y is –0.4 mm, the trajectory of mass center of rotor is shown in Fig. 3(b). The rotor position subsystem of decoupling control for bearingless induction motor has good dynamic and static performance.

4.2 System of speed

The step response of the speed subsystem of bearingless induction motor is shown in Fig. 3(c). The expectation speed is 6 000 r/min, and the simulation results have shown that the overshoot of the system is less than 5% and the adjusting time is less than 0.5s, so the speed subsystem has good performance.

5 Conclusion

1) In this paper, the decoupling control arithmetic based on α-th order inverse system theory has been used successfully in realizing dynamic decoupling control among radial displacement subsystems and torque (speed) subsystem of the 5-degrees-of-freedom bearingless induction motor.

2) Dynamic decoupling control method is realized that not only each subsystem has no coupling but also all subsystems have been linearized, therefore the system we design attains the ideal performance easily.

3) The simulation results have shown that this kind of control strategy can realize dynamic decoupling control among suspension forces and torque of the 5-degrees-of-freedom bearingless induction motor. The rotor can suspend steadily, the speed and 5-degrees-of-freedom displacements can be controlled independently. The whole system has good dynamic and static performance.

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