A Framework of Finite-model Kalman Filter with Case Study: MVDP-FMKF Algorithm

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Abstract Kalman filtering techniques have been widely used in many applications, however, standard Kalman filters for linear Gaussian systems usually cannot work well or even diverge in the presence of large model uncertainty. In practical applications, it is expensive to have large number of high-cost experiments or even impossible to obtain an exact system model. Motivated by our previous pioneering work on finite-model adaptive control, a framework of finite-model Kalman filtering is introduced in this paper. This framework presumes that large model uncertainty may be restricted by a finite set of known models which can be very different from each other. Moreover, the number of known models in the set can be flexibly chosen so that the uncertain model may always be approximated by one of the known models, in other words, the large model uncertainty is "covered" by the "convex hull" of the known models. Within the presented framework according to the idea of adaptive switching via the minimizing vector distance principle, a simple finite-model Kalman filter, MVDP-FMKF, is mathematically formulated and illustrated by extensive simulations. An experiment of MEMS gyroscope drift has verified the effectiveness of the proposed algorithm, indicating that the mechanism of finite-model Kalman filter is useful and efficient in practical applications of Kalman filters, especially in inertial navigation systems.

Key words Finite-model Kalman filter, minimizing vector distance principle, adaptive switching, MEMS gyroscope de-noising

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Inertial navigation systems (INS)[1] are widely used in many applications, and a good inertial navigation system is usually expected to be absolutely independent, all-weather and anti-interference. The gyroscope is a kernel device in the inertial navigation systems. The precision of the INS mainly depends on the precision of inertial devices, such as gyroscopes and accelerometers.

In order to improve the precision of inertial navigation systems, it is important to estimate the errors of the kernel devices and make a compensation for the inertial navigation systems[2]. In practice, one main approach is the so-called Kalman filtering, which was proposed in Kalman′s pioneering work[3], where he formulated one general state estimating problem and investigated the linear filtering and prediction problem.

The standard Kalman filter applies to linear systems with precisely known model and a priori knowledge of statistical nature on process noise and measurement noise. Following the Kalman′s basic work, many extensions and generalizations were developed to deal with the case where the conditions of standard Kalman filter cannot be strictly satisfied[4−5]. Stanley F. Schmidt adopted the idea of linearization around the trajectory and solves filtering and prediction problems of nonlinear systems in the 1960s, which was named as extend Kalman filter (EKF) later[6−7]. Using the idea of Taylor expansion, the EKF tries to approximate nonlinear systems as time-varying linear models and then uses the standard Kalman filter to estimate unknown states. Then Carlson presented a federal Kalman filter (FKF)[8] algorithm to deal with multi-system optimal data fusion problem. Julier et al. and Uhlmann introduced the so-called unscented Kalman filter (UKF)[9−10] for nonlinear estimation in 1997. Compared with the EKF algorithm, the UKF algorithm could obtain a more accurate estimation with a greater computation cost. The robust Kalman filter (RKF)[11] was presented to solve the problem that the system model cannot be obtained accurately. Then the adaptive Kalman filter (AKF)[12−13] was used to deal with the problem that system parameters are uncertain or cannot be accurately obtained.

In all these existing works, the objective is to obtain good de-noising and estimating effects when the system model has small uncertainty or the statistical properties of the process and observation noises are uncertain. The filtering and prediction problems when system models have large uncertainty were seldom addressed. In the presence of large model uncertainty, generally speaking, the standard Kalman filter cannot work well and there is no way to ensure its stability and convergence. Due to this challenging problem, in the applications of Kalman filters, we usually need to make great efforts in the modeling of errors by a large number of high-cost experiments so as to gain a model which is accurate enough for use with the standard Kalman filtering technique. This traditional approach also has the disadvantage of sensitivity to model drifting or model changes due to unexpected or unobserved uncertain issues like system faults.

Motivated by the above challenging problems, based on our previous pioneering work on finite-model adaptive control[14−16], a framework of finite-model Kalman filter (FMKF) is presented in this paper. In this framework, a finite-model Kalman filter may adaptively adjust its active model or parameters so as to reduce the model uncertainty and then make filtering and prediction based on a bank of parallel Kalman filters, which interchange information and share some common information for the purpose of prediction. Within this framework, the finite-model Kalman filter algorithms aim to resolve filtering and prediction problems when the system has large model uncertainty.

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To illustrate the idea of the finite-model Kalman filter, one simple algorithm within the framework of FMKF is presented based on the idea of adaptive switching via the minimizing vector distance principle. The mathematical form of the proposed algorithm is explicitly given in this contribution, which consists of a group of iterations regarding the switching sequence, bank filter prediction, and the measurement update. The simulations and experiments show that the proposed method can effectively deal with MEMS gyroscope noise signal and improve the precision of the measurement data.

The main contributions of this paper are highlighted as follows:

1) A framework of finite-model Kalman filter is presented to resolve the problem of performance degradation of the standard Kalman filter, with the aid of measurement information rather than innovation information alone when the system model is uncertain in a large range.

2) A simple finite-model Kalman filter algorithm based on adaptive switching via the minimizing vector distance principle, termed as FMKF-MVDP, is proposed and mathematically formulated to illustrate the presented framework of finite-model Kalman filter.

3) To verify the effectiveness of the proposed simple finite-model Kalman filter, extensive simulations and experiments are conducted and the results show that this algorithm has satisfactory filtering effects when the system model is inaccurate in a large range.

The rest of this paper is organized as follows: In Section 1, the standard Kalman filter is reviewed with mathematical equations for convenience of later discussions; then, we resolve the challenging filtering problem for state estimation in presence of large model uncertainties, based on the key idea borrowed from our previous studies on finite model adaptive control. Section 2 introduces a general framework of finite-model Kalman filter, within which a simple finite-model Kalman filter, termed as MVDP-FMKF, according to the idea of adaptive switching via the minimizing vector distance principle, is mathematically formulated in Section 3 and verified through simulations in Section 4, which illustrate the usefulness of the proposed algorithm. To further check whether the proposed algorithm can work well in practical navigation systems, some experimental tests on an MEMS gyroscope, MEMS130, are discussed in Section 5, which validate the effectiveness and usefulness of the MVDP-FMKF algorithm. Finally, Section 6 summarizes this paper with some concluding remarks.

1 Kalman filter

In 1960, Kalman published his famous paper describing a recursive solution to the discrete-time linear filtering problem. Since then, due to the advance in digital computing, the Kalman filter has been a subject of extensive research and application, particularly in the area of autonomous or assisted navigation[17].

The Kalman filter is a set of mathematical equations that provides an efficient computational (recursive) means to estimate the state of a process, in a way that minimizes the mean of the squared errors. The filter is very powerful since it can be used to estimate states in the past, at present, and even in the future.

It is the core of the program to design Kalman filter′s observation update and state update. Suppose that the state x_k to be estimated is governed by the following dynamics

$$\begin{cases} x_k = \phi_{k,k-1}x_{k-1} + \Gamma_{k-1}w_{k-1} \\ z_k = H_k x_k + v_k \end{cases} \tag{1}$$

where $\phi_{k,k-1}$ is the transfer matrix, H_k is the observation matrix, w_k is the process noise sequence, v_k is the measurement noise sequence, and z_k represents observable measurements at time t_k.

