Coordination Control of Networked Euler-Lagrange Systems with Possible Switching Topology

MIN Hai-Bo[1] LIU Zhi-Guo[1] LIU Yuan[1] WANG Shi-Cheng[1] YANG Yan-Li[1]

Abstract This paper studies adaptive coordination control of Euler-Lagrange (EL) systems with unknown parameters in system dynamics and possible switching topology. By introducing a novel adaptive control architecture, decentralized controllers are developed, which allow for parametric uncertainties. Based upon graph theory, Lyapunov theory and switching control theory, the stability of the proposed algorithms are demonstrated. A distinctive feature of this work is to address the coordination control of EL systems with unknown parameters and switching topology in a unified theoretical framework. It is shown that both static and dynamic coordinations can be reached even when the communication is switching. Simulation results are provided to demonstrate the effectiveness of the obtained results.

Key words Euler-Lagrange (EL) systems, coordination control, switching topology, adaptive control

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The past few years have witnessed the burgeoning interest in coordination control problems from a variety of fields, including biology, physics, robotics and control engineering. Generally, in a multi-agent system, agents coordinate with each other within the group, which enables lots of tasks that could not be fulfilled by solo systems and significantly promote the efficiency and the reliability of the system. In fact, the coordination control of networked systems can be treated as a special case of general synchronization of complex networks[1−2], where each agent (or node) is a fundamental unit, and can have different meanings in different situations, such as chemical substrates, microprocessors, computers, schools, companies, papers, webs, people, and so on. So far, the coordination control has been used in the field of wireless sensor network, spacecraft formation, unmanned aerial vehicle (UAV) formation, etc.

Euler-Lagrange (EL) systems can describe a large class of physical systems including mechanical and power systems, and the control of EL systems was a classical control problem in the 1980s and has both practical and theoretical interests[3−4]. Inspired by the concept of coordination control of multi-agent systems, recently a lot of efforts have been made on studying the coordination control of a group of networked EL systems using a distributed control law[5−21]. For example, in [5] a distributed controller was proposed to tackle the consensus of EL systems, where Lyapunov theory and Matrosov theory were used to demonstrate the stability. The finite-time coordination control of EL systems was addressed in [6−7] where the sliding mode controller technique was adopted. Various distributed adaptive controllers were proposed in [8−18, 20] to tackle the coordination control problem of EL systems in the presence of parametric uncertainties. In these studies, distributed control laws were dictated by communication topologies which describe the information exchange among different subsystems, and the communication topology played an essential role in system characteristics.

It is well known that the communication network is often unreliable due to the package loss and message dropouts. This property can be conveniently described by a dynamic topology whose links may be disconnected at any time instants. However, up to now, most of the aforementioned studies on coordination control of EL systems are based on the assumption that the communication topology is fixed with an exception of [19−21]. Although the coordination control problem seems to be much more simplified by this assumption, the switching topology may deteriorate the performance of system and even cause instability. On the other hand, the parameters of system dynamics are usually unknown or difficult to obtain, hence it is of great interest to consider this factor when designing the controller.

In this paper, we propose a unified adaptive control architecture which allows for both unknown parameters in the system dynamics and the possible directed switching topology. A distinct feature of this paper is that it comprehensively considers the distributed coordination control of EL systems with both parametric uncertainty and switching topologies. The object is that a networked EL system tracks a dynamic reference signal and reaches consensus with these uncertain factors. The problem has many applications such as spacecraft formation and multiple tele-operators, especially when communication links are severely disturbed by factors such as the sun wind and magnetic storm.

In contrast with [5−18] which addressed coordination of networked EL systems with a fixed communication topology, this paper considers the case where the communication topology can switch. In contrast with [19, 21] which dealt with EL systems with known parameters, this paper allows for systems with parametric uncertainty. In contrast to [20] where undirected and balanced directed topologies were required, our control algorithms allow for the most common directed "quasi-strongly" connected topology which can be easily satisfied in practical applications. Furthermore, instead of rendering the state to asymptotically approach to zero, our algorithm realizes dynamic tracking. We hence extend [20] to a broader application scenario.

This paper is organized as follows. First, we introduce some background on the dynamics of Euler-Lagrange systems and graph theory, and the problem formulation is established. Main results on adaptive state consensus algo-

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Recommended by Associate Editor LV Jin-Hu
1. Hi-Tech Institute of Xi′an, Xi′an 710025, China

rithms are provided in Section 1. In Section 2, a numerical example is given to illustrate the effectiveness of the proposed algorithm, and finally the results are summarized in Section 3.

Notation 1. $\mathbf{R} = (-\infty, \infty)$, $\mathbf{R}_{>0} = (0, \infty)$, $\mathbf{R}_{\geq 0} = [0, \infty)$. $\lambda_m\{A\}$ and $\lambda_M\{A\}$ represent the minimum and maximum eigenvalues of matrix A, respectively. $\|A\|$ is the norm of matrix A. $|\boldsymbol{x}|$ stands for the standard Euclidean norm of vector $\boldsymbol{x}$. For any function $\boldsymbol{f} : \mathbf{R}_{\geq 0} \rightarrow \mathbf{R}^n$, the $\mathcal{L}_\infty$-norm is defined as $\|\boldsymbol{f}\|_\infty = \sup_{t\geq 0} |\boldsymbol{f}(t)|$, and the $\mathcal{L}_2$-norm as $\|\boldsymbol{f}\|_2^2 = \int_0^\infty |\boldsymbol{f}(t)|^2 \mathrm{d}t$. The $\mathcal{L}_\infty$ spaces are defined as the sets $\{\boldsymbol{f} : \mathbf{R}_{\geq 0} \rightarrow \mathbf{R}^n : \|\boldsymbol{f}\|_\infty < \infty\}$ and $\{\boldsymbol{f} : \mathbf{R}_{\geq 0} \rightarrow \mathbf{R}^n : \|\boldsymbol{f}\|_2 < \infty\}$, respectively.

1 Problem formulation and background

1.1 Euler-Lagrange dynamics

We consider a team of N networked EL systems (henceforth called agents) indexed by set $\mathcal{I} = \{1, \cdots, N\}$. EL systems neglecting friction or other disturbances are formulated by

$$M(\boldsymbol{q}_i)\ddot{\boldsymbol{q}}_i + \boldsymbol{C}(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)\dot{\boldsymbol{q}}_i + \boldsymbol{g}(\boldsymbol{q}_i) = \boldsymbol{\tau}_i(t) + \boldsymbol{\tau}_{\mathrm{ext},i}(t) \tag{1}$$

where $\boldsymbol{q}_i \in \mathbf{R}^p$ is the vector of generalized coordinates, $M(\boldsymbol{q}_i) \in \mathbf{R}^{p\times p}$ is the symmetric positive-finite intertial matrix, $C(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)\boldsymbol{q}_i \in \mathbf{R}^p$ is the vector of Coriolis and centrifugal force, $\boldsymbol{g}(\boldsymbol{q}_i)$ is the gravitational force, $\boldsymbol{\tau}_i(t)$ is the vector of inputs associated with the ith system, and $\boldsymbol{\tau}_{\mathrm{ext},i}(t)$ is the disturbance force which is assumed to be unknown but constant. Before proceeding, we give some fundamental properties for system (1) that will be extensively exploited in the following[19].

