Delay-dependent Stability for Systems with Fast-varying Neutral-type Delays via a PTVD Compensation

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Abstract The stability for a class of linear neutral systems with time-varying delays is studied in this paper, where delay in neutral-type term includes a fast-varying case (i.e., the derivative of delay is more than one), which has never been considered in current literature. The less conservative delay-dependent stability criteria for this system are proposed by applying new Lyapunov-Krasovskii functional and novel polynomials with time-varying delay (PTVD) compensation technique. The aim to deal with systems with fast-varying neutral-type delay can be achieved by using the new functional. The benefit brought by applying the PTVD compensation technique is that some useful elements can be included in criteria, which are generally ignored when estimating the upper bound of derivative of Lyapunov-Krasovskii functional. A numerical example is provided to verify the effectiveness of the proposed results.

Key words Linear neutral systems, stability, delay-dependent, fast-varying neutral-type delay, polynomials with time-varying delay (PTVD) compensation technique

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There are two kinds of discrete time delays in systems: retarded-type delay and neutral-type delay. The retarded-type delay means that the delay is in the states of systems, whereas the neutral-type delay means that the delay is in the derivatives of states of systems. In recent years, the neutral systems with delays (i.e., systems with retarded-type delays and neutral-type delays) have received much attention, and can be found in many fields, such as population ecology[1], distributed networks containing lossless transmission lines[2], propagation and diffusion models[3], and partial element equivalent circuits in very large scale integration (VLSI) systems[4]. Thus, the stability of linear neutral systems with delay has developed into a hot topic both in theory and in practice[5]. At present, the stability results for linear neutral systems with delays can be generally classified into two types: delay-independent case which can be applied to delay with arbitrary size, and delay-dependent case which makes use of the size of delay. Generally speaking, the delay-dependent case is less conservative than the delay-independent one. Therefore, researches on delay-dependent stability for linear neutral systems with delays has been extensively carried out. For example, [6−7] proposed a descriptor system approach to deal with linear neutral systems with delays. In [8−9], the Lyapunov-Krasovskii functional with term $\boldsymbol{x}(t)-C\boldsymbol{x}(t-\tau)$ was employed, where $\boldsymbol{x}(t)$ was the system state, C was constant matrix with $\|C\|<1$, and τ was delay. Then, a free-weighting matrix approach[10−11] was proposed to deal with linear delay systems.

Recently, some new techniques have been used in stability analysis of systems with delay. Those results were included in [12−23]. In [12−13], the methods based on characteristic function (or transfer function) were used to deal with linear neutral systems with constant delays. In [14−15], the authors considered the term $-\int_{t-d_M}^{t-d(t)}\dot{\boldsymbol{x}}^{\rm T}(s)Z\dot{\boldsymbol{x}}(s){\rm d}s$ in Lyapunov functional, which was usually neglected in previous studies, where $d(t)$ denoted time-varying delay and d_M denoted the upper bound of $d(t)$, i.e., $d(t)\in[0,\ d_M]$. The augmented Lyapunov-Krasovskii functional was employed to reduce the conservativeness of stability results in [14, 16−18]. In [19−20], the robust stability for the neutral systems with delays and nonlinear perturbations were studied. And then, neutral systems with distributed delays and interval delays were found in [21−22]. In [23], the absolute stability of neutral systems was studied. To the best of our knowledge, there is no stability criterion that can deal with systems with fast-varying neutral-type delays, i.e., the derivative of neutral-type delay is more than one. How to obtain the stability results dealing with fast-varying neutral-type delay and reduce their conservativeness that has motivated the present study.

In this paper, the stability of linear neutral system with time-varying retarded-type delays and time-varying neutral-type delays (including the fast-varying neutral-type delay) is studied. By employing a new Lyapunov-Krasovskii functional and novel polynomials with time-varying delay (PTVD) compensation technique, the less conservative stability criteria are obtained. Compared with previous results, it is the first time to consider the fast-varying neutral-type delay in neutral system with delay, which is achieved by the new functional. Since the PTVD compensation technique is used, some useful terms can be introduced by using some polynomials with time-varying delays to system, which are usually ignored at the process estimating the upper bound of derivative of Lyapunov-Krasovskii functional. Obviously, the criteria are less conservative by applying this novel technique. A numerical example shows that our results are effective and less conservative than other reports in previous literature.

In the following, $D=[d_{ij}]_{n\times n}$ denotes an $n\times n$ real matrix. $D^{\rm T}$ and $\|D\|$ represent the transpose and norm of matrix, respectively, where $\|\cdot\|$ is the Euclidean norm. $D>0$ ($D<0$) denotes that D is a positive (negative) definite matrix. $D\geq 0$ ($D\leq 0$) denotes that D is a positive (negative) semidefinite matrix. I denotes the identity matrix with an appropriate dimension.