If the system noise and observation noise are zero-mean, Gaussian and uncorrelated, then $\hat{x}_k$, the estimate of x_k, can be recursively obtained by the following equations[3]:

$$\begin{cases} \hat{x}_{k,k-1} = \phi_{k,k-1}\hat{x}_{k-1} \\ \hat{x}_k = \hat{x}_{k,k-1} + K_k(z_k - H_k\hat{x}_{k,k-1}) \\ P_{k,k-1} = \phi_{k,k-1}P_{k-1}\phi_{k,k-1}^{\mathrm{T}} + \Gamma_{k-1}Q_{k-1}\Gamma_{k-1}^{\mathrm{T}} \\ K_k = P_{k,k-1}H_k^{\mathrm{T}}(H_kP_{k,k-1}H_k^{\mathrm{T}} + R_k)^{-1} \\ P_k = (I - K_kH_k)P_{k,k-1} \end{cases} \tag{2}$$

2 Framework of finite-model Kalman filter

The standard Kalman filter is optimal in the sense of minimum mean squared errors and maximum likelihood estimation, provided that the system model is linear and precisely known a priori; and the process noise and the measurement noise are jointly Gaussian and uncorrelated with known covariance matrices. However, in practice, these requirements can seldom be satisfied due to the following problems:

1) The practical systems are usually nonlinear although many of them may be approximated by linear systems. Absolutely linear systems seldom exist in practical applications.

2) Even if the practical system in consideration is linear, the system model may not be exactly known with accurate parameters. In practice, model parameters may be approximately identified by applying some system identification methods offline through the data obtained via extensive experiments. However, the cost of this approach is usually expensive and does not guarantee accurate system identification, which may result in poor performance of standard Kalman filter for the identified model. Furthermore, if the practical system is in fact time-varying, then the approach of system identification will fail sometimes.

3) The standard Kalman filter requires that the process noise and the measurement noise be zero-mean and uncorrelated Gaussian random noise. However, in some applications, the noise may be biased and its mean or mathematical expectation may be unknown. In such cases, further noise modeling is often needed and it is possible to use a Kalman filter by augmenting the mean of the noise as an extra state.

4) In most cases, we cannot have the covariances of the unknown process noise and measurement noise a priori. Therefore, to apply the standard Kalman filter, we must first try to obtain the statistical properties of the process noise and the measurement noise, which are usually calculated from extensive experiments. To deal with this problem, an alternative approach is to simply use larger covariances to represent the a priori knowledge on the process noise and measurement noise.

5) In practice, the probability distribution of the process noise or the measurement noise may not be normal distribution, and this case is often termed as non-Gaussian system, which often results in poor performance of the standard Kalman filter.

Due to the above problems, to resolve these practical problems to some extent, some extensions or variants of the standard Kalman filter have been proposed in the literature. For example, for problem 1), the EKF is one approach to apply the technique of Kalman filter to nonlinear systems based on the idea of Taylor expansion, while the UKF[18] and particle filters (PF)[19] are two typical approaches to handle filtering problem for nonlinear/non-Gaussian systems of problem 2) or problem 5). Comparing with others problem 3) may be relatively easy to resolve, and one typical approach in the one mentioned above if we have some physical or a priori knowledge on the sources of possible errors in the sensors. As to problem 4), there exists one approach called adaptive Kalman filter (AKF)[13], whose idea is to adaptively estimate the unknown statistical properties of the noises and combine the Kalman filter with the estimation process. And the Kalman filter based on the support vector machine (SVMKF)[17] may be regarded as another example to deal with problem 4).

In this paper, our general framework of finite-model Kalman filter is mainly motivated by the thoughts to the difficult problem 2), which is less discussed in the literature. It is well known that inaccurate model will degrade the performance of Kalman filter or even make it divergent. To address such a problem, besides the approach of system identification by expensive experiments, one approach called robust Kalman filter (RKF) was discussed in [20]. The basic idea of the RKF is rooted from the research on robust control, whose key concepts are nominal model and stability margin as well as certain performance index like H_∞ norm. Roughly speaking, the robust Kalman filter can only deal with small model uncertainty due to its nature of worst-case analysis.

When the system model is unknown in a large range, the approach of RKF will fail. Noticing this challenging yet critical problem, we present a kind of framework of finite-model Kalman filter (FMKF), which is motivated by our previous study on the finite-model adaptive control (FMAC), whose general framework was firstly established by Ma[21]. The essential idea is that large model uncertainty can be approximated by a finite bank of known models and hence adaptive control law can be constructed from the control signals designed for the known finite models, no matter whether the known models are similar or not. To this end, several typical algorithms of FMAC were proposed, including the LS-like algorithm[14], WLS-like algorithm[15] and other algorithms[16], whose rigorous closed-loop stability analysis can be found. Among these algorithms, the LS-like algorithm generates a switching signal among a bank of models at each step by minimizing the equally weighted sums of pth power of errors, while the WLS-like algorithm generates a switching signal at each step by minimizing a similar weighted performance with a forgetting factor, and other types of FMAC algorithms adopt other ideas like adaptive combination rather than the idea of switching among the models.

Following the idea of FMAC, we present the overall background of FMKF. Suppose that we have a finite bank of known models, and each model can be described by the following state-space equations

$$\mathbb{M}_i : \begin{cases} x_{k+1} = f_i(x_k, u_k, w_k) \\ z_k = h_i(x_k, u_{k-1}, v_k) \end{cases} \tag{3}$$

where x_k is the state at time k, u_k is the control at time k, z_k is the output at time k, w_k is the process noise at time k, and v_k is the measurement noise at time k. Here functions f_i and h_i are known mappings to describe the state evolution and output measurement for model $\mathbb{M}_i$ ($i = 1, 2, \cdots, N$). Hence, model $\mathbb{M}_i$ can be formally described by a triple $(f_i, h_i, \mathbb{N}_i)$, where $\mathbb{N}_i$ denotes the a priori knowledge on w_k and v_k, which may be presented by statistical properties or probability distributions of the process noise and the measurement noise.

For simplicity and brevity, we consider only linear Gaussian models without control like

$$\mathbb{M}_i : \begin{cases} x_{k+1} = \phi^{(i)}_{k+1,k} x_k + \Gamma^{(i)}_k w_k \\ z_k = H^{(i)}_k x_k + v_k \end{cases} \tag{4}$$

where w_k and v_k are zero-mean jointly Gaussian noises with covariances $Q^{(i)}_k$ and $R^{(i)}_k$, respectively. Hence, model $\mathbb{M}_i$ can be presented by matrices $\phi^{(i)}_{k+1,k}$, $\Gamma^{(i)}_k$, $H^{(i)}_k$ and $Q^{(i)}_k$, $R^{(i)}_k$. Obviously, for each fixed model $\mathbb{M}_i$, since its model is accurate with exact parameters, we can design a standard Kalman filter to estimate its state x_k via its output sequence $\{z_k\}$.

Now we consider a practical uncertain system, whose true model is not known a priori. In this case, it is impossible to design a Kalman filter for this system since the implementation of Kalman filter requires that all matrices $\phi_{k+1,k}$, Γ_k, H_k, Q_k, and R_k be available at each time step. For this challenging problem, from the perspective of finite models, suppose that we have enough known different models $\mathcal{M} = \{\mathbb{M}_1, \mathbb{M}_2, \cdots, \mathbb{M}_N\}$. For the uncertain true system, there exists a model $\mathbb{M}_{\mathbb{I}} \in \mathcal{M}$, where subscript $\mathbb{I}$ is not known a priori, such that the input-output behavior of model $\mathbb{M}_{\mathbb{I}}$ can approximate that of the true system. The key idea of the FMKF is to use Kalman filters for known models to construct state estimates for the true unknown system by making full use of posterior data at each step.