Property 1. The inertial matrix $M_i(\boldsymbol{q}_i)$ is lower and upper bounded, i.e.,

$$0 < \lambda_m\{M_i(q_i)\}I \leq M_i(\boldsymbol{q}_i) \leq \lambda_M\{M_i(\boldsymbol{q}_i)\}I < \infty \tag{2}$$

Property 2. The Lagrangian dynamics is linearly parameterizable, i.e.,

$$\begin{aligned} M(\boldsymbol{q}_i)\ddot{\boldsymbol{q}}_i + C(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)\dot{\boldsymbol{q}}_i + \boldsymbol{g}(\boldsymbol{q}_i) - \boldsymbol{\tau}_{\mathrm{ext},i}(t) = \\ Y(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i, \ddot{\boldsymbol{q}}_i)\boldsymbol{\theta}_i = \boldsymbol{\tau}_i(t) \end{aligned}$$

where $\boldsymbol{\theta}_i$ is a constant p-dimensional vector of parameters whose elements include the link masses, moments of inertial, etc., and $Y(\cdot) \in \mathbf{R}^{p\times p}$ is the matrix of known functions of the generalized coordinates and their higher derivatives.

Property 3. Under an appropriate definition of matrix $C(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)$, matrix $\dot{M}(\boldsymbol{q}_i) - 2C(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)$ is skew-symmetric. Therefore, for a given vector $\boldsymbol{r} \in \mathbf{R}^p$, it is easy to verify that

$$\boldsymbol{r}^{\mathrm{T}}(\dot{M}(\boldsymbol{q}_i) - 2C(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i))\boldsymbol{r} = 0$$

Property 4[8]. Consider a mechanical system of the form (1) and assume that $\dot{\boldsymbol{q}}_i, \ddot{\boldsymbol{q}}_i \in \mathcal{L}_\infty$. Then, the time derivative of its Coriolis matrix $\dot{C}_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)$ is bounded.

1.2 Graph theory

Graph can be conveniently used to represent the information flow between agents. Let $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ be an undirected graph or directed graph (digraph) of order n with the set of nodes $\mathcal{V}(\mathcal{G}) = \{v_1, v_2, \cdots, v_n\}$, the set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $\mathcal{A} = \{a_{ij}\}$ with nonnegative adjacency elements a_{ij}. The node indices belong to a finite index set $\mathcal{I} = \{1, 2, \cdots, n\}$. An edge of $\mathcal{G}$ is denoted by $e_{ij} = (v_i, v_j)$ and it is said to be incoming with respect to v_j and outgoing with respect to v_i. For a undirected graph, $\forall\, i, j \in \mathcal{I}$, if $(v_i, v_j) \in \mathcal{E}(\mathcal{G})$, then $(v_j, v_i) \in \mathcal{E}(\mathcal{G})$, but this does not hold for digraph. The set of neighbors of node v_i is the set of all nodes which point to (communicate with) v_i, denoted by $\mathcal{N}_i = \{v_j \in \mathcal{V} : (v_i, v_j) \in \mathcal{E}\}$. The graph adjacency matrix $\mathcal{A} = [a_{ij}]$, $\mathcal{A} \in \mathbf{R}^{N\times N}$, is such that $a_{ij} = 1$ if $e_{ij} \in \mathcal{E}$ and $a_{ij} = 0$ if $e_{ij} \notin \mathcal{E}$. The in-degree of vertex v_i is denoted by $d_i = \sum_{j=1}^N a_{ji}$. Similarly, the out-degree of a vertex $v_i \in \mathcal{G}$ is denoted by $d_i = \sum_{j=1}^N a_{ij}$. If the in-degree equals the out-degree for all $v_i \in \mathcal{V}(\mathcal{G})$, then the graph is said to be balanced. $\mathcal{D} = \{d_{ij}\} \in \mathbf{R}^{N\times N}$ is called the degree matrix of $\mathcal{G}$. The Laplacian of $\mathcal{G}$ is the matrix $\mathcal{L} = \mathcal{D} - \mathcal{A}$. When $\mathcal{G}$ contains a spanning tree, the Laplacian has a single zero eigenvalue and the corresponding eigenvector is the vector of ones, $\mathbf{1}$; Moreover, all the other non-zero eigenvalues are in the open right half plane[22].

1.3 Instrumental lemma

Define

$$\begin{aligned} &\boldsymbol{e}(t) = \boldsymbol{x}_d(t) - \boldsymbol{x}(t) \\ &\dot{\boldsymbol{x}}_r(t) = \dot{\boldsymbol{x}}_d(t) + \Lambda\boldsymbol{e}(t) \\ &\boldsymbol{r}(t) = \dot{\boldsymbol{x}}_r(t) - \dot{\boldsymbol{x}}(t) = \dot{\boldsymbol{e}}(t) + \Lambda\boldsymbol{e}(t) \end{aligned} \tag{3}$$

where $\boldsymbol{x}_d(t)$, $\boldsymbol{x}(t) \in \mathbf{R}^n$, $\Lambda \in \mathbf{R}^{n\times n}$ is a positive definite matrix. With the following lemma, the stability of $\boldsymbol{e}(t)$ and $\dot{\boldsymbol{e}}(t)$ can be concluded by studying $\boldsymbol{r}(t)$.

Lemma 1[23]. Let $\boldsymbol{e}(t) = h(t) * r(t)$, where "$*$" denotes convolution product and $h(t) = L^{-1}(H(s))$ with $H(s)$ being an $n \times n$ strictly proper, exponentially stable transfer function, $L^{(-1)}$ denotes the inverse transformation of the Laplace manipulator. Then, $r \in L_2^n$ implies that $e \in L_2^n \cap L_\infty^n$, $\dot{e} \in L_2^n$, e is continuous and $|\boldsymbol{e}(t)| \rightarrow 0$ as $t \rightarrow \infty$. If, in addition, $|\boldsymbol{r}(t)| \rightarrow 0$ as $t \rightarrow \infty$, then $|\dot{\boldsymbol{e}}(t)| \rightarrow 0$.

For finite function families

$$\mathcal{F} = \{f_p(x), p \in P\} \tag{4}$$

where $P = 1, \cdots, N$ and each $f_p(x)$ is a continuous vector field of $\mathbf{R}^n$ such that $f_p(0) = 0$,

Definition 1[24]. Let Ω be an open subset of $\mathbf{R}^n$ containing the origin. We say that $V(x) : \Omega \rightarrow [0, +\infty)$ is a common weak Lyapunov function for (4) if it is of class $\mathcal{C}^1$, positive definite, and the following holds:

$$\Delta V(x) \cdot f_p(x) \leq 0 \tag{5}$$

for each $x \in \Omega$ and each $p \in P$. We say that $V(x)$ is a common strict Lyapunov function for (4) if in (5) the strict ineuality holds for each $x \in \Omega\backslash\{0\}$.