1 System description and preliminaries

Consider the following linear neutral system with time-varying delays:

$$\begin{aligned}&\dot{\boldsymbol{x}}(t)=A\boldsymbol{x}(t)+B\boldsymbol{x}(t-d(t))+C\dot{\boldsymbol{x}}(t-\tau(t)), \quad t\geq 0\\&\boldsymbol{x}(t)=\boldsymbol{\phi}(t), \quad \forall t\in[-\max(d_M,\tau_M),0]\end{aligned} \tag{1}$$

where $\boldsymbol{x}(\cdot)=[x_1(\cdot)\ \ x_2(\cdot)\ \ \cdots\ \ x_n(\cdot)]^{\rm T}$ is the state vector of system, A, B, and C are constant matrices, and $\|C\|<1$. The initial condition $\boldsymbol{\phi}(t)$ is a continuous and differentiable vector-valued function of $t\in[-\max(d_M,\tau_M),\ 0]$. The time-delays $d(t)$ and $\tau(t)$ are two irrelevant differentiable functions that satisfy

$$0\leq d(t)\leq d_M, \quad \dot{d}(t)\leq\mu \tag{2}$$

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and

$$0 \leq \tau(t) \leq \tau_M, \quad \eta_1 \leq \dot{\tau}(t) \leq \eta_2 \tag{3}$$

where $d_M > 0$ and $\tau_M > 0$. For parameters η_1 and η_2 in (3), only two cases are considered in this paper, which are 1) $\eta_1 \leq \eta_2 < 1$ (slowly varying delay, i.e., $\dot{\tau}(t) < 1$) and 2) $1 < \eta_1 \leq \eta_2$ (fast varying delay, i.e., $\dot{\tau}(t) > 1$). Especially, for case 2), it is the first time to discuss the systems with neutral-type time-varying delays.

The following lemmas will be used to prove the results of this paper.

Lemma 1 (Jensen's inequality[24]). For any constant matrix $\Omega > 0$, vector function $\boldsymbol{\chi}(t)$ with appropriate dimensions, and function $\sigma(t) \in \mathbf{R}$ satisfies $0 < \sigma(t) \leq \delta$, we have

$$\left[\int_{t-\sigma(t)}^{t} \boldsymbol{\chi}(s)\mathrm{d}s\right]^{\mathrm{T}} \Omega \left[\int_{t-\sigma(t)}^{t} \boldsymbol{\chi}(s)\mathrm{d}s\right] \leq \sigma(t)\int_{t-\sigma(t)}^{t} \boldsymbol{\chi}^{\mathrm{T}}(s)\Omega\boldsymbol{\chi}(s)\mathrm{d}s$$

Lemma 2. The following inequalities

$$\begin{cases}\Delta + \beta X_1 < 0\\ \Delta + \beta X_2 < 0\end{cases} \tag{4}$$

are equivalent to the following condition

$$\Delta + zX_1 + (\beta - z)X_2 < 0 \tag{5}$$

where X_1, X_2, and Δ are constant matrices with appropriate dimensions, variable $z \in [0,\ \beta] \in \mathbf{R}$, and $\beta > 0$.

Proof. See Appendix. □

Remark 1. Lemma 2 is proposed based on the idea of convex combination[25]. Since the proof was not given in [25], the detailed proof is provided in this paper. Some similar results were employed in [26−28].

2 Main results

In this section, the new stability criteria will be proposed to deal with linear neutral systems with time-varying delay. An augmented Lyapunov-Krasovskii functional and PTVD compensation technique will be used in the proposed criteria. First, the case of slow-varying neutral-type delay will be considered, i.e., $\eta_1 \leq \eta_2 < 1$.

Theorem 1. System (1) with time-varying delays $d(t)$ and $\tau(t)$ satisfying (2) and (3) is asymptotically stable, for the given scalar parameters d_M, μ, τ_M, and $\eta_1 \leq \eta_2 < 1$, if there exist some matrices:

$$P = P^{\mathrm{T}} = \begin{bmatrix} P_{11} & P_{12} & P_{13}\\ P_{12}^{\mathrm{T}} & P_{22} & P_{23}\\ P_{13}^{\mathrm{T}} & P_{23}^{\mathrm{T}} & P_{33}\end{bmatrix} > 0$$

$$Q_1 = Q_1^{\mathrm{T}} > 0, Q_2 = Q_2^{\mathrm{T}} > 0, R_1 = R_1^{\mathrm{T}} > 0, R_2 = R_2^{\mathrm{T}} > 0$$