In the framework of FMKF, there may exist different methods for the purpose of constructing state estimates from the parallel Kalman filters for known models. At each time step k, the Kalman filter for model $\mathbb{M}_i$ involves two steps: the prediction step is to propagate $\hat{x}^{(i)}_{k-1,k-1}$ to $\hat{x}^{(i)}_{k,k-1}$ according to the state equation of model i, while the correction step is to correct $\hat{x}^{(i)}_{k,k-1}$ to $\hat{x}^{(i)}_k$ with the available new measurement z_k using the optimal Kalman gain $K^{(i)}_k$, computed from known system matrices. Now, one natural problem will arise: How do we determine the best estimate $\hat{x}_k$ of the true system state x_k from the N different estimates $\hat{x}^{(i)}_k$ $(i = 1, 2, \cdots, N)$? In other words, can we have some reasonable or useful rules, termed as model switching for convenience, to decide which $\hat{x}^{(i)}_k$ can serve as an estimate $\hat{x}_k$ of x_k? Or can we have some reasonable or useful rules, termed as model combination/fusion for convenience, to combine or fuse all $\hat{x}^{(i)}_k$ $(i = 1, 2, \cdots, N)$ so as to yield

$\hat{x}_k$? Once $\hat{x}_k$ can be determined at time k, then in the next step, the Kalman filters for all models may reset their initial conditions $\hat{x}_k^{(i)}$ to the same $\hat{x}_k$ and use it to give new estimates $\hat{x}_{k+1,k}^{(i)}$ and $\hat{x}_{k+1,k+1}^{(i)}$, which continues the overall loop of filtering. The research of the FMKF is to figure out such possible rules, to verify its usefulness or effectiveness, to analyze its closed-loop properties, and to explore its wide applications. Obviously, the FMKF architecture, which is briefly illustrated in Fig. 1, involves a kind of online learning/adaptation mechanism, and such mechanism will make it possible to cope with the challenging problems of filtering, prediction, and control for systems with large model uncertainty or time-varying model drift.

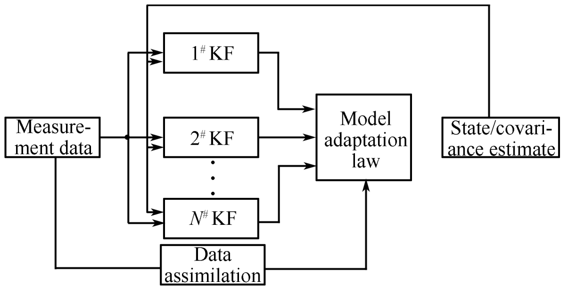

Fig. 1 Architecture of finite-model Kalman filter system

Remark 1. In the general framework of the FMKF, the true model can be time-invariant or slow time-varying yet unknown. Note that in practice we usually cannot find out a governing rule or models transition probability matrix to describe the switching. And furthermore, in the framework of FMKF, the unknown true model need not be one member of the bank of known models. In our framework, the known models need not have similar structures or same dimensions. The essential idea of FMKF borrows from the so-called "finite cover" concept, which makes it possible to approximate the true model with the known finite models.

Remark 2. As depicted in Fig. 1, there may exist many ways to approximate the true model with the known finite models, that is to say, the so-called model adaptation law is not unique. In certain situations, several known existing algorithms like autonomous multiple models (AMM)[21] and interacting multiple models (IMM)[14] may be also regarded as special cases of the FMKF. For example, the AMM algorithm requires that the true operating mode, which does not change in time, matches a model among the ones in the set. For another example, the key idea behind the IMM algorithm is to assume that the mode jump process is a Markov process, with known mode transition probabilities. The AMM algorithm and IMM algorithm are two ways of combining the estimates from known finite models using the concepts of model likelihood and mode probability.

Remark 3. In the framework of FMKF algorithm, we try to make full use of the measurement sequences to judge the system model. Traditional multiple models (MM) algorithms, such as AMM and IMM, always try to judge the system model according to the innovation sequence. In this section, we give a new framework that judges the system model according to measurement sequence rather than the innovation sequence. Note that in Fig. 1, there is a certain block data assimilation, i.e., collection and processing of the measurement data for the purpose of model adaptation, which distinguishes the FMKF from the traditional MM framework since the MM only uses innovation sequence. Such an idea will be demonstrated by an example of an FMKF-MVDP algorithm in the next section. This design is because in practice we believe that the system model information is embedded in the input/output sequence and a correct system model should match the measurement data, and generally speaking, measurement sequence may contain more information than the innovation sequence.

3 MVDP finite-model Kalman filter algorithm

In this section, within the general framework of FMKF given in Section 3, a kind of FMKF algorithm, called finite-model Kalman filter based on the minimum vector distance principle (MVDP-FMKF), will be presented.

Suppose that the true system model is $\mathbb{S} \in \mathcal{M}$, where $\mathcal{M} = \{\mathbb{M}_1, \mathbb{M}_2, \cdots, \mathbb{M}_N\}$. The finite known models can be very different in terms of their related matrices. Since we do not know which model can represent the true system, we try to guess which model may match the output data best at each step. As to the Kalman filter for model $\mathbb{M}_i$, at time step k it will generate a state estimate $\hat{x}_k^{(i)}$, consequently, we can define an output prediction $\hat{z}_k^{(i)}$ as

$$\hat{z}_k^{(i)} = H_k^{(i)} \hat{x}_k^{(i)} \tag{5}$$

Suppose that there exists such a function $d_i(\cdot)$ defined as

$$d_i(k) = \left\| f_{z_k} - f_{\hat{z}_k^{(i)}} \right\|^2, \quad i = 1, 2, \cdots, N \tag{6}$$

where f_{z_k} and $f_{\hat{z}_k^{(i)}}$ are to be discussed later by example. Intuitively, we construct f_{z_k} and $f_{\hat{z}_k^{(i)}}$ at each time step k, which can be used to measure the discrepancy of the actual output data and the estimated output data using model $\mathbb{M}_i$. If we introduce

$$i_k = \arg \min_{1 \le i \le N} \|d_i(k)\| \tag{7}$$

then the i_kth system model $\mathbb{M}_{i_k}$ can be treated as the active model, i.e., the most matching model in a certain sense, at time step k. That is to say, we could try to find a function of the measurement z_k and the filtering output $\hat{z}_k^{(i)}$ of the ith system model, which is used to measure the extent of matching between the true system and the chosen ith model.

The system block diagram is shown in Fig. 2.

The rest of this section is to give an example of such a function $d_i(k)$ and present the new algorithm. Note that there exist alternative meaningful approaches to construct $d_i(k)$, f_{z_k} and $f_{\hat{z}_k^{(i)}}$, hence the principle introduced in this paper can be flexibly used in various applications and further investigation in the future is necessary to explore more possible suitable schemes.

3.1 Derivation of d_i

First, expressions for f_{z_k} and $f_{\hat{z}_k^{(i)}}$ are given below. Let $j = 1, \cdots, 2\Delta + 1$ and let $z_k(j) = z_{k-j+1}$ be the measurement data in the time window of length $2\Delta+1$ before time step k. A second-order polynomial has the form

$$f_{z_k}(j)=\alpha_0+\alpha_1 j+\alpha_2 j^2 \tag{8}$$

and the coefficients α_0, α_1, and α_2 are determined in a least squares sense by minimizing

$$\sum_{j=1}^{2\Delta+1}\left(z_k(j)-(\alpha_0+\alpha_1 j+\alpha_2 j^2)\right)^2 \tag{9}$$

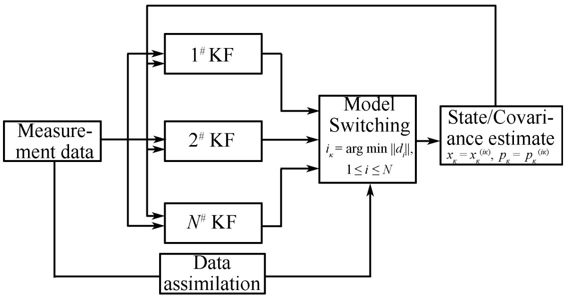

Fig. 2 Structure of MVDP finite-model Kalman filter system

Remark 4. The choices of f_{z_k} are not unique, and other forms except for the second-order polynomial may be also feasible. The intuitive meaning of f_{z_k} is to approximate local data of $z_k(j)$ in a small time window. Note that in practice the sampling interval of measured data is very small, hence locally speaking, the data of $z_k(j)$ will be smooth in the certain sense. Considering the trade-off between the computation cost and approximation effect, we choose a second order polynomial to capture the basic nature of the data, and in this way only three parameters are needed to determine.

