Estimation, Intervention and Interaction of Multi-agent Systems

HU Jiang-Ping[1] LIU Zhi-Xin[2] WANG Jin-Huan[3] WANG Lin[4] HU Xiao-Ming[5]

Abstract In this paper we provide a brief survey on recent research on multi-agent systems. We focus on results in three areas of the research, namely, estimation and filtering, intervention by external means, and interactive control.

Key words Distributed estimation, cooperative filter, soft control, state-topology coevolution, nonlinear consensus control

Citation Hu Jiang-Ping, Liu Zhi-Xin, Wang Jin-Huan, Wang Lin, Hu Xiao-Ming. Estimation, intervention and interaction of multi-agent systems. *Acta Automatica Sinica*, 2013, **39**(11): 1796−1804

DOI 10.3724/SP.J.1004.2013.01796

There has been a tremendous research interest in multi-agent systems in the control field since the last decade[1−3]. Multi-agent control systems are spatially distributed systems consisting of a number of interacting agents in which sensing, communication, and control are carried out locally in a distributed fashion. These networked multi-agent systems may have many advantages compared to single-agent (centralized) systems, including improved flexibility and reliability, and cost efficiency. These advantages are due to the capability of self-organization or coordination, namely, of a global ordered behavior of the networked system that can emerge by implementing properly designed local control protocols. These properties are intrinsically robust against malfunctioning or perturbations in individual agents. The emergence of global effects[4−5] as a result of microscopic local interactions could be seen as a generalization of the framework of statistical mechanics to ensembles of man-made microscopic objects capable of sensing and decision making.

As is well understood, the distributed nature of networked systems and their need to adapt to varying conditions, however, also pose great challenges to the mature approaches and theories in the literature, in particular in terms of emergence and scalability. Emergent behavior has been studied for quite some time in computer science and biology[4−5]. For networked mobile systems, it is clear that emergent behavior is a very promising direction, given that the capability of a single agent is quite limited so far. However, this leads also to the challenging issue of how to prevent undesired emergent behaviors from undermining the reliability of the system.

In distributed coordination of multi-agent systems, a critical aspect is that information can mostly be collected locally by the agents. When it comes to linear multi-agent systems, the leaderless consensus problem is already well understood. However, by adding leaders that has more information or abilities the system can give rise to many new applications[3, 5−7]. In most of the literature, some connectivity of the associated graph is needed. However, how to guarantee the connectivity of the neighbor graphs when the connectivity is distance induced, in particular by exploiting the deployment of leaders, is an important and still unsolved issue. Furthermore, when the size of the group is large, how to use leaders to influence and modify the statistical properties of the group as time evolves, remains an open issue despite some recent study[8], in which a leaderless multi-agent system without repulsive forces is considered.

In this survey we consider the following type of multi-agent systems (MAS):

$$\begin{aligned}\dot{x}_i &= f_i(x, u_i)\\ y_i &= h_i(x_i)\end{aligned}$$

where $x_i \in \mathbf{R}^n,\ i = 1, \cdots, N$, is the state of agent i, u_i the control, y_i the output, and x the stacked vector of vectors x_i.

One can also consider the discrete time case:

$$\begin{aligned}x_i(t+1) &= f_i(x, u_i)\\ y_i &= h_i(x_i)\end{aligned}$$

In this paper we will focus on three areas of the research on multi-agent systems, namely on estimation and filtering, intervention by external means, and interactive control.

Sensor networks are multi-agent systems that consist of a large number of inexpensive wireless devices densely distributed over the region of interest. Sensor network technology can be potentially applied in many areas including manufacturing, agriculture, construction, transportation and so on. Target tracking problem is a very important research topic in multi-sensor monitoring and has attracted the attention of many researchers in robotic systems and control theory till today. Target tracking algorithms usually focus on the cooperative information processing through the sensors′ interaction with a target after the target has already been detected within the area that the sensors cover. There are two critical issues, the designs of deployment strategies and tracking filters, in target tracking for sensor networks. Many results have been obtained to solve these two problems. Though, sometimes, sensor deployment and filter design, seem independent in purposes and methods, in fact, they can be integrated together to improve the information processing quality during target tracking for sensors with limited communication and computation resources[9−10].

Intervention of multi-agent systems by external means such as leaders is a relatively new topic, and the current study mainly focuses on the conceptual framework, experiments and computer simulations. On the other hand, in

Manuscript received July 1, 2013; accepted August 29, 2013
Supported by National Natural Science Foundation of China (61104104, 60804043, 61273221, 61104137, 61203142)
Invited Articles for the Special Issue for the 50th Anniversary of Acta Automatica Sinica
1. School of Automation Engineering, University of Electronic Science and Technology of China, Chengdu 611731, China 2. Key Laboratory of Systems and Control, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China 3. School of Sciences, Hebei University of Technology, Tianjin 300401, China 4. Department of Automation, Shanghai Jiao Tong University, Shanghai 200240, China 5. Department of Mathematics, Royal Institute of Technology (KTH), Stockholm 10044, Sweden

many biological systems, such as fish schooling and honeybee groups, at least some agents (leaders) have pertinent information about where to go, who can in turn help guide the whole group. This phenomenon has inspired researchers to investigate the intervention of multi-agent systems by adding special agents[5−7]. Another motivation came from the desire to understand and intervene crowd behavior during an emergency, which has become increasingly important in the study of multi-agent social systems. Various pedestrian models such as social force model[11−12] have been proposed to gain insight into the crowd dynamics, in particular when in panic[11].

How we in general control multi-agent systems based on local information only such that the overall system exhibits the desired collective behavior is a fundamental and challenging problem. Design of such control for multi-agent systems has been focusing on distributed control, that is, design of local rules for agents such that the system self-organizes to the desired behavior through interaction. Multi-agent systems with mutual interaction between agents′ state and networks′ topology are called multi-agent systems with state-topology coevolution. In the field of complex network, such kinds of systems are also known as coevolutionary or adaptive network.

The paper is organized as follows: Section 1 presents some recent results on distributed estimation design for consensus control of multi-agent systems and target tracking in sensor networks. Two intervention methods of multi-agent systems are reviewed in Section 2. Some interesting results on the coevolution control of multi-agent systems or complex networks are provided in Section 3.

1 Distributed estimation for networked systems

With the rapid development of communication, computation and network theory and technology, control design for a networked system has become a relatively challenging and also inspiring research direction in modern control theory. Distributed computation and decentralized feedback are two distinct characteristics of networked systems control. Thus, control of networked systems generally depends on local information within the neighborhood, even to achieve just a group coordination. Distributed estimation for networked systems becomes greatly interesting when only partial measurements, noisy measurements or unknown disturbances exist in the networked systems.