Lemma 2[24]. Let $V(x) : \Omega \rightarrow [0, +\infty)$ be a weak common Lyapunov function for (4). Let $l > 0$ and let Ω_l be the connected component of the level set $\{x \in \Omega : V(x) < l\}$ such that $0 \in \Omega_l$. Assume that Ω_l is bounded, and let

$$Z = \{x \in \Omega : \exists p \in P \text{ such that } \Delta V(x) \cdot f_p(x) = 0\}$$

Finally, let M be the union of all the compact, weakly invariant sets which are contained in $Z \cap \Omega_l$. Then every solution $\phi \in \mathcal{S}_{\mathrm{dwell}}$ such that $\phi(0) \in \Omega_l$ is attracted by M.

2 Coordination control under fixed communication topology

In this section, we establish an adaptive architecture for coordination control within fixed communication topologies. We will point out in Section 3 that this architecture is also well suited for the switching topology case.

Define the state error between the ith and the jth agents as

$$\boldsymbol{e}_{ij}(t) = \boldsymbol{q}_j(t) - \boldsymbol{q}_i(t), \quad \forall\, i \in \{1, \cdots, N\}, \forall\, j \in \mathcal{N}_i(\mathcal{G}) \quad (6)$$

We choose the input for the ith agent as

$$\boldsymbol{\tau}_i = \hat{M}_i(\boldsymbol{q}_i)\lambda \sum_{j\in\mathcal{N}_i(\mathcal{G})} \dot{\boldsymbol{e}}_{ij} + \hat{C}_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)\lambda \sum_{j\in\mathcal{N}_i(\mathcal{G})} \boldsymbol{e}_{ij} + \bar{\boldsymbol{\tau}}_i \quad (7)$$

where $\hat{M}_i(\boldsymbol{q}_i)$, $\hat{C}_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)$, $i \in \mathcal{I}$ are the estimates of the respective matrices available at that instant, $\lambda \in \mathbf{R}_{>0}$, $\hat{\boldsymbol{\tau}}_{\text{ext},i}$ is the estimated external disturbance and $\boldsymbol{\tau}_i = Y_i\hat{\boldsymbol{\theta}}_i + \bar{\boldsymbol{\tau}}_i$ is the synchronization force that will be defined in the sequel. As the dynamics is linearly parameterizable, according to Property 2, the following equation holds:

$$Y_i\hat{\boldsymbol{\theta}}_i = \hat{M}_i(\boldsymbol{q}_i)\lambda \sum_{j\in\mathcal{N}_i(\mathcal{G})} \dot{\boldsymbol{e}}_{ij} + \hat{C}_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)\lambda \sum_{j\in\mathcal{N}_i(\mathcal{G})} \boldsymbol{e}_{ij} - \hat{\boldsymbol{\tau}}_{\text{ext},i} \quad (8)$$

where $Y_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i, \sum_{j\in\mathcal{N}_i(\mathcal{G})} \dot{\boldsymbol{e}}_{ij}, \sum_{j\in\mathcal{N}_i(\mathcal{G})} \boldsymbol{e}_{ij})$ is a known function of the generalized coordinates, and $\hat{\boldsymbol{\theta}}_i$ is the time-varying estimates of the agent′s actual constant p-dimensional inertial parameters given by $\boldsymbol{\theta}_i$. The force term can therefore be written as $\boldsymbol{\tau}_i = Y_i\hat{\boldsymbol{\theta}}_i + \bar{\boldsymbol{\tau}}_i$. For convenience, we define the synchronization signal of the ith agent as

$$\boldsymbol{\epsilon}_i = -\dot{\boldsymbol{q}}_i + \lambda \sum_{j\in\mathcal{N}_i(\mathcal{G})} \boldsymbol{e}_{ij} \quad (9)$$

Substituting (7) into (1) yields

$$M_i(\boldsymbol{q}_i)\dot{\boldsymbol{\epsilon}}_i + C_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)\boldsymbol{\epsilon}_i = Y_i\tilde{\boldsymbol{\theta}}_i - \bar{\boldsymbol{\tau}}_i \quad (10)$$

where $\tilde{\boldsymbol{\theta}}_i(t) = \boldsymbol{\theta}_i - \hat{\boldsymbol{\theta}}_i(t)$ is the estimation errors of parameters, and it evolves as

$$\dot{\hat{\boldsymbol{\theta}}}_i = \Gamma_i Y_i^{\mathrm{T}} \boldsymbol{\epsilon}_i \quad (11)$$

with Γ_i being a constant positive definite matrix.

Up to now, we have established an adaptive coordination control architecture. We show next that this architecture is suitable for both the static regulation and the dynamic tracking cases.

2.1 Regulation case

In order to regulate and coordinate the states of agents, the following condition has to be fulfilled:

$$\begin{aligned} &\lim_{t\to\infty} |\boldsymbol{q}_i(t) - \boldsymbol{q}_j(t)| = 0 \\ &\lim_{t\to\infty} |\dot{\boldsymbol{q}}_i(t)| \to 0, \ \forall\, i, j \in \mathcal{I} \end{aligned} \quad (12)$$

To this end, we choose $\bar{\boldsymbol{\tau}}_i = K_i\boldsymbol{\epsilon}_i$ ($K_i \in \mathbf{R}_{>0}$). Then the result follows.

Theorem 1. Under the adaptive control architecture (10) and Assumption 1, by choosing $\boldsymbol{\tau}_i = Y_i\hat{\boldsymbol{\theta}}_i + \bar{\boldsymbol{\tau}}_i$, the coordination control in the sense of (12) is achieved with (7) and (11).

Proof. Define a positive semi-definite Lyapunov candidate function $V : \mathcal{C} \to \mathbf{R}^+$ for the system as

$$V(\boldsymbol{\epsilon}_i, \tilde{\boldsymbol{\theta}}_i, \boldsymbol{e}_{ij}) = \frac{1}{2}\sum_{i=1}^{N} \boldsymbol{\epsilon}_i^{\mathrm{T}} M_i(\boldsymbol{q}_i)\boldsymbol{\epsilon}_i + \frac{1}{2}\sum_{i=1}^{N} \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}} \Gamma^{-1} \tilde{\boldsymbol{\theta}}_i \quad (13)$$