Using the notation (9), it can be obtained that

$$\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & 4\\ \vdots & \vdots & \vdots\\ 1 & 2\Delta+1 & (2\Delta+1)^2\end{bmatrix}\begin{bmatrix}\alpha_0\\ \alpha_1\\ \alpha_2\end{bmatrix}=\begin{bmatrix}z_k(1)\\ z_k(2)\\ \vdots\\ z_k(2\Delta+1)\end{bmatrix} \tag{10}$$

Introducing the notation

$$M=\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & 4\\ \vdots & \vdots & \vdots\\ 1 & 2\Delta+1 & (2\Delta+1)^2\end{bmatrix} \tag{11}$$

$$\mathbb{Z}_k=\begin{bmatrix}z_k(1)\\ z_k(2)\\ \vdots\\ z_k(2\Delta+1)\end{bmatrix} \tag{12}$$

and noting that $[\alpha_0,\alpha_1,\alpha_2]^{\rm T}=(M^{\rm T}M)^{-1}M^{\rm T}\mathbb{Z}_k$, we can get

$$f_{z_k}=M(M^{\rm T}M)^{-1}M^{\rm T}\mathbb{Z}_k \tag{13}$$

Similarly, we construct a second-order polynomial of the form

$$f_{\hat{z}_k}(j)=\hat{\alpha}_0+\hat{\alpha}_1 j+\hat{\alpha}_2 j^2 \tag{14}$$

where coefficients $\hat{\alpha}_0$, $\hat{\alpha}_1$, and $\hat{\alpha}_2$ are determined in a least squares sense by minimizing

$$\sum_{j=1}^{2\Delta+1}(\hat{z}_k(j)-(\hat{\alpha}_0+\hat{\alpha}_1 j+\hat{\alpha}_2 j^2))^2 \tag{15}$$

Consequently, we have similar results

$$[\hat{\alpha}_0,\hat{\alpha}_1,\hat{\alpha}_2]^{\rm T}=(M^{\rm T}M)^{-1}M^{\rm T}\hat{\mathbb{Z}}_k \tag{16}$$

and

$$f_{\hat{z}_k}=M(M^{\rm T}M)^{-1}M^{\rm T}\hat{\mathbb{Z}}_k \tag{17}$$

where

$$\hat{\mathbb{Z}}_k=\begin{bmatrix}\hat{z}_k(1)\\ \hat{z}_k(2)\\ \vdots\\ \hat{z}_k(2\Delta+1)\end{bmatrix} \tag{18}$$

After that, we can write

$$d_i(k)=\left\|M(M^{\rm T}M)^{-1}M^{\rm T}(\mathbb{Z}_k-\hat{\mathbb{Z}}_k)\right\|^2 \tag{19}$$

3.2 Two-model MVDP-FMKF algorithm

In this subsection, we first discuss the MVDP-FMKF algorithm when we have two alternative models. The process could be divided into two parts.

1) Initialization. The first step is initializing the filter parameters,

$$\begin{array}{l}P_0^{(1)}=P_0\\ P_0^{(2)}=P_0\\ \hat{x}_0=\bar{x}_0\end{array} \tag{20}$$

where P_0 is a positive definite matrix and $\bar{x}_0$ can be taken as ${\rm E}[x_0]$ (if ${\rm E}[x_0]$ is available a priori) or any vector of proper dimension (e.g. zero vector) (if we have not any knowledge on ${\rm E}[x_0]$).

2) Filtering loop. The second step is filtering the original signals using the known linear system models for $k=1,2,3,\cdots$:

$$\left\{\begin{array}{l}\hat{x}_{k,k-1}^{(1)}=\phi_{k,k-1}^{(1)}\hat{x}_{k-1}\\ P_{k,k-1}^{(1)}=\phi_{k,k-1}^{(1)}P_{k-1}\phi_{k,k-1}^{(1){\rm T}}+\Gamma_{k-1}^{(1)}Q_{k-1}^{(1)}\Gamma_{k-1}^{(1){\rm T}}\\ K_k^{(1)}=P_{k,k-1}^{(1)}H_k^{(1){\rm T}}(H_k^{(1)}P_{k,k-1}^{(1)}H_k^{(1){\rm T}}+R_k^{(1)})^{-1}\\ P_k^{(1)}=(I-K_k^{(1)}H_k^{(1)})P_{k,k-1}^{(1)}\\ \hat{x}_k^{(1)}=\hat{x}_{k,k-1}^{(1)}+K_k^{(1)}(z_k-H_k^{(1)}\hat{x}_{k,k-1}^{(1)})\\ \hat{z}_k^{(1)}=H_k^{(1)}\hat{x}_k^{(1)}\end{array}\right. \tag{21}$$

$$\left\{\begin{array}{l}\hat{x}_{k,k-1}^{(2)}=\phi_{k,k-1}^{(2)}\hat{x}_{k-1}\\ P_{k,k-1}^{(2)}=\phi_{k,k-1}^{(2)}P_{k-1}\phi_{k,k-1}^{(2){\rm T}}+\Gamma_{k-1}^{(2)}Q_{k-1}^{(2)}\Gamma_{k-1}^{(2){\rm T}}\\ K_k^{(2)}=P_{k,k-1}^{(2)}H_k^{(2){\rm T}}(H_k^{(2)}P_{k,k-1}^{(2)}H_k^{(2){\rm T}}+R_k^{(2)})^{-1}\\ P_k^{(2)}=(I-K_k^{(2)}H_k^{(2)})P_{k,k-1}^{(2)}\\ \hat{x}_k^{(2)}=\hat{x}_{k,k-1}^{(2)}+K_k^{(2)}(z_k-H_k^{(2)}\hat{x}_{k,k-1}^{(2)})\\ \hat{z}_k^{(2)}=H_k^{(2)}\hat{x}_k^{(2)}\end{array}\right. \tag{22}$$

Then we can calculate $d_i(k)$ with the output of a different filter $\hat{z}_k^i$ as follows

$$d_i(k)=\left\|f_{z_k}-f_{\hat{z}_k^i}\right\|^2,\quad i=1,2 \tag{23}$$

Then, we choose the active model by the minimizing vector distance principle and update $\hat{x}_k$ and P_k as follows

a) If

$$d_1(k) \leq d_2(k) \tag{24}$$

then $i_k = 1$ and we set

$$\begin{aligned} P_k &= P_k^{(1)} \\ \hat{x}_k &= \hat{x}_k^{(1)} \end{aligned} \tag{25}$$

b) Otherwise, we have $i_k = 2$ since

$$d_1(k) > d_2(k) \tag{26}$$

Then, we set

$$\begin{aligned} P_k &= P_k^{(2)} \\ \hat{x}_k &= \hat{x}_k^{(2)} \end{aligned} \tag{27}$$