1.1 Distributed estimation based consensus control of multi-agent systems

Multi-agent system is an important kind of networked systems, where agents are interconnected over an undirected or directed network topology and coordinated through a decentralized feedback control law. The information flow between each pair of linked agents may be subject to uncertainty and noise. The consensus control of multi-agent systems has to be designed with only partial, disturbed or noisy measurements. There are two main approaches to deal with the distributed estimation based consensus control. One is distributed estimation via observers design for multi-agent coordination, and the other is distributed output regulation based consensus control.

Till now, there have been some important results on distributed observer based consensus control of multi-agent systems[1, 13−15]. In [1], the agent dynamics is described by an identical linear system (1) and each agent receives two measurements: the internal state measurements y_i and the external state measurements z_{ij} relative to other agents.

$$\begin{cases}\dot{x}_i = Ax_i + Bu_i\\ y_i = C_1 x_i\\ z_{ij} = C_2(x_i - x_j), \ \ j \in \mathcal{N}_i\end{cases} \tag{1}$$

where x_i is the state of agent i and $\mathcal{N}_i$ is the index set of the neighbors of agent i. Since both the measurements may contain only partial information of the states of the agent and its neighbors, a dynamical output feedback control

$$\begin{cases}\dot{v}_i = K_1 v_i + K_2 y_i + K_3 z_i\\ u_i = G_1 v_i + G_2 y_i + G_3 z_i\end{cases} \tag{2}$$

where $z_i = \frac{1}{|\mathcal{N}_i|}\sum_{j\in\mathcal{N}_i} z_{ij}$, has been designed for multi-agent systems with some special interconnection topologies. In [13−14], as a special case of the system (1), when a self-active leader is involved in the multi-agent consensus problem,

$$\begin{cases}\dot{p}_0 = q_0\\ \dot{q}_0 = u_0\\ y_0 = p_0\end{cases} \tag{3}$$

a consensus tracking control u_i has to be designed for follower agents, whose dynamics is described by a second-order differential equation

$$\begin{cases}\dot{p}_i = q_i + \delta_i^1\\ \dot{q}_i = u_i + \delta_i^2\end{cases} \tag{4}$$

where p_i, q_i are the state variables of agent i, δ_i^1 and δ_i^2 are the disturbances. Under the assumptions that only the position of the leader can be measured and there exist disturbances in the follower dynamics, a decentralized observer (5) was firstly proposed to estimate the velocity of the leader,

$$\dot{\hat{q}}_i = u_0 - \frac{l}{k}\Bigg[\sum_{j\in\mathcal{N}_i}(p_i - p_j) + b_i(p_i - p_0)\Bigg] \tag{5}$$

and a tracking control

$$u_i = u_0 - k(q_i - \hat{q}_i) - l\Bigg[\sum_{j\in\mathcal{N}_i}(p_i - p_j) + b_i(p_i - p_0)\Bigg] \tag{6}$$

was then designed with the relative position measurements and the velocity estimations. In [15], a class of second-order consensus problem was considered, where all agents are required to reach consensus on both the position and the velocity. Two auxiliary systems were assigned to each agent with only position measurements, which was implemented to generate intermediary reference trajectories. When multiple dynamic leaders exist in a multi-agent system, a containment problem was considered in [16], where the follower agents are required move into the convex hull spanned by the dynamic leaders under the constraints that the velocities and the accelerations of both the leaders and the followers are not available. A finite-time containment control has been built by using a dynamic feedback strategy. In [17], an observer based consensus tracking control has been given for a leader-follower system with noisy measurements via a novel velocity decomposition technique. In [18], a dynamic feedback consensus control has been proposed for a group of autonomous agents with unicycle dynamics and a virtual leader.

For multi-agent systems with an exogenous disturbance system, distributed output regulation based consensus control have been studied in recent years. In [19], the synchronized output regulation problem of linear networked systems has been considered under the scenario where only leaders have the information of the state of the exogenous system. A distributed synchronous protocol has been given for the follower agents to regulate its output by the estimation of the state of exogenous system. An extension has been made to the solvability of the regulator equation in [20] by proposing a new assumption on the state matrix of the exogenous system. References [21] and [22] dealt with the leaderless consensus/synchronization problem and realized that all the outputs or states are equal. The objective does not care and cannot dictate the asymptotic behavior of the output/state of each agent. Reference [21] makes use of the internal model design while [22] employs the feedforward approach. In [23], the parameter uncertainties are allowed in the system matrices associated with the agent dynamics. Then distributed output regulation was proposed via state and output feedback using an internal model principle. Reference [24] further relaxed the no-cycle constraint on the interaction network and simplified the control design and the stability analysis.

1.2 Cooperative filtering for sensor networks

As another important kind of networked systems, sensor networks have broad applications in surveillance and monitoring of an environment, collaborative processing of information, and gathering scientific data from spatially distributed sources for environmental modeling and protection. A fundamental problem in sensor networks is to solve detection and estimation problems, for example, target localization and tracking.

Most of the past research on target tracking has been focused on the use of centralized algorithms that run on static sensor networks[25]. Centralized Kalman filtering plays a crucial role in such target tracking algorithms. In [26], the sensing model of the sensor i was

$$y_i(t) = r(t) + \omega_i(t) \tag{7}$$

where $r(t)$ is the target signal and $\omega_i(t)$ is a zero-mean white Gaussian noise. Then, a consensus filter

$$\dot{r}_i = \sum_{j\in\mathcal{N}_i}(r_j - r_i) + \sum_{j\in\mathcal{M}_i}(y_j - r_i) \tag{8}$$

where $\mathcal{N}_i$ is the index set of the neighbors of sensor i and $\mathcal{M}_i = \mathcal{N}_i \cup \{i\}$, was introduced as a tool for distributed sensor fusion in sensor networks. The consensus filter is a dynamic version of average-consensus algorithm that has been extensively used for sensor fusion. In [9], a solution was proposed to the problem of collision-free tracking of a mobile target via mobile sensor networks using a combination of the flocking and Kalman-consensus filtering algorithms.