The derivative of $V(\epsilon_i, \tilde{\boldsymbol{\theta}}_i, \boldsymbol{e}_{ij})$ along the trajectory of (10) is given by

$$\begin{aligned} &\dot{V}(\boldsymbol{\epsilon}_i, \tilde{\boldsymbol{\theta}}_i, \boldsymbol{e}_{ij}) = \\ &\quad \frac{1}{2}\sum_{i=1}^{N} \epsilon_i^{\mathrm{T}} \dot{M}_i(\boldsymbol{q}_i)\epsilon_i + \sum_{i=1}^{N} \boldsymbol{\epsilon}_i^{\mathrm{T}} \boldsymbol{M}_i(\boldsymbol{q}_i)\dot{\boldsymbol{\epsilon}}_i + \sum_{i=1}^{N} \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}} \Gamma^{-1} \dot{\tilde{\boldsymbol{\theta}}}_i = \\ &\quad \frac{1}{2}\sum_{i=1}^{N} \boldsymbol{\epsilon}_i^{\mathrm{T}} \dot{M}_i(\boldsymbol{q}_i)\boldsymbol{\epsilon}_i + \sum_{i=1}^{N} \boldsymbol{\epsilon}_i^{\mathrm{T}} \left(Y_i\tilde{\boldsymbol{\theta}}_i - \bar{\boldsymbol{\tau}}_i - C_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)\boldsymbol{\epsilon}_i\right) - \\ &\quad \sum_{i=1}^{N} \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}} Y_i^{\mathrm{T}} \boldsymbol{\epsilon}_i = \sum_{i=1}^{N} \boldsymbol{\epsilon}_i^{\mathrm{T}} \left(-\bar{\boldsymbol{\tau}}_i\right) = -\sum_{i=1}^{N} \boldsymbol{\epsilon}_i^{\mathrm{T}} K_i \boldsymbol{\epsilon}_i \end{aligned} \quad (14)$$

where Property 3, (10) and (11) are used in the above derivation. Since $V \geq 0$ and $\dot{V} \leq 0$, $\forall\, i \in \mathcal{I}$, $\boldsymbol{\epsilon}_i(t) \in \mathcal{L}_2$ and $\epsilon_i(t), \tilde{\boldsymbol{\theta}}_i(t) \in \mathcal{L}_\infty$. Next, we show that $|\boldsymbol{\epsilon}_i(t)| \to 0$. To achieve this, we only need to show that $\dot{\boldsymbol{\epsilon}}_i(t) \in \mathcal{L}_\infty$ according to Barbalat′s Lemma. To this end, we rewrite the second equation in (9) as

$$\dot{\boldsymbol{q}}(t) = -\lambda(\mathcal{L} \otimes I_3)\boldsymbol{q}(t) + I_{3\times N} \otimes \boldsymbol{\epsilon}(t) \quad (15)$$

where I_N is the identity matrix with dimension N, $\boldsymbol{q} = [\boldsymbol{q}_1^{\mathrm{T}}(t), \cdots, \boldsymbol{q}_N^{\mathrm{T}}(t)]^{\mathrm{T}}$, $\boldsymbol{\epsilon} = [\boldsymbol{\epsilon}_1^{\mathrm{T}}(t), \cdots, \boldsymbol{\epsilon}_N^{\mathrm{T}}(t)]^{\mathrm{T}}$, "$\otimes$" is the standard Kronecker product, and $\mathcal{L}$ corresponds to the Laplacian matrix. Under the assumption that the communication graph contains a spanning tree, it can be concluded that $-\mathcal{L}$ has exactly one zero eigenvalue and all the other non-zero eigenvalues are in the open left half plane. On the other hand, the transfer function between $\boldsymbol{\epsilon}(t)$ and $\dot{\boldsymbol{q}}(t)$ of system (15) can be written as

$$T(s) = \frac{sI_{3\times N}}{sI_{3\times N} + \lambda\mathcal{L} \otimes I_3} \quad (16)$$

Note that the characteristic poles of $T(s)$ are just the eigenvalues of $-\lambda\mathcal{L}$. Therefore, we conclude that $T(s)$ has exactly one zero pole and the other poles are located in the open left-hand complex plane. By considering the zero-pole cancelation in $T(s)$, we conclude that this system is stable. Since $\boldsymbol{\epsilon} \in \mathcal{L}_\infty$, we have $\dot{\boldsymbol{q}} \in \mathcal{L}_\infty$ and thus $\sum_{j\in\mathcal{N}_i(\mathcal{G})} \dot{\boldsymbol{e}}_{ij}(t) \in \mathcal{L}_\infty$. By observing (9), since $\boldsymbol{\epsilon}_i(t)$, $\dot{\boldsymbol{q}}_i(t) \in \mathcal{L}_\infty$, one has $\sum_{j\in\mathcal{N}_i(\mathcal{G})} \boldsymbol{e}_{ij}(t) \in \mathcal{L}_\infty$. In addition, by (8), one can show that the boundedness of Y_i depends on the boundedness of $\hat{M}_i(\boldsymbol{q}_i)$, $\sum_{j\in\mathcal{N}_i(\mathcal{G})} \dot{e}_{ij}$, $\hat{C}_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)$, $\sum_{j\in\mathcal{N}_i(\mathcal{G})} \boldsymbol{e}_{ij}$, $\hat{\boldsymbol{\tau}}_{\text{ext},i}$ and $\hat{\boldsymbol{\theta}}_i$, and it is independent of $\boldsymbol{q}_i$. Then, by Property 1, Property 4, the boundedness of $\dot{\boldsymbol{q}}_i$, $\tilde{\boldsymbol{\theta}}_i$, $\sum_{j\in\mathcal{N}_i(\mathcal{G})} \dot{\boldsymbol{e}}_{ij}(t)$ and $\sum_{j\in\mathcal{N}_i(\mathcal{G})} \boldsymbol{e}_{ij}(t)$, we conclude that Y_i is also bounded. Then, we further conclude that $\dot{\boldsymbol{\epsilon}}(t) \in \mathcal{L}_\infty$ according to (10). Therefore, $|\boldsymbol{\epsilon}_i(t)| \to 0$, which implies $|\dot{\boldsymbol{q}}_i(t)| \to 0$. Then, we conclude that $|\boldsymbol{q}_i(t) - \boldsymbol{q}_j(t)| \to 0$, $\forall\, i, j \in \mathcal{I}$ according to Theorem 3 in [25]. □

2.2 Dynamic tracking case

In this case, the control objective can be formulated as

$$\begin{aligned} &\lim_{t\to\infty} |\boldsymbol{q}_i(t) - \boldsymbol{q}_d(t)| = 0 \\ &\lim_{t\to\infty} |\dot{\boldsymbol{q}}_i(t) - \dot{\boldsymbol{q}}_d(t)| = 0, \ \forall\, i \in \mathcal{I} \end{aligned} \quad (17)$$

where $\boldsymbol{q}_d(t), \dot{\boldsymbol{q}}_d(t) \in \mathcal{L}_\infty$. We redefine a new adjacency matrix $\mathcal{W} = [w_{ij}] \in \mathbf{R}^{(N+1)\times(N+1)}$, where $w_{ij} = a_{ij}$, $\forall\, i, j \in \mathcal{I}$ and $w_{i(N+1)} = 1$ if agent i has access to the reference and