3.3 General MVDP-FMKF algorithm

Similarly, we can introduce the general MVDP-FMKF algorithm as follows:

1) Initialization.

$$\begin{aligned} &P_0^{(i)} = P_0, \quad i = 1, 2, \cdots, N \\ &\hat{x}_0 = \bar{x}_0 \end{aligned} \tag{28}$$

2) Filtering loop. At time step k, for each model $\mathbb{M}_i$ (i = 1, 2, $\cdots$, N), the following Kalman iterations are made first:

$$\begin{aligned} \hat{x}_{k,k-1}^{(i)} &= \phi_{k,k-1}^{(i)} \hat{x}_{k-1} \\ P_{k,k-1}^{(i)} &= \phi_{k,k-1}^{(i)} P_{k-1} \phi_{k,k-1}^{(i)\mathrm{T}} + \Gamma_{k-1} Q_{k-1} \Gamma_{k-1}^{\mathrm{T}} \\ K_k^{(i)} &= P_{k,k-1}^{(i)} H_k^{\mathrm{T}} (H_k P_{k,k-1}^{(i)} H_k^{\mathrm{T}} + R_k)^{-1} \\ P_k^{(i)} &= (I - K_k^{(i)} H_k) P_{k,k-1}^{(i)} \\ \hat{x}_k^{(i)} &= \hat{x}_{k,k-1}^{(i)} + K_k^{(i)} (z_k - H_k \hat{x}_{k,k-1}^{(i)}) \\ \hat{z}_k^{(i)} &= H_k \hat{x}_k^{(i)} \end{aligned} \tag{29}$$

Then we can calculate $d_i(k)$ as follows

$$d_i(k) = \left\| f_{z_k} - f_{\hat{z}_k^i} \right\|^2, \quad i = 1, 2 \tag{30}$$

where f_{z_k} and $f_{\hat{z}_k^i}$ are introduced before. Then, according to the minimizing vector distance principle, $\hat{x}_k$ and P_k can be updated by

$$\begin{aligned} P_k &= P_k^{(i_k)} \\ \hat{x}_k &= \hat{x}_k^{(i_k)} \end{aligned} \tag{31}$$

where

$$i_k = \arg \min_{i \in \{1, 2, \cdots, N\}} d_i(k) \tag{32}$$

Repeat the above steps for each time step k.

4 Simulation of the MVDP-FMKF algorithm

In this section, some simulation results are reported to illustrate the effects of the MVDP-FMKF algorithm.

4.1 One-dimensional simulation

We consider the following simple scalar system

$$\begin{cases} x_k = \theta x_{k-1} + w_{k-1} \\ z_k = x_k + v_k \end{cases} \tag{33}$$

where $\mathrm{E}(w_k) = \mathrm{E}(v_k) = 0$, $\mathrm{Var}(w_k) = \mathrm{Var}(v_k) = 0.1$, and $\theta = 1$.

Suppose that the precise value of θ is unknown a priori, and that we only know $\theta \in \{0.1, 1, 5\}$. Obviously, a single Kalman filter cannot make sure to generate reliable state estimates. In this simulation, we have 3 models, and Kalman filters are designed for $\theta = 0.1, 1, 5$, respectively. The true value of θ is randomly taken from the set $\{0.1, 1, 5\}$ and then fixed for simulation. The simulation results of the MVDP-FMKF algorithm compared with KF, AMM, and IMM are shown in Figs. 3 ∼ 5, where the true value of θ is 1. For comparison, besides the results of the MVDP-FMKF algorithm, the ideal results of the ideal Kalman filter using the true system parameter ($\theta = 1$), the non-ideal results of the Kalman filter using the wrong system parameter ($\theta = 0.1$), the non-ideal results of the Kalman filter using the wrong system parameter ($\theta = 5$), AMM and IMM algorithms are also illustrated.

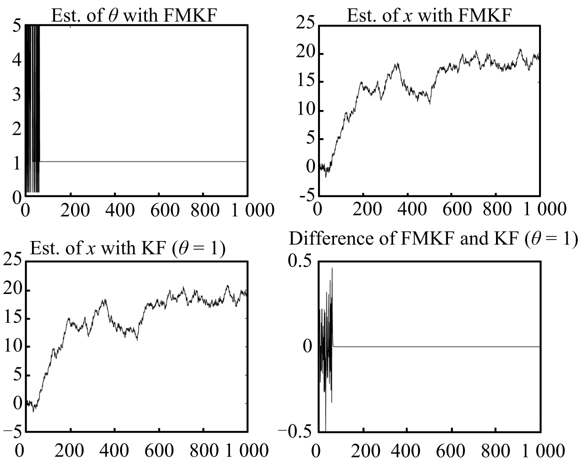

Fig. 3 Simulation results of one-dimensional MVDP-FMKF algorithm

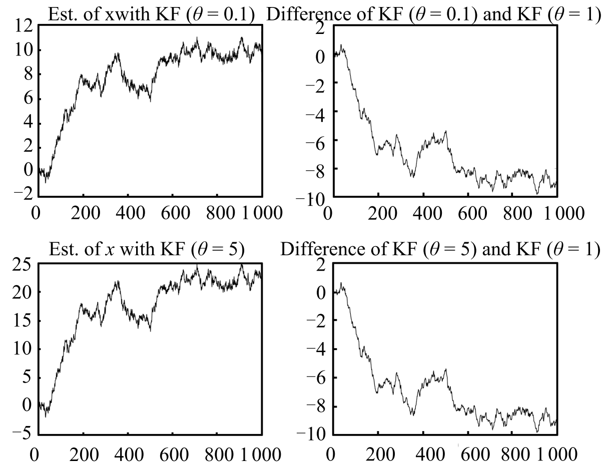

Fig. 4 Traditional Kalman filter using wrong model parameter θ can lead to growth of estimation errors

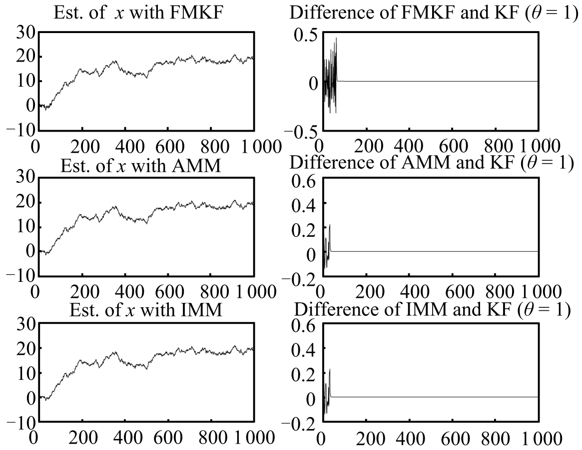

Fig. 5 Performance comparison of MVDP-FMKF, AMM and IMM algorithms

From Figs. 3 and 4, it can be seen that the de-noising effect of MVDP-FMKF is almost equal to the ideal Kalman filter with precise system parameters, and the state estimate errors will diverge with wrong system models ($\theta = 0.1$ and $\theta = 5$). That is to say, the MVDP-FMKF can choose the right system model with a large probability and lead to acceptable de-noising effect when the system parameters cannot be obtained precisely.

From Fig. 5, it can be seen that the de-noising effect of the MVDP-FMKF is comparable with those of the AMM and IMM algorithms. However, the MVDP-FMKF algorithm does not need to calculate the model likelihood and mode probability, especially exponential functions. That is to say, the MVDP-FMKF algorithm has a smaller computation cost compared with the AMM and IMM algorithms. Besides, the MVDP-FMKF does not need to know the so-called TPM (transition probability matrix).