However, when a target is tracked by using a group of static or mobile nonlinear sensors such as range and direction sensors, Kalman filter or Kalman-consensus filter will not be valid any longer. For example, when the target is moving with a nonlinear dynamics

$$\dot{r}(t) = f(r,t) + g(t)\xi(t) \tag{9}$$

and the measurement of sensor i is given by

$$y_i(t) = h_i(r) + \theta(t)\omega_i(t) \tag{10}$$

where $\xi(t)$, $\omega_i(t)$ are unrelated zero-mean white Gaussian noise. In order to make an estimation of $r(t)$ for the sensor network, some popular nonlinear filtering approaches have been widely used in the case of Gaussian models, such as the extended Kalman filter (EKF)[27], unscented Kalman filter (UKF)[28], and in the case of non-Gaussian models, such as particle filter[29]. For the nonlinear systems (9) and (10), an EKF filter is given by

$$\dot{\hat{r}}(t) = f(\hat{r},t) + K(t)\big[y(t) - h(\hat{r})\big] \tag{11}$$

where the Kalman gain matrix $K(t) = P(t)C^{\mathrm{T}}(t)R^{-1}(t)$, the error covariance matrix $P(t)$ satisfies

$$\begin{aligned}\dot{P}(t) = A(t)P(t) + P(t)A^{\mathrm{T}}(t) + Q(t) - \\ P(t)C^{\mathrm{T}}(t)R^{-1}(t)C(t)P(t)\end{aligned}$$

and $A(t)$, $C(t)$ are the Jacobian matrices of $f(r,t)$, $h(r)$, respectively. Though it seems difficult to show the convergence of EKF since the uniform complete controllability cannot be assumed generally[27, 30], EKF is still a practical nonlinear filter in engineering applications. Recently, the works[10, 31] considered a nonlinear target tracking problem, where the target is moving with

$$\begin{cases}\dot{x} = Ax + Bu\\ \dot{\zeta} = \Gamma\zeta + \nu(t)\\ u = D\zeta\end{cases} \tag{12}$$

and the measurement function of sensor i is

$$y_i(t) = h_i(p, s_i) + \omega_i(t) \tag{13}$$

Here, $s_i(t)$ denotes the position of sensor i. An observer-based filter (14) was proposed for static or mobile sensors.

$$\begin{cases}\dot{\hat{x}} = Ax + BD\hat{\zeta} + K_1(y - h(\hat{x}, s))\\ \dot{\hat{\zeta}} = \Gamma\hat{\zeta} + K_2(y - h(\hat{x}, s))\end{cases} \tag{14}$$

The nonlinear observers were constructed for sensor information fusion to track a moving target and the convergence of the filter was analyzed as well. However, the nonlinear filtering algorithm proposed in [10] suffers from one weakness: all sensors are awake and active during the whole target tracking process, which may violate limited energy constraints in practice, even could result in information redundancy. In [31], a formation control based sensor deployment strategy was proposed to guarantee the feasibility of the nonlinear observer-based target tracking filter.

2 Intervention of multi-agent systems

As we know, the research on multi-agent systems can be classified into three categories:

1) Analysis: Given the local rule of the agents, what is the collective behavior of the system?

2) Distributed control: Given the desired collective behavior, how do we design the rules of the agents such that the system exhibit the desired behavior?

3) Intervention: Given the desired behavior, how do we control or intervene in the system without destroying the local rule of the system?

Intervention of MAS is a very important issue, since in many practical situations, it is not allowed to change the local rule of the agents, for example, the flying birds, the people in panic, but we need to control the behavior of the whole system, such as, guiding the bird flocks, leading the people in panic to escape from the fire. Then what can we do to intervene in the system such that the desired behavior emerges from the system?

We note that intervention is different from distributed control[6] and pinning control[32]. In distributed control, each agent can be regarded as a control system, and the control law of each agent can be designed based on local information. In pinning control, we need to design the local feedback control law of some (not all) agents selected with some special properties. In fact, the pinning control can be regarded as a special case of distributed control. However, for intervention of MAS, one of the key points is that the local interaction between agents cannot be changed.

In this section, we will introduce two kinds of intervention methods: soft control and adding "information" agents.

2.1 Soft control of multi-agent systems

Soft control, put forward by Han et al.[7], is a novel method to intervene in the collective behavior of MAS. The central idea of soft control is to add one (or some) special agent(s) (called shill) into the original systems to guide the system to the desired behavior, but without changing the local rules of the existing agents. The special agent(s) can be controlled or designed by us, and cannot be identified by the existing agents. The existing agents take them as ordinary agents, so the shill agent(s) will not destroy the local rules of the ordinary agents, but can affect the behavior of other agents in its neighborhood. The property of local interactions between agents makes the influence of the shill spread out, so adding shill agent(s) may control the behavior of the whole system. In the following part, we will introduce a case study[7] to show that it is feasible to intervene in the MAS.

First, we will introduce the multi-agent model to be controlled, which is proposed by Vicsek et al.[33]. This model can be used to investigate the properties of nonequilibrium systems, such as gathering, transport and phase transition, and also has potential applications in biological systems involving clustering and migration.

The Vicsek model is a discrete-time MAS model composed of n autonomous agents, labeled $1, 2, \cdots, n$. Each member moves in the plane with a constant speed v, but with heading updated according to the averaged velocity of neighbors plus the noise effect. Here the neighbors are defined via a circle of radius r, we use $\mathcal{N}_i(t)$ to denote the neighbor set of agent i ($i = 1, 2, \cdots, n$) at discrete time t ($t = 0, 1, 2, \cdots$), that is

$$\mathcal{N}_i(t) = \{j | d_{ij}(t) < r,\ j = 1, 2, \cdots, n\} \tag{15}$$

where $d_{ij}(t) = \|X_i(t) - X_j(t)\|$ with $\|\cdot\|$ being the Euclidean norm, and $X_i(t) \in \mathbf{R}^2$ is the position of the agent i at time t. Each agent moves with a constant speed, so the position is updated according to the following equation:

$$X_i(t+1) = X_i(t) + v(\cos\theta_i(t), \sin\theta_i(t))^{\mathrm{T}},\quad i = 1, 2, \cdots, n,\quad t = 0, 1, 2, \cdots \tag{16}$$

where $\theta_i(t)$ is the heading of the agent i at time t, which is updated according to the following equation:

$$\theta_i(t+1) = \arctan \frac{\sum\limits_{j\in\mathcal{N}_i(t)} \sin\theta_j(t)}{\sum\limits_{j\in\mathcal{N}_i(t)} \cos\theta_j(t)} + \xi_i(t),\ \ i = 1, 2, \cdots, n \tag{17}$$

with $\xi_i(t)$ being the noise. Vicsek et al.[33] show that when the density is large and the noise is small, the system can reach consensus, that is, all agents move with the same heading eventually. This model looks simple, but possesses some key features of MAS, such as local interactions, dynamical neighborhood. Thus, it attracts much attention of researchers in recent years, and many results focus on seeking the consensus conditions, see [34−37] from many others. However, the behavior resulting from self organization may not be what we want. In the following, we will show that soft control is a feasible way to intervene in the system, such that the system exhibits the expected behavior.