0 otherwise, and $w_{(N+1)k} = 0$, $\forall\, k \in \{1, \cdots, N+1\}$. The synchronization signal is defined as

$$\boldsymbol{\eta}_i = \frac{1}{d_i}\left[\sum_{j=1}^{N+1} w_{ij}(\dot{\boldsymbol{q}}_j - \dot{\boldsymbol{q}}_i) + \sum_{j=1}^{N+1} w_{ij}(\boldsymbol{q}_j - \boldsymbol{q}_i)\right], \quad i = 1, \cdots, N \tag{18}$$

where d_i denotes the in-degree of the ith agent, $\boldsymbol{q}_{N+1} = \boldsymbol{q}_d(t)$, and $\dot{\boldsymbol{q}}_{N+1} = \dot{\boldsymbol{q}}_d(t)$, namely, the reference is viewed as a "virtual leader", and is indexed by $N+1$. Correspondingly, Y_i is defined as

$$Y_i = M_i(\boldsymbol{q}_i)\left[\sum_{j=1}^{1+N} w_{ij}\ddot{\boldsymbol{q}}_j + \sum_{j=1}^{N+1} w_{ij}(\dot{\boldsymbol{q}}_j - \dot{\boldsymbol{q}}_i)\right] + C_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)\left[\sum_{j=1}^{N+1} w_{ij}\dot{\boldsymbol{q}}_j + \sum_{j=1}^{N+1} w_{ij}(\boldsymbol{q}_j - \boldsymbol{q}_i)\right] - \boldsymbol{\tau}_{\mathrm{ext},i} \tag{19}$$

Note that Y_i depends on the states of agents and their first and second-order derivatives. In practical implementation, the second-order derivatives of the neighbors' states can be calculated by numerical differentiation. By choosing $\boldsymbol{\tau}_i = Y_i\hat{\boldsymbol{\theta}}_i + \bar{\boldsymbol{\tau}}_i$, it is easy to verify that the following equation holds:

$$M_i(\boldsymbol{q}_i)\dot{\boldsymbol{\eta}}_i + C_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)\boldsymbol{\eta}_i = \frac{Y\tilde{\boldsymbol{\theta}}_i - \bar{\boldsymbol{\tau}}_i}{d_i} \tag{20}$$

where $\tilde{\boldsymbol{\theta}}_i(t) = \boldsymbol{\theta}_i - \hat{\boldsymbol{\theta}}_i(t)$ is the estimation errors of parameters, and $\hat{\boldsymbol{\theta}}_i(t)$ is updated by

$$\dot{\hat{\boldsymbol{\theta}}}_i = \frac{\Gamma_i Y_i^{\mathrm{T}}\boldsymbol{\eta}_i}{d_i} \tag{21}$$

where Γ_i is a positive definite matrix. Then, we have the following result.

Corollary 1. Under the adaptive control architecture (18) ~ (21) and if the directed graph $\mathcal{W}$ contains a spanning tree, by choosing $\bar{\boldsymbol{\tau}}_i = K_i\boldsymbol{\eta}_i$ ($K_i \in \mathbf{R}_{>0}, i \in \mathcal{N}^f$), then the coordination control in the sense of (17) is reached.

Proof. By choosing the similar Lyapunov function with (13)

$$V = \frac{1}{2}\sum_{i=1}^{N}\boldsymbol{\eta}_i^{\mathrm{T}} M_i(\boldsymbol{q}_i)\boldsymbol{\eta}_i + \frac{1}{2}\sum_{i=1}^{N}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\Gamma^{-1}\tilde{\boldsymbol{\theta}}_i \tag{22}$$

we can get its derivative as

$$\dot{V} = -\sum_{i=1}^{N}\frac{\boldsymbol{\eta}_i^{\mathrm{T}} K_i\boldsymbol{\eta}_i}{d_i} \tag{23}$$

Since $d_i > 0$, we have $\boldsymbol{\eta}_i \in L_\infty \cap \mathcal{L}_2$ according to the proof of Theorem 1. It follows from Lemma 1 that $\sum_{j=1}^{N+1} a_{ij}(\boldsymbol{q}_j - \boldsymbol{q}_i) \in \mathcal{L}_\infty$, $\sum_{j=1}^{N+1} a_{ij}(\dot{\boldsymbol{q}}_j - \dot{\boldsymbol{q}}_i) \in \mathcal{L}_\infty$ and

$$\sum_{j=1}^{N+1} a_{ij}(\boldsymbol{q}_j - \boldsymbol{q}_i) \rightarrow 0, \;\; i = 1, \cdots, N \tag{24}$$

We rewrite (24) in matrix form as $(\mathcal{L}_{N+1} \otimes I_p)\boldsymbol{q} \rightarrow 0$, where $\boldsymbol{q} = [\boldsymbol{q}_1^{\mathrm{T}}, \cdots, \boldsymbol{q}_N^{\mathrm{T}}]$, the square matrix $\boldsymbol{\mathcal{L}}_{N+1} = [l_{ij}] \in \mathbf{R}^{(N+1)\times(N+1)}$ is defined as $l_{ii} = \sum_{j\neq i} a_{ij}$, $l_{ij} = -a_{ij}$, $\forall\, i \in \mathcal{I}$, and $l_{(N+1)i} = 0$, $\forall\, i$. Following the same procedure as the proof of Theorem 3.3 in [26], it can be concluded that $\boldsymbol{q}_i \rightarrow \boldsymbol{q}_j(t)$, $\forall\, i, j \in \{1, \cdots, N+1\}$ under Assumption 1. Then we get $\boldsymbol{q}_i \rightarrow \boldsymbol{q}_d(t)$ since $\boldsymbol{q}_{N+1} = \boldsymbol{q}_d(t)$. On the other hand, following the same line as proof of Theorem 1, it can be concluded that $\boldsymbol{\eta}_i \rightarrow 0$. From Lemma 1, it follows that $\sum_{j=1}^{N+1} a_{ij}(\dot{\boldsymbol{q}}_j - \dot{\boldsymbol{q}}_i) \rightarrow 0$. It turns out that the same procedure to prove $\boldsymbol{q}_i \rightarrow \boldsymbol{q}_d(t)$ can be used again to show $\dot{\boldsymbol{q}}_i \rightarrow \dot{\boldsymbol{q}}_d(t)$. Therefore, the coordination control in the sense of (17) is reached. □

Remark 1. The above discussions are based on the assumption that the external disturbance $\boldsymbol{\tau}_{\mathrm{ext},i}$ is constant and bounded. If this assumption is relaxed to unknown time-varying but bounded disturbances, it can be shown that the coordination can also be obtained by adopting the sliding mode like technique. We present a brief discussion as follows.

Suppose that the external disturbance $\boldsymbol{\tau}_{\mathrm{ext},i} \in \mathcal{L}_\infty$; according to Property 2, we redefine $Y_i\hat{\boldsymbol{\theta}}_i$ as

$$Y_i\hat{\boldsymbol{\theta}}_i = \hat{M}_i(\boldsymbol{q}_i)\lambda\sum_{j\in N_i(\mathcal{G})}\dot{\boldsymbol{e}}_{ij} + \hat{C}_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)\lambda\sum_{j\in N_i(\mathcal{G})}\boldsymbol{e}_{ij} \tag{25}$$

and $\bar{\boldsymbol{\tau}}_i$ as

$$\bar{\boldsymbol{\tau}}_i(t) = K_i\boldsymbol{\epsilon}_i(t) + \boldsymbol{k}_i\mathrm{sgn}(\boldsymbol{\epsilon}_i(t)) \tag{26}$$

where $\boldsymbol{\epsilon}_i(t)$ is defined in (9), $\boldsymbol{k}_i \in \mathbf{R}_+^p$ and sgn$(\cdot)$ is the sign function. Then, the closed-loop dynamics becomes

$$M_i(\boldsymbol{q}_i)\dot{\epsilon}_i + C_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)\epsilon_i = Y_i\tilde{\boldsymbol{\theta}}_i - \bar{\tau}_i - \hat{\tau}_{\mathrm{ext},i} \tag{27}$$