Remark 5. Computation cost is very important for a practical system to deal with the signal a real-time. Especially, in an integrated inertial navigation system, there are many filtering blocks, and the computation effort is more important than the precision in some aspects. In an integrated inertial navigation system, the MVDP-FMKF can be used to deal with a gyroscope output signal. With a low computation cost, the filter has been proved in practice to be able to obtain acceptable results and satisfy the real-time performance.

Besides, the AMM and IMM algorithms cannot deal with the estimating problem when the system models $\mathbb{M}_i$ have different dimensions.

Remark 6. Nowadays, the multiple models used by the existing multiple model estimators need not have the same dimension or structure. A very typical example in the target tracking area is the use of the constant velocity motion model and constant acceleration model. For sure, these two models have different dimensions. But they just use the common state variable information to get estimating state and abandon private state variable information. That is to say, the multiple models (MM) algorithms were designed to deal with system model set with the same dimensions although they can obtain good results in the model set with different dimensions. In the framework of FMKF, generally it is not necessary to require that all models share some common states.

For the system shown in (33), suppose that the precise value of θ is unknown a priori, and that we only know $\theta \in \{0.1, 5\}$ or the system model is as follows

$$\begin{bmatrix} x_k \\ \delta_k \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_{k-1} \\ \delta_{k-1} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w_{k-1}$$
$$z_k = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ \delta_k \end{bmatrix} + v_k \tag{34}$$

Remark 7. The model in (34) is a different system model. The MVDP-FMKF demands that the model set should have at least one or more models that can describe the system motion exactly or roughly at every moment. But in this example, the first and second models cannot describe the system motion. So we build a model (34) to describe the system motion to satisfy the MVDP-FMKF demand. We use this example to demonstrate that the MVDP-FMKF method can be applied in the cases of multiple different types of models.

Under this situation, the AMM algorithm and IMM algorithm cannot generate system state estimates because the system model set contains models with different dimensions, which obviously makes it difficult to introduce and calculate the likelihood function for each model candidate. And the transition probability matrix of the system model set is also unknown, which is another factor to limit the use of the AMM and IMM algorithms. Whereas, the MVDP-FMKF can deal with these estimation problems since the principle of MVDP optimization can be applied to models of any dimensions and the transition probability matrix is not necessary.

The simulation results are shown in Fig. 6, where the index in the top-left subfigure takes values of 1 ($\theta = 0.1$), 2 ($\theta = 1$), and 3 ($\theta = 5$), respectively. It can be seen that the MVDP-FMKF can deal with mixing-dimension system state estimating problems. And it can also yield an acceptable filtering performance even when the system model does not contain the precise system model.

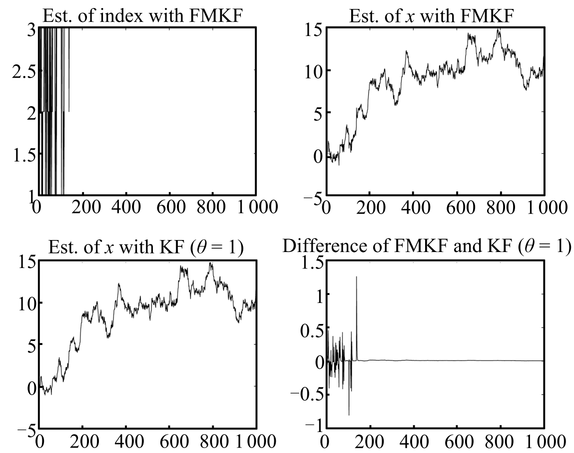

Fig. 6 Simulation results of mixing-dimensional MVDP-FMKF algorithm

4.2 Multi-dimensional simulation

In this part, a multi-dimensional MVDP-FMKF simulation is presented. The true system model is given as follows:

$$\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k-1) \\ x_2(k-1) \end{bmatrix} + \begin{bmatrix} w_1(k) \\ w_2(k) \end{bmatrix}$$
$$\begin{bmatrix} z_1(k) \\ z_2(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} v_1(k) \\ v_2(k) \end{bmatrix} \tag{35}$$

where $\mathrm{E}(w_i) = \mathrm{E}(v_i) = 0$, and $\mathrm{Var}(w_i) = \mathrm{Var}(v_i) = 0.1$ $(i = 1, 2)$. Then the finite known models include three models, among which one is the true system model, and the other ones are similar to the true system model except that its system transition matrix $\phi_{k,k-1}$ is $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$ or $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ rather than $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. The filtering problem is to estimate the unknown states $x_1(k)$ and $x_2(k)$ in the presence of large model uncertainty since the exact system model is completely not known a priori. The simulation results are shown in Figs. 7 ~ 12, where the results of ideal Kalman filter using the precise system model are also plotted for the purpose of comparison.

From the simulation results, it can be seen that in this case of multi-dimensional simulation, the new algorithm

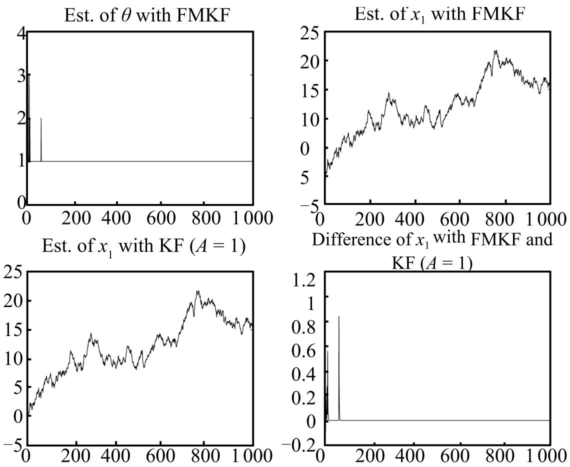

Fig. 7 Simulation results of multi-dimensional FMKF algorithm for state $x_1(k)$

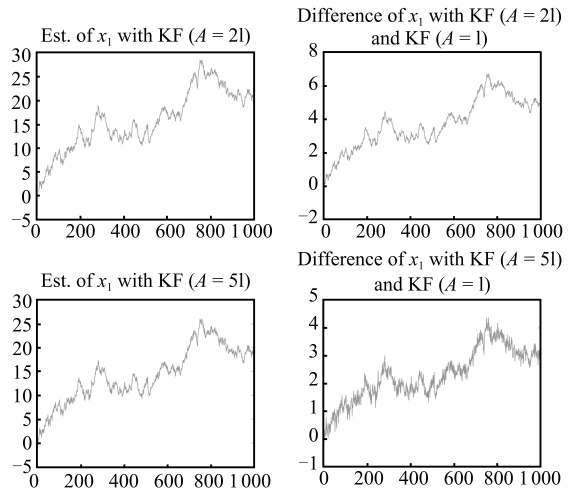

Fig. 8 Traditional Kalman filter using a wrong model matrix A can lead to growth of estimation errors for state $x_1(k)$

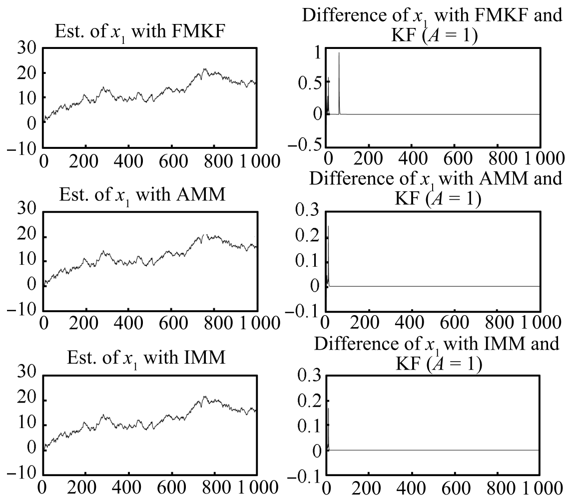

Fig. 9 Performance comparison of MVDP-FMKF, AMM and IMM algorithms for state $x_1(k)$