In [7], the authors considered the case where one special agent (called shill) is added to intervene in the MAS (15) ∼ (17) without noise effect such that all agents move with the same expected heading π. The position and heading of the shill, denoted as $X_0(t)$ and $\theta_0(t)$, can be controlled. The control law is designed based on the information of all agents:

$$\begin{cases} X_0(t) = x_{S(t)}(t) \\ \theta_0(t) = \begin{cases} \theta_{S(t)} + \beta, & \theta_{S(t)}(t) \le \pi - \beta \\ \pi, & \theta_{S(t)}(t) > \pi - \beta \end{cases} \end{cases} \tag{18}$$

where $\beta \in (0, \pi)$ is a constant, and $S(t) = \arg\min_{1\le i\le n}\{\theta_i(t)\}$ denotes the "worst" agent in terms of the heading error.

According to the above control law of the shill agent, the following result was obtained in [7]:

Theorem 1. For the MAS (15) ∼ (17) without noise effect, for any $r \ge 0$, $v \ge 0$ and any initial configuration $\{X_i(0) \in \mathbf{R}^2, \theta_i(0) \in [0, \pi), i = 1, 2, \cdots, n\}$, the shill agent which obeys the control law (18) can guide all agents to the expected heading π.

Remark 1. In control law (18), the moving direction and the heading of the shill is inconsistent. In [38], Han and Wang provide a new strategy to overcome it. Moreover, in [39], the authors considered the soft control of the MAS (15) ∼ (17) with noise.

Remark 2. The idea of soft control can also be used to intervene in other MAS, for example, in [40], the shill agents added can promote the rate of cooperation in repeated multi-player prisoner′s dilemma game.

2.2 Leader-follower model

It is known that in many biological systems, such as fish schooling and honeybee groups, most agents make navigation decision according to social interaction. However, there are a small number of agents with pertinent information about the destination. For example, they know the location of food, these agents can help to guide the group. The "information" agents are called leaders, and the other ordinary agents without information are called followers. Although the leaders are not assumed to be able to change the local rules of the followers, the local interaction between agents does make the information spread within the group.

Inspired by this, some researchers focus on the intervention of MAS by adding leaders. For example, Couzin et al.[5] investigated how the proportion of leaders affects the behavior in decision-making. They built a MAS model, where each agent obeys the three local interaction rules: repulsion, alignment and attraction. By computer simulations, the authors show that the larger the population size, the smaller the proportion of leaders is needed to guide the group. Also they studied how the system behaves when there are two kinds of leaders with different information.

In [41], by experiments on vast oceanic fish shoals, the authors pointed out that small sets of leaders significantly influence the actions of much larger groups.

To explain the effectiveness of the leaders, some theoretical results arise. For example, Jadbabaie et al.[3] studied the MAS with only one leader, they show that if the neighbor graph formed by leader and all followers are connected in some sense, then the leader can guide all agents to move with the same expected headings. However, how we guarantee the connectivity of the neighbor graph is an unresolved issue. In [42], the authors present a necessary and sufficient condition for the stable convergence to a collective decision of a continuous-time model, where each agent′s heading is updated according to the well-known Kuramoto model for populations of coupled oscillators. Actually, the leader-follower model is widely studied in cooperative control of engineering MAS, see [3, 13, 43−44] among many others.

It is worth mentioning that Liu et al.[44] provide quantitative results for the proportion of leaders needed for the expected consensus. In [45], a simpler model is used, where the follower agents update their position and heading according to (16) and (17), while the leaders update their position and heading according to

$$X_i(t+1) = X_i(t) + v(\cos\bar{\theta}_0, \sin\bar{\theta}_0)^{\mathrm{T}}$$

where $\bar{\theta}_0$ is the expected heading. Here for simplicity, at each time step, the leaders take $\bar{\theta}_0$ as their moving direction.

To analyze it, the authors introduce the following random framework.

Assumption 1. 1) The initial positions of all agents are uniformly and independently distributed in the unit square; 2) The initial headings of the ordinary agents are uniformly and independently distributed in $[-\pi, \pi)$, and the initial positions of all agents and the initial headings of the ordinary agents are independent.

The main result can be stated as follows:

Theorem 2. Let the radius be a positive constant, and the speed satisfy the condition $v \leq \frac{\pi(\min\{1,r\})^2}{512 \cdot 8r}$. Under Assumption 1, if the proportion of leaders α_n satisfies the following condition:

$$\alpha_n \geq C\sqrt[4]{\frac{\log n}{n}}$$

where C is positive constant independent of n, then the leaders can guide the system to the expected heading $\bar{\theta}_0$ eventually when the population size n is large enough.

Many problems deserve to be further investigated, for example, what is the necessary condition for the proportion of leaders? If there are two classes of leaders with different information, what will the results be? For the model proposed in [5], how do we analyze it?

3 Multi-agent systems with nonlinear interaction

Multi-agent systems consist of a number of interacting agents, where the specific pattern of interaction is represented by a network. The interaction among agents can be governed by linear local rules, which is so-called linear multi-agent systems. At present, there is a large amount of literatures considering linear multi-agent systems from first order to second order as well as higher order systems. Refer to [46−49] and the references therein, to name a few. But in reality, the interaction among agents may be nonlinear. Typical examples are the dynamics of complex network. On the other hand, the mutual interaction between agents′ state and networks′ topology can also lead to the nonlinearity. In this section, we will review the research progress on multi-agent systems with nonlinear interaction from two aspects: general nonlinear multi-agent systems and those with state-topology co-evolution.

3.1 General nonlinear multi-agent systems

In real systems, nonlinear intrinsic dynamics or nonlinear interaction among agents are inevitable. Examples are systems of coupled oscillators. On nonlinear multi-agent systems, a pioneering work is [50], in which Moreau considered a general nonlinear discrete system

$$x_i(t+1) = f_i(t, x_1(t), \cdots, x_n(t)), \quad i = 1, 2, \cdots, n \quad (19)$$

where $x_i \in X$, X is a Euclidean space of arbitrary finite dimension and the map $f_i : \mathbf{N} \times X^n \to X$ is continuous. This general model can include many special systems such as synchronization of coupled oscillators[51], swarming of Vicsek model[33], consensus of linear and nonlinear multi-agent systems, and so on.