$$Y_1 = Y_1^{\mathrm{T}} > 0, Y_2 = Y_2^{\mathrm{T}} > 0, Z_1 = Z_1^{\mathrm{T}} > 0, Z_2 = Z_2^{\mathrm{T}} > 0$$

$$S_1 = S_1^{\mathrm{T}} > 0, S_2 = S_2^{\mathrm{T}} > 0, S_3 = S_3^{\mathrm{T}} > 0$$

such that the following matrix inequalities hold:

$$\Phi + \bar{A}^{\mathrm{T}}(Y_1 + d_M^2 Z_1 + \tau_M^2 Z_2)\bar{A} - e_1^{\mathrm{T}} Z_1 e_1 - e_3^{\mathrm{T}} Z_2 e_3 < 0 \tag{6}$$

$$\Phi + \bar{A}^{\mathrm{T}}(Y_1 + d_M^2 Z_1 + \tau_M^2 Z_2)\bar{A} - e_1^{\mathrm{T}} Z_1 e_1 - e_4^{\mathrm{T}} Z_2 e_4 < 0 \tag{7}$$

$$\Phi + \bar{A}^{\mathrm{T}}(Y_1 + d_M^2 Z_1 + \tau_M^2 Z_2)\bar{A} - e_2^{\mathrm{T}} Z_1 e_2 - e_3^{\mathrm{T}} Z_2 e_3 < 0 \tag{8}$$

$$\Phi + \bar{A}^{\mathrm{T}}(Y_1 + d_M^2 Z_1 + \tau_M^2 Z_2)\bar{A} - e_2^{\mathrm{T}} Z_1 e_2 - e_4^{\mathrm{T}} Z_2 e_4 < 0 \tag{9}$$

where Φ is shown at the bottom of this page,

$$\begin{aligned}
\Phi_1 =& P_{11}A + A^{\mathrm{T}}P_{11} + Q_1 + Q_2 + R_1 - Z_1 - Z_2 + \eta_2 S_1\\
\Phi_2 =& -(1-\mu)Q_1 - 2Z_1\\
\Phi_3 =& -Q_2 - Z_1\\
\Phi_4 =& -(1-\eta_2)R_1 + (1-\eta_1)R_2 - 2Z_2 + \eta_2 S_2\\
\Phi_5 =& -R_2 - Z_2 + \eta_2 S_3\\
\Phi_6 =& -(1-\eta_2)Y_1 + (1-\eta_1)Y_2 + \eta_2 P_{12}^{\mathrm{T}} S_1^{-1} P_{12} +\\
& \eta_2 P_{22} S_2^{-1} P_{22} + \eta_2 P_{23} S_3^{-1} P_{23}^{\mathrm{T}}\\
\bar{A} =& \begin{bmatrix} A & B & 0 & 0 & 0 & C & 0\end{bmatrix}\\
e_1 =& \begin{bmatrix} I & -I & 0 & 0 & 0 & 0 & 0\end{bmatrix}\\
e_2 =& \begin{bmatrix} 0 & I & -I & 0 & 0 & 0 & 0\end{bmatrix}\\
e_3 =& \begin{bmatrix} I & 0 & 0 & -I & 0 & 0 & 0\end{bmatrix}\\
e_4 =& \begin{bmatrix} 0 & 0 & 0 & I & -I & 0 & 0\end{bmatrix}
\end{aligned}$$

and $*$ denotes the symmetric terms in a symmetric matrix.

Proof. Construct the following Lyapunov-Krasovskii functional:

$$V(\boldsymbol{x}(t)) = V_1(\boldsymbol{x}(t)) + V_2(\boldsymbol{x}(t)) + V_3(\boldsymbol{x}(t)) + V_4(\boldsymbol{x}(t)) \tag{10}$$

where

$$\begin{aligned}
V_1(\boldsymbol{x}(t)) =& \boldsymbol{\delta}^{\mathrm{T}}(t)P\boldsymbol{\delta}(t)\\
V_2(\boldsymbol{x}(t)) =& \int_{t-d(t)}^{t} \boldsymbol{x}^{\mathrm{T}}(s)Q_1\boldsymbol{x}(s)\mathrm{d}s + \int_{t-d_M}^{t} \boldsymbol{x}^{\mathrm{T}}(s)Q_2\boldsymbol{x}(s)\mathrm{d}s\\
V_3(\boldsymbol{x}(t)) =& \int_{t-\tau(t)}^{t} \left(\boldsymbol{x}^{\mathrm{T}}(s)R_1\boldsymbol{x}(s) + \dot{\boldsymbol{x}}^{\mathrm{T}}(s)Y_1\dot{\boldsymbol{x}}(s)\right)\mathrm{d}s +\\
& \int_{t-\tau_M}^{t-\tau(t)} \left(\boldsymbol{x}^{\mathrm{T}}(s)R_2\boldsymbol{x}(s) + \dot{\boldsymbol{x}}^{\mathrm{T}}(s)Y_2\dot{\boldsymbol{x}}(s)\right)\mathrm{d}s\\
V_4(\boldsymbol{x}(t)) =& d_M\int_{-d_M}^{0}\int_{t+\theta}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)Z_1\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta +\\
& \tau_M\int_{-\tau_M}^{0}\int_{t+\theta}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)Z_2\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta
\end{aligned}$$