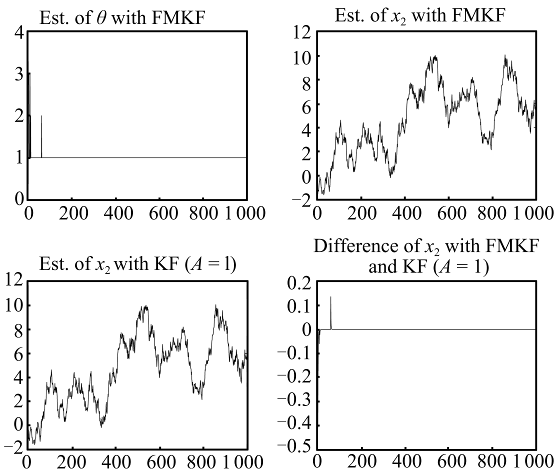

Fig. 10 Simulation results of multi-dimensional FMKF algorithm for state $x_2(k)$

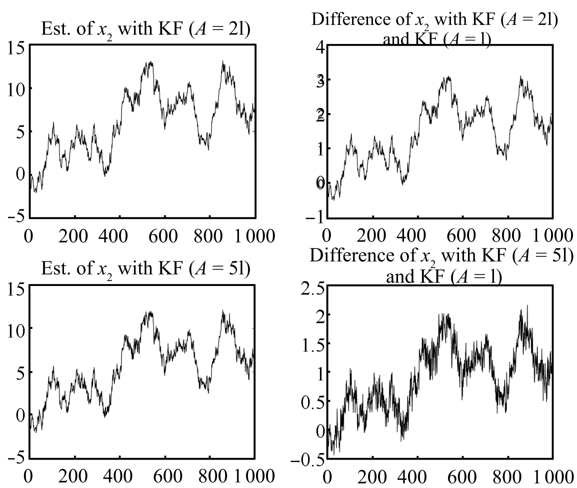

Fig. 11 Traditional Kalman filter using a wrong model matrix A can lead to growth of estimation errors for state $x_2(k)$

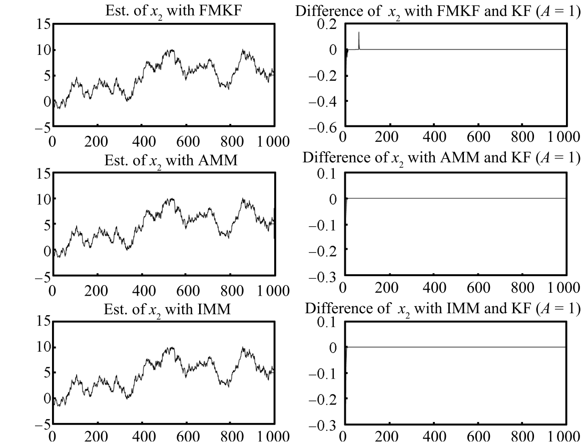

Fig. 12 Performance comparison of MVDP-FMKF, AMM and IMM algorithms for state $x_2(k)$

could choose the right system model ($A = I$) as the active one with a big probability and obtain state estimates almost as precisely as the Kalman filter with the accurate system model ($A = I$). Besides, the state estimate errors have a diverging trend with wrong system models ($A = 2I$ and $A = 5I$). In other words, the new algorithm can also yield acceptable de-noising effect when the true system model is not precisely available.

5 Experimental tests

In this section, we choose an MEMS gyroscope to test the proposed MVDP-FMKF algorithm. For the MEMS gyroscope, the existence of random errors is the most important factor to influence its precision because uncertain random errors cannot be simply compensated like unknown yet constant drift error. Therefore, it is particularly critical to reduce random errors to improve the measurement accuracy of MEMS gyroscope.

We used an MEMS130 gyroscope in the condition of 25 °C to test the practical performance of the MVDP-FMKF algorithm. The shaft angle rate information of the output data of the MEMS gyroscope was used as the original data. The sampling period was $T = 5\,\mathrm{ms}$.

In this experiment, two different models were used to test the MVDP-FMKF algorithm. The first model was taken as an ARMA(2,2) (auto-regressive and moving average) model based on the zero drift data. And the second model was an ARMA(2,2) model based on the constant output and swing data.

The real-time filtering results of the gyroscope zero drift are depicted in Fig. 13, where it can be seen that compared with the first model and the second model, the MVDP-FMKF has good de-noising effect for the gyroscope zero drift output. From the value of θ, it can be seen that the MVDP-FMKF algorithm switched between the first and second system models. For the gyroscope signal generated with zero drift (i.e., the first model), the MVDP-FMKF algorithm has performance similar to the corresponding optimal Kalman filter using the first system model, while the de-noised signal by the Kalman filter using the wrong model (second model) is almost incorrect from its magnitude.

To test the gyroscope constant output, the real-time filtering results are shown in Fig. 14. It can be seen that when the gyroscope has a high constant output, the second model has amplitude attenuation. But the MVDP-FMKF does not have amplitude attenuation. The better filtering results are automatically chosen by the MVDP-FMKF. From the value of θ, we can see that the MVDP-FMKF algorithm can adaptively choose the first system model when the second model has big amplitude attenuation. For the zero drift output signal, the algorithm can choose the second model to improve de-noising effect at some time.

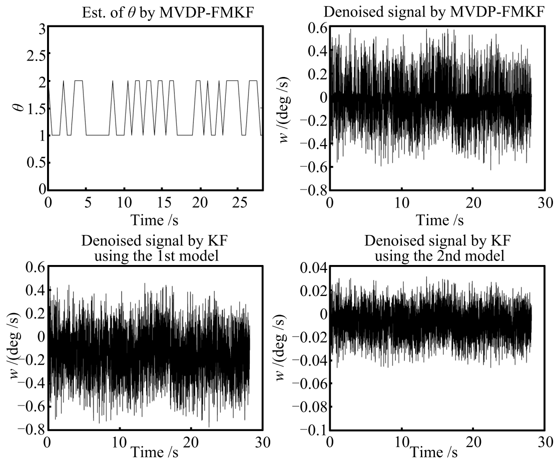

Fig. 13 For gyroscope constant angle rate output data, the on-line filtering results depict the consistence of the MVDP-FMKF algorithm and the Kalman filter based on the first (correct) model

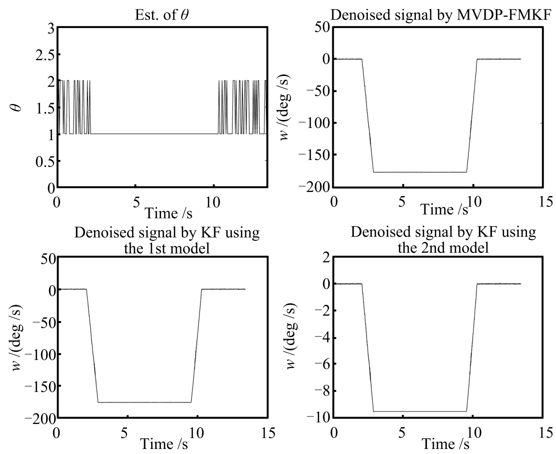

Fig. 14 For gyroscope constant angle rate output data, the on-line filtering results depict the better performance of MVDP-FMKF algorithm compared with the Kalman filtering based on the second (wrong) model

To test the gyroscope swing by 20 Hz frequency of 0.3° amplitude, the real-time filtering results are shown in Figs. 15 and 16. It can be seen that when the gyroscope wings by 20 Hz frequency of 0.3° amplitude, the second model has amplitude attenuation. But the MVDP-FMKF does not have amplitude attenuation and it can follow the gyroscope real-time output. That is to say, the MVDP-FMKF has good real-time performance and the bandwidth of the algorithm is more than or equal to 20 Hz at least.