To reach certain consensus defined in [50], each f_i is assumed to satisfy a strict convexity condition, which means that for each agent i, the updated state $x_i(t+1)$ is a strict convex combination of the current states of agent i and its neighbors, that is,

$$x_i(t+1) = \mathrm{conv}\{x_i(t), x_j(t),\ j \in \mathcal{N}_i(t)\}$$

hold for any $i = 1, 2, \cdots, n$ and $t \geq 0$. Under this convexity assumption, several necessary and/or sufficient conditions on the communication topology guaranteeing the convergence of the individual agents′ states to a common value were presented using graph theory and the set-value Lyapunov method.

Then, Lin et al. generalized the work of Moreau to nonlinear continuous multi-agent systems. Using nonsmooth analysis, they proved that if the vector fields satisfy a certain subtangentiality condition, asymptotic state agreement is achieved if and only if the dynamic interaction digraph being sufficiently connected over time[52].

As we know, the key condition in [50] is the convexity of the nonlinear functions f_i. This condition was improved by Angeli and Bliman in [53] as follows: each agent moves towards the relative interior of a set of the present and past states of neighbor agents, which is not necessarily a convex hull. Arbitrary bounded time delays in the communication channels were also considered.

Recently, [54] further investigated consensus of a discrete-time MAS with nonlinear transmission and time-varying delays, which relaxed the commonly assumed convexity condition and hence generalized several well known results. The evolution of agent i complies with

$$x_i(t+1) = \sum_{j=1}^{n} a_{ij}(t) f_{ij}(x_j(t - \tau_j^i(t))), \quad i = 1, 2, \cdots, n \quad (20)$$

where $x_i \in \mathbf{R}^m$, $a_{ij}(t)$ are the weights in the graph $G(t)$ at time t. Here, the term $f_{ij}(x_j(t - \tau_j^i(t)))$ represents the nonlinearity and time delays in the transmission of information from agent j to agent i. Instead of the convexity condition in [50], this paper assumes that all $f_{ij} \in \mathcal{F}$ and share two common sets $\mathcal{B}$ and $\mathcal{U}$, where the set $\mathcal{F}$ is defined as follows.

A function f belongs to $\mathcal{F}$ if the following conditions are satisfied:

1) $f : \mathbf{R}^m \to \mathbf{R}^m$ is continuous.

2) There exists a nonempty compact convex set $\mathcal{B} \subset \mathbf{R}^m$ and a nonempty bounded convex set $\mathcal{U} \subset \mathcal{B}$ such that: a) $f(x) \in \mathcal{B}$ for all $x \in \mathcal{B}$; b) $f(x) = x$ for all $x \in \mathcal{U}$, and $d(f(x),\mathcal{U}) < d(x,\mathcal{U})$ for all $x \in \mathcal{B}$ such that $f(x) \neq x$.

For example, let $f(x) = x^3$ with $\mathcal{B} = [-1, 1]$ and $\mathcal{U} = \{0\}$. Then, it is obvious that $f \in \mathcal{F}$.

Under the assumption about f_{ij} above and the bounded communication delays, the consensus results of system (20) were given when the topology graph $G(\infty) = \lim_{k\to\infty} \cup_{t\geq k} G(t)$ is connected and $\{G(t)\}_{t\geq 0}$ is jointly connected, respectively.

Some other stabilization problems on nonlinear multi-agent systems can be referred to [55−58] and the references therein.

3.2 Multi-agent systems with state-topology coevolution

Multi-agent systems with mutual interaction between agents′ state and networks′ topology are called multi-agent systems with state-topology coevolution. In the field of complex network, such kinds of systems are also known as coevolutionary or adaptive network. A number of papers have recently appeared on the modeling of coevolutinary networks, such as coevolution of behavior and structure in Web[59], influence of behavior on the spread of diseases[60], co-emergence of cooperation and hierarchical structure in games[61], evolution of opinion formation on adaptive network[62], see [63−64] and http://adaptive-networks.wikidot.com/publications for more references. In this paper, we will focus on the theoretical study on the multi-agent systems with state-topology coevolution.

A classical flocking model with state-topology coevolution is the so-called Vicsek model[65], as introduced in Subsection 2.1. The communication networks are determined by the positions of the agents, thus are state-dependent and dynamic. A large number of theoretical results have shown that the connectivity of the dynamic networks is crucial for the consensus[3, 66−67]. For the Vicsek model, once given the initial conditions and the system parameters, the dynamical process of the network topology will be driven by the evolution of the agents′ states. Therefore, what kinds of initial conditions and system parameters can lead to consensus turns out to be a difficult and challenging issue. Toward this issue, Liu and Guo presented a preliminary theoretical analysis, and developed a sufficient condition for the synchronization that the speed v should be sufficiently slow with $v \leq \frac{d}{\triangle_0}(\frac{\cos\bar{\theta}}{n})^n$, where n is the size of population and d, $\triangle_0$, $\bar{\theta}$ are determined by the initial states[35]. This result implies that the speed v should decrease exponentially as the population increases, which is restrictive for large population. In [68], the condition on the speed v is improved from $\mathrm{O}(n^{-n})$ to $\mathrm{O}(n^{-\beta})$, where β is a constant independent of n. Tang and Guo made a major advance by putting the model in a random framework, and under uniformly distributed initial conditions, they proved that for any given speed v and communication radius r the linearized Vicsek model will synchronize with large probability as long as the size of the population is large enough[69]. Liu and Guo obtained a similar condition for synchronization of the original Vicsek model in [36]. Furthermore, in [37], Chen et al. investigated the smallest possible radius for synchronization of the linearized Vicsek model, and proved that, in a certain sense, it approximately equals the critical radius for connectivity of random geometric graphs, i.e. $\log n/(\pi n)$.

The interaction in the Vicsek model is defined based on the metric distance between agents, that each agent interacts with all agents within a fixed metric distance. By means of empirical observations and simulations, Ballerini et al. have shown that this kind of metric interaction is less efficient to maintain cohesion comparing with topological interaction[70]. Here, topological interaction is qualified by how many intermediate individuals separate two agents, not how far apart they are. In the topological case, the strength of the interaction could remain the same at different densities. Since the topology interaction is not symmetrical, the neighbor graph is direct. Wang et al. presented a preliminary theoretical study for the synchronization of such topological interaction, and developed a sufficient condition which showed the relationship between the speed, the heading and the density of the group[71]. In a random framework, Chen et al. proved that if the number of the topological neighbors is proportional to the population size, then for any speed, the system can synchronize with large probability[72].