Using the same Lyapunov function as (13), we get its derivative as

$$\dot{V}(\epsilon_i, \tilde{\boldsymbol{\theta}}_i, \boldsymbol{e}_{ij}) = -\sum_{i=1}^{N}\boldsymbol{\epsilon}_i^{\mathrm{T}}(K_i\boldsymbol{\epsilon}_i) - \sum_{i=1}^{N}\boldsymbol{\epsilon}_i^{\mathrm{T}}(\boldsymbol{\tau}_{\mathrm{ext},i} + \boldsymbol{k}_i\mathrm{sgn}(\boldsymbol{\epsilon}_i)) \tag{28}$$

Denote $\boldsymbol{v}^{(m)}$ as the mth element of vector $\boldsymbol{v}$. Suppose that the disturbances are bounded by $\left|\boldsymbol{\tau}_{\mathrm{ext},i}^{(m)}\right| \leq \Upsilon_i^m$, $m = 1, \cdots, p$. By choosing $k_i^m = \Upsilon_i^m + \eta_i^m$, $m = 1, \cdots, p$, where $\eta_i^m \in \mathbf{R}^+$, we obtain

$$\dot{V}(\boldsymbol{\epsilon}_i, \tilde{\boldsymbol{\theta}}_i, \boldsymbol{e}_{ij}) \leq -\sum_{i=1}^{N}\boldsymbol{\epsilon}_i^{\mathrm{T}}(K_i\boldsymbol{\epsilon}_i) - \sum_{i=1}^{N}\sum_{m=1}^{p}\eta_i^m\left|\epsilon_i^m\right| \tag{29}$$

Then, similar proof procedures as in Theorem 1 can be employed to demonstrate the effectiveness of this method to deal with the time-varying bounded disturbances. Also note that in order to avoid the undesirable control chatting, saturation functions sat(ϵ_i^m/ϕ_i^m) can be used in place of the switching function sat(ϵ_i^m), with the ϕ_i^m representing the thicknesses of the corresponding "boundary layers". However, when saturation functions are used, $\boldsymbol{\epsilon}_i$ is only guaranteed to converge to the bounded layers with corresponding small tracking errors rather than zero. The precise coordination will not be reached with the the time-varying bounded disturbances.

3 Coordination control under switching topology

In this section, we deal with coordination control of EL systems with switching topology. To this end, we assume a finite set of graphs $\{\mathcal{G}_p\}$ with $p \in \mathcal{P} = \{1, v, P\}$. At any time t, one of the graphs $\{\mathcal{G}_p\}$ represents the topology of the communication network between agents. The switching signal $\sigma : [0, \infty) \to \mathcal{P}$ determines the index of the active graph at time t. First, we make some assumptions on interconnection graphs and switching signal as follows.

Assumption 1. The switching signal σ is piecewise constant from the right and non-chattering. If there are infinitely many switching times, there exists a $\tau > 0$ such that for every $T \geq 0$ we can find a positive integer i for which $t_{i+1} - \tau \geq t_i \geq T$. In other words, we persistently encounter intervals of length at least τ between switching times.

Assumption 2. The communication topology $\{\mathcal{G}_p\}$ is quasi-strongly connected for each $p \in \mathcal{P} = \{1, \cdots, P\}$.

Under the adaptive control architecture (10), let the synchronization force be

$$\bar{\boldsymbol{\tau}}_{\sigma i}(x, t) = K_i \boldsymbol{\epsilon}_i \tag{30}$$

where $\boldsymbol{\epsilon}_i$ is defined by (9). Then, we are ready to show that even when the communication graph $\mathcal{G}$ is dynamic, the coordination of the group is also reachable.

Theorem 2. Under Assumptions 1 and 2, and the adaptive control architecture (10), the coordination control in the sense of (12) is achieved with (30), (7), and (11) for all initial conditions.

Proof. The proof is based on the existence of a common weak Lyapunov function candidate for the switched networked EL systems. Once existence of such a function is shown, by invoking Lemma 1 one can conclude convergence of $\boldsymbol{\epsilon}$ to zero in the presence of a non-vanishing dwell time between any two sequential switches.

Consider the following positive definite radially unbounded continuously differentiable Lyapunov function candidate for the ith communication topology, where $i \in \mathcal{I}$, namely,

$$V(t) = \frac{1}{2}\sum_{i=1}^{N} \boldsymbol{\epsilon}_i^{\mathrm{T}}(t) M_i(\boldsymbol{q}_i)\boldsymbol{\epsilon}_i(t) + \frac{1}{2}\sum_{i=1}^{N} \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}(t)\Gamma^{-1}\tilde{\boldsymbol{\theta}}_i(t) \tag{31}$$

According to the proof of Theorem 1, the time derivative of the above function is obtained as

$$\dot{V}(t) = -\sum_{i=1}^{N} \boldsymbol{\epsilon}_i^{\mathrm{T}} K_i \boldsymbol{\epsilon}_i \tag{32}$$

The above function is negative semi-definite with respect to $\boldsymbol{\epsilon}_i$ since the Laplacian matrix of the communication graph is a positive semi-definite matrix. Consequently, $\boldsymbol{\epsilon}_i, \forall i$ are stable and bounded. Noting the fact that the Lyapunov function candidate is identical for all communication topologies one concludes that the Lyapunov function (31) is a weak common Lyapunov function for the considered switched system. Let us define set $\mathcal{H} = \{\boldsymbol{\epsilon}_i \in \mathbf{R}^k, \forall i \in I, \dot{V} = 0\}$. Note that when $\dot{V} = 0$ we have $\boldsymbol{\epsilon}_i = 0$. Now, let set $\bar{\mathcal{H}}$ be the largest weakly invariant set in $\mathcal{H}$. On $\bar{\mathcal{H}}$ we have $\boldsymbol{\epsilon}_i = 0$. By invoking Lemma 2, we conclude that under non-vanishing dwell time $\boldsymbol{\epsilon}_i$ converges to $\mathcal{H}$. Consequently, for the networked EL systems we have $\boldsymbol{\epsilon}_i \to 0$ as $t \to 0$. Then similar procedures can be adopted as in the proof of Theorem 1 to show that $|\boldsymbol{q}_i(t) - \boldsymbol{q}_j(t)| \to 0$, and $\dot{\boldsymbol{q}}_i \to 0$, $\forall i, j \in \mathcal{I}$. □

Remark 2. In contrast to [20] where balanced topology is required, we only need quasi-strongly connected topology. In addition, the dynamic tracking can be fulfilled by our method whereas the controller architecture only guarantees asymptotically stable in terms of both the state and its first derivative.

Similar to Corollary 1, Theorem 2 can be easily extended to the dynamic tracking case by noting that (22) is also a weak common Lyapunov function.