To test the gyroscope swing by 10 Hz frequency of 1° amplitude, the real-time filtering results are shown in Figs. 17

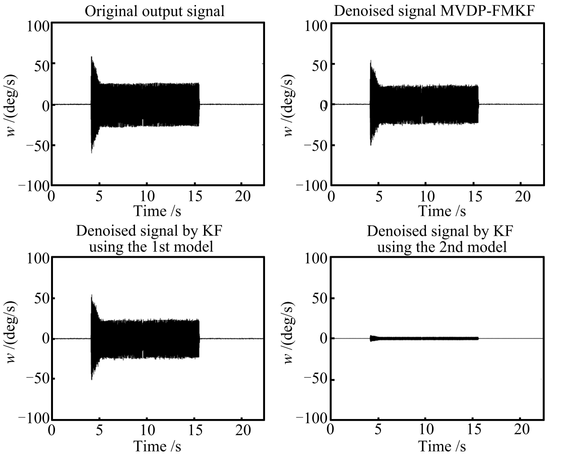

Fig. 15 For gyroscope angle swing output data (by 20 Hz frequency of 0.3° amplitude), the on-line filtering results depict the successful tracking of the MVDP-FMKF algorithm for the original output data

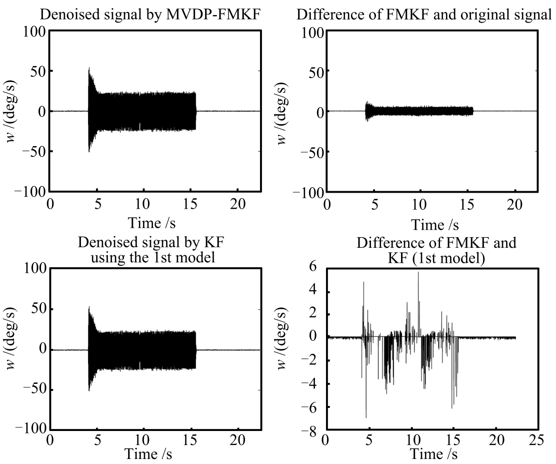

Fig. 16 For gyroscope angle swing output data (by 20 Hz frequency of 0.3° amplitude), the tracking performance of MVDP-FMKF algorithm is similar to that of Kalman filter based on the first model

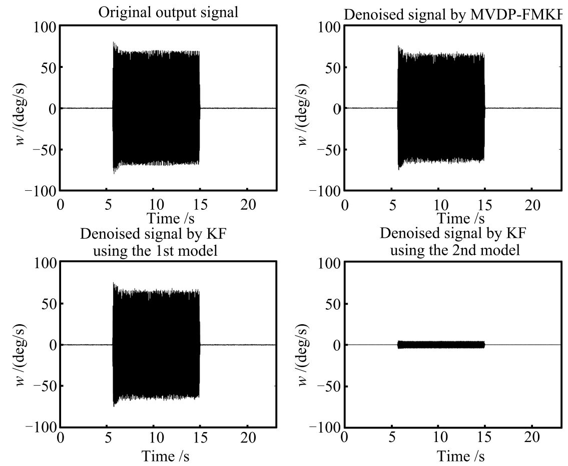

Fig. 17 For gyroscope angle swing output data (by 10 Hz frequency of 1° amplitude), the on-line filtering results depict the successful tracking of MVDP-FMKF algorithm for the original output data

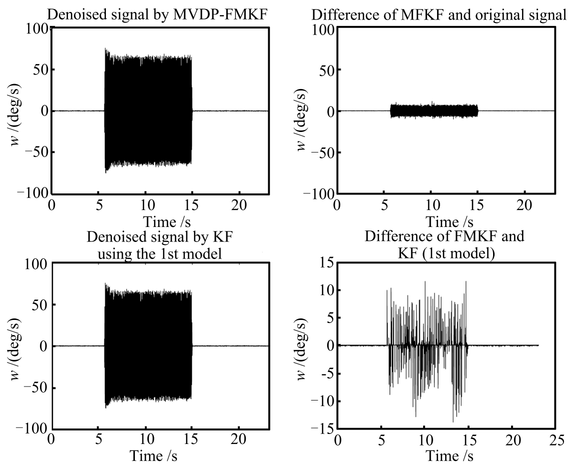

Fig. 18 For gyroscope angle swing output data (by 10 Hz frequency of 1° amplitude), the tracking performance of MVDP-FMKF algorithm is similar to that of Kalman filter based on the first model

and 18, where it can be seen that when the gyroscope swings by 10 Hz frequency of 1° amplitude, the second model has amplitude attenuation. The MVDP-FMKF does not have amplitude attenuation and it can follow the large-amplitude high-frequency real-time output.

6 Conclusion

The Kalman filtering techniques have been widely used in many applications, especially the navigation problems. However, successful applications of standard Kalman filters often require that the state-space model in use should be accurate enough with exact system matrices or parameters as well as proper a priori information of the process noise and the measurement noise, which in fact requires people to make great efforts in the modeling procedure before the real application of Kalman filter. Generally speaking, exact or perfect modeling is impossible or expensive in most practical applications, and an inaccurate model often results in that the performance of Kalman filter is very poor or completely not acceptable.

To address the crucial filtering degradation problem due to the large model uncertainty, motivated by our previous pioneering work on finite-model adaptive control, we have introduced the idea of finite-model Kalman filtering, which presumes that the large model uncertainty is restricted by a finite set of known models. This assumption is not as restrictive as it looks, since the known models can be very different from each other, and the number of known models can be flexibly chosen so that the uncertain model may always be approximated by one of the known models, although the matched model is not a priori known.

Within the presented framework of finite-model Kalman filter, we have mathematically formulated and illustrated a simple finite-model Kalman filter, termed as MVDP-FMKF, which adopts the idea of adaptive switching via minimizing vector distance principle. Despite of the intuitive and easy-to-implement algorithm, its closed-loop system is rather complex, which needs further investigation in the future.

In this contribution, extensive simulations and an experiment of gyroscope drift have verified the effectiveness of the proposed algorithm. Hence, the mechanism of finite-model Kalman filter is useful and efficient in dealing with large model uncertainties which may be frequently seen in practical applications of Kalman filters and may degrade the performance or even destroy the convergence or stability of the standard Kalman filter. In summary, this introduced method of finite-model Kalman filter has practical significance in the inertial navigation systems, and may be extended to other types of practical systems where Kalman filtering techniques are required yet the system model is not accurately known a priori.

In the future, to gain a deeper understanding of finite-model Kalman filters, it is necessary to investigate more FMKF algorithms, mathematically establish their stability properties, and explore their wide applications.

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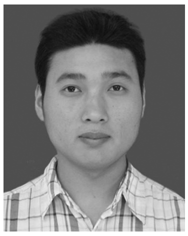

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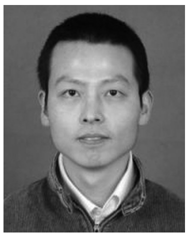

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