There are also some other kinds of interaction modes. In [34], Cucker and Smale made use of global interactions with weights decaying according to the distances among agents. They proved that when the decay rate is less than 1/2, convergence of the flock to a common velocity is guaranteed, while some conditions on the initial positions and velocities should be added for the case that the decay rate is larger than 1/2. A number of local potential functions are developed to generate attractive interactions to guarantee connectivity, such as the Laplacian matrix based function[73], the edge-tension functions[74−78], and the navigation functions[79−82]. Most of the potential functions in the above mentioned results are unbounded, except those in [79−80] for single-integrator agents, and those in [78, 81] for double-integrator agents, and that in [82] for unicycles.

4 Conclusions

In this paper, we have reviewed some new research and development in multi-agent systems from the viewpoint of control theory. Both theoretical results and experiments are reviewed for the following main issues: distributed estimation and cooperative filtering, soft control and informed agent intervention, and nonlinear interaction dynamics, in multi-agent systems. There are some other interesting yet challenging problems not covered in this paper, such as constrained communication, optimal consensus control, competition and cooperation, and so on.

References

1 Fax J A, Murray R M. Information flow and cooperative control of vehicle formations. *IEEE Transactions on Automatic Control*, 2004, **49**(9): 1465−1476

2 Olfati-Saber R, Fax J A, Murray R M. Consensus and cooperation in networked multi-agent systems. *Proceedings of IEEE*, 2007, **95**(1): 215−233

3 Jadbabaie A, Lin J, Morse A S. Coordination of groups of mobile autonomous agents using nearest neighbor rules. *IEEE Transactions on Automatic Control*, 2003, **48**(6): 988−1001

4 Buhl J, Sumpter D J T, Couzin D, Hale J J, Despland E, Miller E R, Simpson S J. From disorder to order in marching locusts. *Science*, 2006, **312**(5778): 1402−1406

5 Couzin I D, Krause J, Franks N R, Levin S A. Effective leadership and decision-making in animal groups on the move. *Nature*, 2005, **433**(7025): 513−516

6 Olfati-Saber R. Flocking for multi-agent dynamic systems: algorithms and theory. *IEEE Transactions on Automatic Control*, 2006, **51**(3): 401−420

7 Han J, Li M, Guo L. Soft control on collective behavior of a group of autonomous agents by a shill agent. *Journal of Systems Science and Complexity*, 2006, **19**(1): 54−62

8 Yang P, Freeman R, Lynch K. Multi-agent coordination by decentralized estimation and control. *IEEE Transactions on Automatic Control*, 2008, **53**(11): 2480−2496

9 Olfati-Saber R, Jalalkamali P. Coupled distributed estimation and control for mobile sensor networks. *IEEE Transactions on Automatic Control*, 2012, **57**(10): 2609−2614

10 Hu J P, Hu X M. Optimal target trajectory estimation and filtering using networked sensors. *Journal of Systems Science and Complexity*, 2008, **21**(3): 325−336

11 Helbing D, Farkas I, Vicsek T. Simulating dynamical features of escape panic. *Nature*, 2000, **407**(6803): 487−490

12 Moussard M, Helbing D, Theraulaz G. How simple rules determine pedestrian behavior and crowd disasters. *Proceedings of the National Academy of the Sciences of the United States of America*, 2011, **108**(17): 6884−6888

13 Hong Y G, Hu J P, Gao L X. Tracking control for multi-agent consensus with an active leader and variable topology. *Automatica*, 2006, **42**(7): 1177−1182

14 Hong Y G, Chen G R, Bushnell L. Distributed observers design for leader-following control of multi-agent networks. *Automatica*, 2008, **44**(3): 846−850

15 Ren W. On consensus algorithms for double-integrator dynamics. *IEEE Transactions on Automatic Control*, 2008, **53**(6): 1503−1509

16 Li J Z, Ren W, Xu S Y. Distributed containment control with multiple dynamic leaders for double-integrator dynamics using only position measurements. *IEEE Transactions on Automatic Control*, 2012, **57**(6): 1553−1559

17 Hu J P, Feng G. Distributed tracking control of leader-follower multi-agent systems under noisy measurement. *Automatica*, 2010, **46**(8): 1382−1387

18 Gustavi T, Hu X. Observer-based leader-following formation control using onboard sensor information. *IEEE Transactions on Robotics*, 2008, **24**(6): 1457−1462

19 Xiang J, Wei W, Li Y J. Synchronized output regulation of linear networked systems. *IEEE Transactions on Automatic Control*, 2009, **54**(6): 1336−1341

20 Huang J. Remarks on synchronized output regulation of linear networked systems. *IEEE Transactions on Automatic Control*, 2011, **56**(3): 630−631

21 Kim H, Shim H, Seo J H. Output consensus of heterogeneous uncertain linear multi-agent systems. *IEEE Transactions on Automatic Control*, 2011, **56**(1): 200−206

22 Wieland P, Sepulchre R, Allgower F. An internal model principle is necessary and sufficient for linear output synchronization. *Automatica*, 2011, **47**(5): 1068−1074

23 Wang X L, Hong Y G, Huang J, Jiang Z P. A distributed control approach to a robust output regulation problem for multi-agent linear systems. *IEEE Transactions on Automatic Control*, 2010, **45**: 2891−2895

24 Su Y, Hong Y, Huang J. A general result on the robust cooperative output regulation for linear uncertain multi-agent systems. *IEEE Transactions on Automatic Control*, 2013, **58**(5): 1275−1279

25 Bar-Shalom Y, Li X R. *Multitarget-Multisensor Tracking: Principles and Techniques*. Storrs, CT: YBS Publishing, 1995

26 Olfati-Saber R, Shamma J S. Consensus filters for sensor networks and distributed sensor fusion. *Proceedings of the 44th IEEE Conference on Decision and Control and 2005 European Control Conference. Seville, Spain*, 2005, 6698−6703

27 Jazwinski A H. *Stochastic Processes and Filtering Theory*. Mineola NY: Dover Publications, 2007

28 Julier S J, Uhlmann J K, Durrant-Whyte H F. A new approach for filtering nonlinear system. In: Proceedings of the 1995 American Control Conference. Washington, DC: IEEE, 1995: 1628−1632