Corollary 2. Under Assumptions 1 and 2, and the adaptive architecture (18) ~ (21) and the assumption that the directed graph $\mathcal{W}$ contains a spanning tree, by choosing $\bar{\boldsymbol{\tau}}_{\sigma i} = K_i \boldsymbol{\eta}_i$ ($K_i \in \mathbf{R}_{>0}$, $i \in \mathcal{N}^f$) where $\boldsymbol{\eta}$ is defined in (18), the coordination control in the sense of (17) is reached.

Proof. The proof of Corollary 2 can follow the same as Corollary 1 and Theorem 2 and is omitted here. □

4 Numerical simulation

In this section, we simulate scenarios where three two-link manipulators reach state consensus through local communication. We assume that the dynamicses of the manipulators are identical. Note that the algorithms proposed in this paper can also handle networked EL systems with a different dynamics. The dynamicses of the two-link manipulators are given as follows.

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -h\dot{q}_2 & -h(\dot{q}_1 + \dot{q}_2) \\ h\dot{q}_1 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

where

$$H_{11} = a_1 + 2a_3 \cos q_2 + 2a_4 \sin q_2$$
$$H_{12} = H_{21} = a_2 + a_3 \cos q_2 + a_4 \sin q_2$$
$$H_{22} = a_2$$
$$h = a_3 \sin q_2 - a_4 \cos q_2$$

with

$$a_1 = I_1 + m_l l_{c1}^2 + I_e + m_e l_{ce}^2 + m_e l_1^2$$
$$a_2 = I_e + m_e l_{ce}^2$$
$$a_3 = m_e l_1 l_{ce} \cos \delta_e$$
$$a_4 = m_e l_1 l_{ce} \sin \delta_e$$

In this simulation, we use $m_1 = 1$, $l_1 = 1$, $m_e = 2$, $\delta_e = 30°$, $I_1 = 0.12$, $l_{c1} = 0.5$, $I_e = 0.25$, and $l_{ce} = 0.6$. For simplicity, the communication topology is illustrated in Fig. 1. According to Property 2, we choose $\boldsymbol{\theta} = [a_1, a_2, a_3, a_4]^{\mathrm{T}}$, and $\lambda = I_{4\times4}$. The corresponding $Y(\boldsymbol{q}, \boldsymbol{r}) = [y_{ij}] \in \mathbf{R}^{2\times4}$ is then defined as

$$y_{11} = \dot{\epsilon}_1,\ y_{12} = \dot{\epsilon}_2,\ y_{21} = 0,\ y_{22} = \dot{\epsilon}_1 + \dot{\epsilon}_2$$
$$y_{23} = \dot{\epsilon}_1 \cos(\dot{q}_2) + \dot{\epsilon}_1 \dot{q}_1 \sin(q_2)$$
$$y_{24} = -\dot{\epsilon}_1 q_1 \cos(q_2) + \epsilon_1 \dot{q}_1 \sin(q_2)$$
$$y_{13} = (2\dot{\epsilon}_1 + \dot{\epsilon}_2)\cos(q_2) - (\dot{q}_2\epsilon_1 + \dot{q}_1\epsilon_2 + \dot{q}_2\epsilon_2)\sin(q_2)$$
$$y_{14} = (2\dot{\epsilon}_1 + \dot{\epsilon}_2)\sin(q_2) + (\dot{q}_2\epsilon_1 + \dot{q}_1\epsilon_2 + \dot{q}_2\epsilon_2)\cos(q_2)$$

We set the parameters to be 70 % of their real values. The initial values of $\boldsymbol{q}_i$ and $\dot{\boldsymbol{q}}_i$ are chosen randomly within $[-2.5, 2.5]$. The reference signal $\boldsymbol{q}_d(t)$ is chosen as $\boldsymbol{q}_d(t) = [1.25\cos^2(0.25t), 0.8\sin(0.3t)]$, and the control gains are chosen as $K = 6$, $\Gamma = 2I_{4\times4}$, and $\Lambda = \Gamma$.

The simulation was conducted in Wolfram Mathematica© 7, and the case of dynamic tracking in the leader-follower framework was studied, where agent ref is designated as the virtual leader. We first simulated the case of dynamic tracking under a static topology which is shown in Fig. 1. Figs. 2 ∼ 5 show respectively the joint angels and their derivatives of the agents. It can be seen from the results that the dynamic tracking is attained.

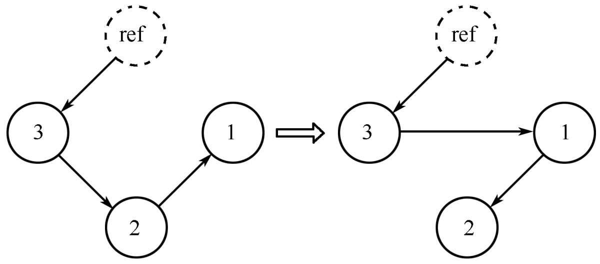

Fig. 1 Communication topology

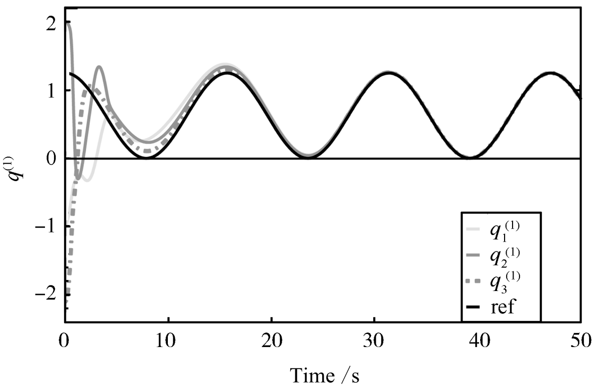

Fig. 2 Convergence of $q^{(1)}$ within static topology

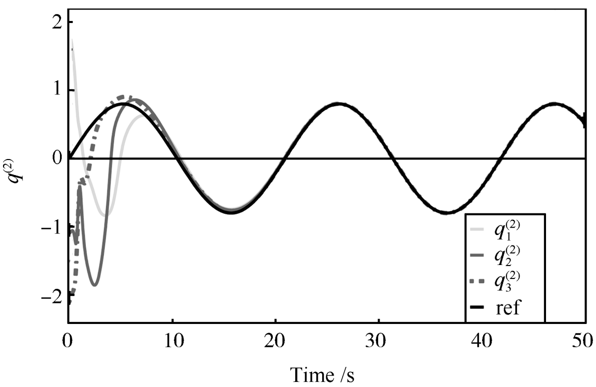

Fig. 3 Convergence of $q^{(2)}$ within static topology

For the switching topology case, we assume that the communication topology switches periodically between Figs. 1 (a) and 1 (b) with 5 Hz. The simulation results are shown in Figs. 6 ∼ 9. Note that compared with the static topology case, the switching property was revealed within the first 8 seconds, then the dynamic tracking was achieved.

5 Conclusion

We have investigated the problem of coordination control

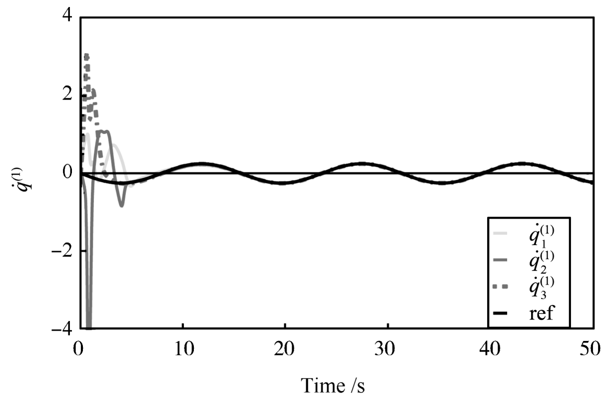

Fig. 4 Convergence of $\dot{q}^{(1)}$ within static topology

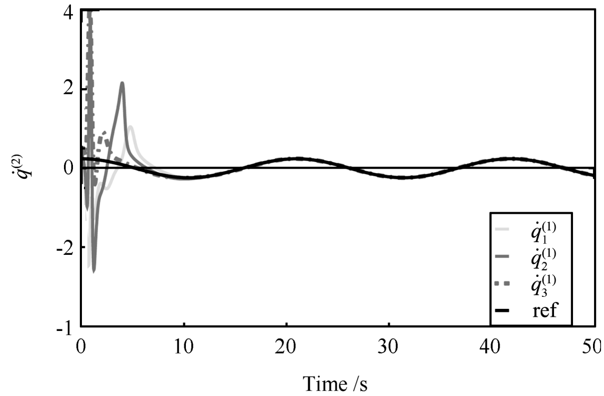

Fig. 5 Convergence of $\dot{q}^{(2)}$ within static topology

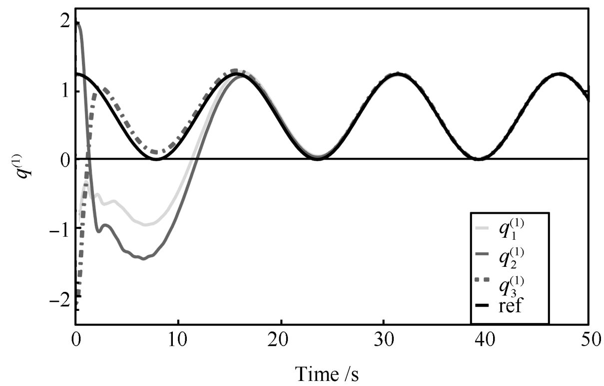

Fig. 6 Convergence of $q^{(1)}$ within switching topology

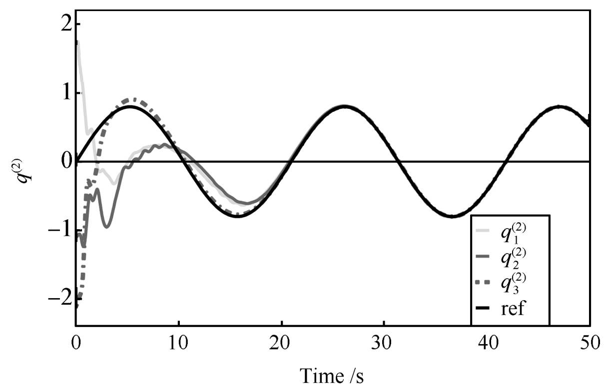

Fig. 7 Convergence of $q^{(2)}$ within switching topology

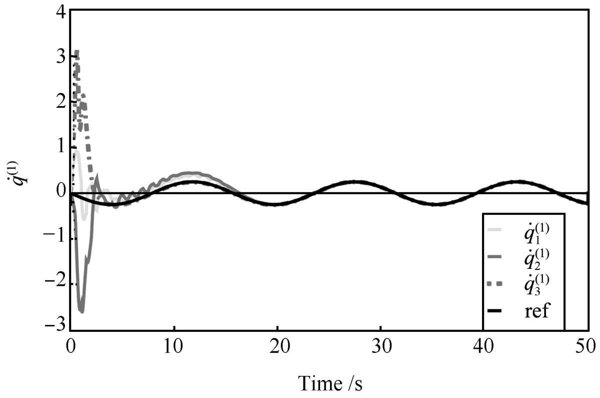

Fig. 8 Convergence of $\dot{q}^{(1)}$ within switching topology

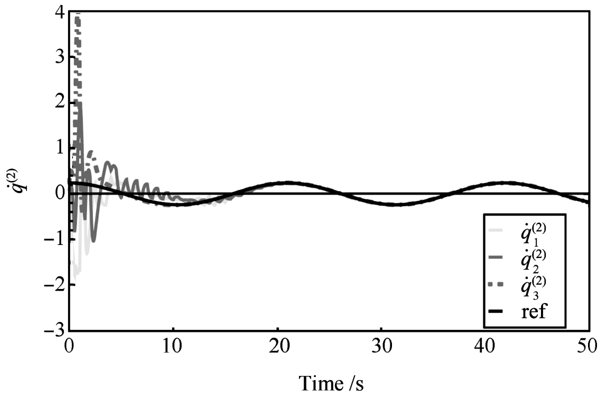

Fig. 9 Convergence of $\dot{q}^{(2)}$ within switching topology

of EL systems with parametric uncertainty and possible switching communication topology. Under the assumption that parameters in system dynamics and external disturbances are unknown, we established a unified adaptive control architecture. Then, based on this architecture, coordination controllers were developed. It is proved by Lyapunov theory and the switching version LaSalle-like theory that these controllers can guarantee the coordination in both static and switching topologies. Numerical simulation was conducted to demonstrate the effectiveness of the proposed controllers. Our future work will be the implementation of experiment on our hardware-in-the-loop platform.

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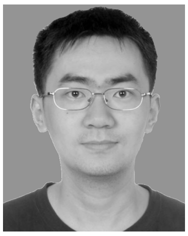
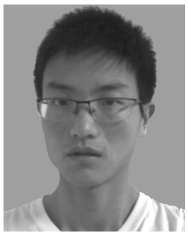


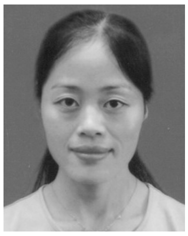
MIN Hai-Bo Ph. D., lecturer at Hi-Tech Institute of Xi′an. His research interest covers spacecraft formation and multi-agent coordination control. Corresponding author of this paper.
E-mail: haibo.min@gmail.com

LIU Zhi-Guo Ph. D., lecturer at Hi-Tech Institute of Xi′an. His research interest covers satellite navigation, precision guidance and control.
E-mail: lzgc@163.com



LIU Yuan Ph. D. candidate at Hi-Tech Institute of Xi′an. His research interest covers multi-agent coordination control and spacecraft attitude control.
E-mail: liuyuan0123@gmail.com

WANG Shi-Cheng Ph. D., professor at the Second Artillery Engineering University. His research interest covers navigation, guidance and control, control theory and engineering.
E-mail: wshcheng@vip.163.com



YANG Yan-Li Ph. D., lecturer at Hi-Tech Institute of Xi′an. Her research interest covers network based control system, navigation guidance and control.
E-mail: yanli0244@sina.com