29 Doucet A, Freitas N D, Gordon N J. *Sequential Monte Carlo Methods in Practice*. New York: Springer, 2001

30 Reif K, Gunther S, Yaz E, Unbehauen R. Stochastic stability of the continuous-time extended Kalman filter. *IEE Proceedings Control Theory and Applications*, 2000, **147**(1): 45−52

31 Hu J P, Hu X M. Nonlinear filtering in target tracking using cooperative mobile sensors. *Automatica*, 2010, **46**(12): 2041−2046

32 Wang X F, Chen G R. Pinning control of scale-free dynamical networks. *Physica A*, 20002, **310**(3−4): 521−531

33 Vicsek T, Czirók A, Ben-Jacob E, Cohen I, Sochet O. Novel type of phase transition in a system of self-driven particles. *Physical Review Letters*, 1995, **75**(6): 1226−1229

34 Cucker F, Smale S. Emergent behavior in flocks. *IEEE Transactions on Automatic Control*, 2007, **52**(5): 852−862

35 Liu Z X, Guo L. Connectivity and synchronization of vicsek model. *Science in China: Series F*, 2008, **51**(7): 848−858

36 Liu Z X, Guo L. Synchronization of multi-agent systems without connectivity assumptions. *Automatica*, 2009, **45**(12): 2744−2753

37 Chen G, Liu Z X, Guo L. The smallest possible interaction radius for flock synchronization. *SIAM Journal of Control and Optimization*, 2012, **50**(4): 1950−1970

38 Han J, Wang L. Nondestructive intervention to multi-agent systems through an intelligent agent. *PLoS ONE*, 2013, **8**(5): e61542. DOI: 10.1371/journal.pone.0061542

39 Wang L, Guo L. Robust consensus and soft control of multi-agent systems with noises. *Journal of Systems Science and Complexity*, 2008, **21**(3): 406−415

40 Wang X, Han J, Han H. Special agents can promote cooperation in the population. *PLoS ONE*, 2011, **6**(12): e29182. DOI: 10.1371/journal.pone.0029182

41 Makris N, Ratilal P, Jagannathan S, Gong Z, Andrews M, Bertsatos I. Critical population density triggers rapid formation of vast oceanic fish shoals. *Science*, 2009, **323**(5922): 1734−1737

42 Leonard N E, Shen T, Nabet B, Scardovi L, Couzinc I D, Levin S A. Decision versus compromise for animal groups in motion. *Proceedings of the National Academy of the Sciences of the United States of America*, 2012, **109**(1): 227−232

43 Wang P K C, Hadaegh F Y. Coordination and control of multiple microspacecraft moving in formation. *The Journal of the Astronautical Sciences*, 1996, **44**(3): 315−355

44 Kapila V, Sparks A G, Buffington J M, Yan Q G. Spacecraft formation flying: dynamics and control. *Journal of Guidance, Control, Dynamics*, 2000, **23**(3): 561−564

45 Liu Z X, Han J, Hu X M. The proportion of leaders needed for the expected consensus. *Automatica*, 2011, **47**(12): 2697−2703

46 Olfati-Saber R, Murray R M. Consensus problems in networks of agents with switching topology and time-delays. *IEEE Transactions Automatic Control*, 2004, **49**(9): 1520−1533

47 Cheng D Z, Wang J H, Hu X M. An extension of LaSalle′s invariance principle and its application to multi-agent consensus. *IEEE Transactions Automatic Control*, 2008, **52**(7): 1765−1770

48 Li Z, Duan Z, Chen G, Huang L. Consensus of multi-agent systems and synchronization of complex networks: a unified viewpoint. *IEEE Transactions Circuits and Systems-I Regular Papers*, 2010, **57**(1): 213−224

49 Tang Y T, Hong Y G. Hierarchical distributed control design for multi-agent systems using approximate simulation. *Acta Automatica Sinica*, 2013, **39**(6): 868−874

50 Moreau L. Stability of multiagent systems with time-dependent communication links. *IEEE Transactions Automatic Control*, 2005, **50**(2): 169−182

51 Jadbabaie A, Motee N, Barahona M. On the stability of the Kuramoto model of coupled nonlinear oscillators. *Proceedings of the 2004 American Control Conference. Boston, MA*, 2004, 0743−1619

52 Lin Z Y, Francis B, Maggiore M. State agreement for continuous-time coupled nonlinear systems. *SIAM Journal on Control and Optimization*, 2007, **46**(1): 288−307

53 Angeli D, Bliman P A. Stability of leaderless discrete-time multi-agent systems. *Mathematics of Control, Signals, and Systems*, 2006, **18**(4): 293−322

54 Chen Y, Lv J H, Lin Z L. Consensus of discrete-time multi-agent systems with transmission nonlinearity. *Automatica*, 2013, **49**(6): 1768−1775

55 Lorenz J, Lorenz D A. On conditions for convergence to consensus. *IEEE Transactions on Automatic Control*, 2010, **55**(7): 1651−1656

56 Hui Q, Haddad W M. Distributed nonlinear control algorithms for network consensus. *Automatica*, 2008, **44**(9): 2375−2381

57 Li H, Liao X F, Lei X Y, Huang T W, Zhu W. Second-order consensus seeking in multi-agent systems with nonlinear dynamics over random switching directed networks. *IEEE Transactions on Circuits and Systems — I: Regular Papers*, 2013, **60**(6): 1595−1607

58 Yang H, Jiang B, Cocquempot V, Zhan H G. Stabilization of switched nonlinear systems with all unstable modes: application to multi-agent systems. *IEEE Transactions Automatic Control*, 2011, **56**(9): 2230−2235

59 McCabe C, Watson R, Prichard J S, Hall W. The web as an adaptive network: coevolution of web behavior and web structure. In *Web Science Conference 2011, Koblenz German*, 2011, 1−7.

60 Funk S, Salathe M, Jansen V A A. Modelling the influence of human behavior on the spread of infectious diseases: a review. *Journal of the Royal Society Interface*, 2010, **7**(50): 1247−1256

61 Lee S M, Holme P, Wu Z X. Emergent hierarchical structures in multiadaptive games. *Physical Review Letters*, 2011, **106**(2): 028702

62 Schmittmann B, Mukhopadhyay A. Opinion formation on adaptive networks with intensive average degree. *Physical Review E*, 2010, **82**(6): 066104

63 Gross T, Blasius B. Adaptive coevolutionary networks: a review. *Journal of the Royal Society: Interface*, 2008, **5**(20): 259−271

64 Alberts B. Special issue: complex systems and networks. *Science*, 2009, **325**(5939): 405−432

65 Vicsek T, Zafiris A. Collective motion. *Physics Reports*, 2012, **517** (3−4), 71−140

66 Ren W, Cao Y C. *Distributed Coordination of Multi-Agent Networks: Emergent Problems, Models, and Issues.* New York: Springer, 2011

67 Yan W S, Li J B, Wang Y T. Consensus for damaged multi-agent system. *Acta Automatica Sinica*, 2012, **38**(11): 1127−1133

68 Wang L, Wang X F. New conditions for synchronization in dynamical communication networks. *Systems and Control Letters*, 2011, **60**(4): 219−225

69 Tang G G, Guo L. Convergence of a class of multi-agent systems in probabilistic framework. *Journal of Systems Science and Complexity*, 2007, **20**(2): 173−197

70 Ballerini M, Cabibbo N, Candelier R, Cavagna A, Cisbani E, Giardina I. Interaction ruling animal collective behavior depends on topological rather than metric distance: evidence from a field study. *Proceedings of the National Academy of the Sciences of the United States of America* , 2008, **105**(4): 1232−1237

71 Wang L, Wang X F, Hu X M. Synchronization of multi-agent systems with topological interaction. *Proceedings of the 18th IFAC World Congress (IFAC), Milan: IFAC*, 2011, 14642−14647

72 Chen C, Chen G, Guo L. Synchronization of mobile autonomous agents with M-nearest-neighbor rule. *Proceedings of the 31st Chinese Control Conference. Hefei* 2102. 1147−1152

73 Zavlanos M M, Pappas G J. Potential fields for maintaining connectivity of mobile networks. *IEEE Transactions on Robotics*, 2007, **23**(4): 812−816

74 Ji M, Egerstedt M. Distributed coordination control of multi-agent systems while preserving connectedness. *IEEE Transactions on Robotics*, 2007, **23**(4): 693−703

75 Dimarogonas D V, Kyriakopoulos K J. Connectedness preserving distributed swarm aggregation for multiple kinematic robots. *IEEE Transactions Robotics*, 2008, **24**(5): 1213−1223

76 Su H S, Wang X F, Chen G R. A connectivity-preserving flocking algorithm for multi-agent systems based only on position measurements. *International Journal of Control*, 2009, **82**(7): 1334−1343

77 Zavlanos M M, Tanner H G, Jadbabaie A, Pappas G J. Hybrid control for connectivity preserving flocking. *IEEE Transactions on Automatic Control*, 2009, **54**(12): 2869−2875

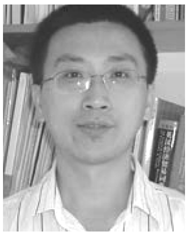
78 Su H S, Wang X F, Chen G R. Rendezvous of multiple mobile agents with preserved network connectivity. *Systems and Control Letters*, 2010, **59**(5): 313−322

79 Ajorlou A, Momeni A, Aghdam A G. A class of bounded distributed control strategies for connectivity preservation in multi-agent systems. *IEEE Transactions on Automatic Control*, 2010, **55**(12): 2828−2833

80 Dimarogonas D V, Johansson K H. Bounded control of network connectivity in multi-agent systems. *IET Control Theory and Applications*, 2010, **4**(8): 1330−1338

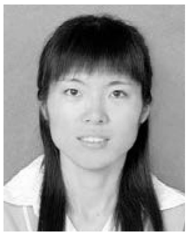
81 Wang L, Wang X F, Hu X M. Connectivity preserving flocking without velocity measurement. *Asian Journal of Control*, 2013, **15**(2): 521−532

82 Ajorlou A, Aghdam A G. Connectivity preservation in nonholonomic multi-agent systems: a bounded distributed control strategy. *IEEE Transactions on Automatic Control*, 2013, **58**(9): 2366−2371



HU Jiang-Ping Received his bachelor degree in applied mathematics and master degree in computational mathematics from Lanzhou University, China, in 2000 and 2004, respectively. He received his Ph. D. degree in complex system modelling and control from the Academy of Mathematics and Systems Science (AMSS), Chinese Academy of Sciences (CAS), China, in 2007. From September 2007 to August 2008, he was a postdoctor in the Department of Mathematics, Royal Institute of Technology (KTH), Sweden. Since 2008, he has been an associate professor at School of Automation Engineering, University of the Electronic Science and Technology of China (UESTC), China. His research interest covers multi-agent systems, sensor networks, social dynamics, and air traffic control. Corresponding author of this paper. E-mail: hjp_lzu@163.com

LIU Zhi-Xin Received her bachelor degree in mathematics from Shandong University, China, in 2002, and Ph. D. degree in control theory from AMSS, CAS, China, in 2007. She is an associate professor at AMSS, CAS, China. From February 2009 to April 2009, she was with Royal Institute of Technology, Sweden, as a visiting researcher. Her research interest covers complex systems and multi-agent systems.
E-mail: lzx@amss.ac.cn



WANG Jin-Huan Received her bachelor and master degrees in the School of Mathematical Sciences, Shandong Normal University in 2002 and 2005, respectively, and Ph. D. degree from AMSS, CAS in 2008. She is currently a lecturer of the School of Sciences, Hebei University of Technology. Her research interest covers complex systems control and switched systems. E-mail: wjhuan228@163.com

WANG Lin Received her bachelor and master degrees from the School of Mathematical Sciences, Shandong Normal University in 2003 and 2006, respectively. She received her Ph. D. degree in operations research and control theory from AMSS, CAS, in 2009. She is now an associate professor with the Department of Automation, Shanghai Jiao Tong University. Her research interest covers analysis and control of multi-agent systems, adaptive complex networks, and coordination of multiple manipulators.
E-mail: wanglin@sjtu.edu.cn

HU Xiao-Ming Received his bachelor degree from University of Science and Technology of China, China, in 1983, and his master and Ph. D. degrees from Arizona State University, USA, in 1986 and 1989, respectively. He served as a research assistant at the Institute of Automation, CAS, China, from 1983 to 1984. From 1989 to 1990 he was a Gustafsson postdoctoral fellow at the Royal Institute of Technology, Sweden, where he is currently a professor of optimization and systems theory. His research interest covers nonlinear control systems, nonlinear observer design, sensing and active perception, motion planning, control of multi-agent systems, and mobile manipulation. E-mail: hu@kth.se