where $\boldsymbol{\delta}^{\mathrm{T}}(t) = \begin{bmatrix}\boldsymbol{x}^{\mathrm{T}}(t) & \boldsymbol{x}^{\mathrm{T}}(t-\tau(t)) & \boldsymbol{x}^{\mathrm{T}}(t-\tau_M)\end{bmatrix}$, $P = P^{\mathrm{T}} = \begin{bmatrix} P_{11} & P_{12} & P_{13}\\ P_{12}^{\mathrm{T}} & P_{22} & P_{23}\\ P_{13}^{\mathrm{T}} & P_{23}^{\mathrm{T}} & P_{33}\end{bmatrix} > 0$, $Q_k = Q_k^{\mathrm{T}} > 0$, $R_k = R_k^{\mathrm{T}} > 0$, $Y_k = Y_k^{\mathrm{T}} > 0$, $Z_k = Z_k^{\mathrm{T}} > 0$, and $k = 1, 2$.

$$\Phi = \begin{bmatrix}
\Phi_1 & P_{11}B + Z_1 & 0 & A^{\mathrm{T}}P_{12} + Z_2 & A^{\mathrm{T}}P_{13} & P_{11}C + P_{12} & P_{13}\\
* & \Phi_2 & Z_1 & B^{\mathrm{T}}P_{12} & B^{\mathrm{T}}P_{13} & 0 & 0\\
* & * & \Phi_3 & 0 & 0 & 0 & 0\\
* & * & * & \Phi_4 & Z_2 & P_{12}^{\mathrm{T}}C + P_{22} & P_{23}\\
* & * & * & * & \Phi_5 & P_{13}^{\mathrm{T}}C + P_{23}^{\mathrm{T}} & P_{33}\\
* & * & * & * & * & \Phi_6 & 0\\
* & * & * & * & * & * & -Y_2
\end{bmatrix}$$

Calculating the time derivatives of $V_i(\boldsymbol{x}(t))$ $(i=1,2,3,4)$ along the trajectories of system (1) yields

$$\dot V_1(\boldsymbol{x}(t)) = 2\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{x}(t-\tau(t))\\ \boldsymbol{x}(t-\tau_M)\end{bmatrix}^{\rm T}\begin{bmatrix}P_{11} & P_{12} & P_{13}\\ P_{12}^{\rm T} & P_{22} & P_{23}\\ P_{13}^{\rm T} & P_{23}^{\rm T} & P_{33}\end{bmatrix}\times \begin{bmatrix}A\boldsymbol{x}(t)+B\boldsymbol{x}(t-d(t))+C\dot{\boldsymbol{x}}(t-\tau(t))\\ (1-\dot\tau(t))\dot{\boldsymbol{x}}(t-\tau(t))\\ \dot{\boldsymbol{x}}(t-\tau_M)\end{bmatrix} \tag{11}$$

$$\begin{aligned}\dot V_2(\boldsymbol{x}(t)) \le\ & \boldsymbol{x}^{\rm T}(t)(Q_1+Q_2)\boldsymbol{x}(t)+\\ & (1-\mu)\boldsymbol{x}^{\rm T}(t-d(t))Q_1\boldsymbol{x}(t-d(t)) -\\ & \boldsymbol{x}^{\rm T}(t-d_M)Q_2\boldsymbol{x}(t-d_M)\end{aligned} \tag{12}$$

$$\begin{aligned}\dot V_3(\boldsymbol{x}(t)) \le\ & \boldsymbol{x}^{\rm T}(t)R_1\boldsymbol{x}(t)-\boldsymbol{x}^{\rm T}(t-\tau_M)R_2\boldsymbol{x}(t-\tau_M) -\\ & (1-\eta_2)\boldsymbol{x}^{\rm T}(t-\tau(t))R_1\boldsymbol{x}(t-\tau(t))+\\ & (1-\eta_1)\boldsymbol{x}^{\rm T}(t-\tau(t))R_2\boldsymbol{x}(t-\tau(t))+\\ & \dot{\boldsymbol{x}}^{\rm T}(t)Y_1\dot{\boldsymbol{x}}(t)-\dot{\boldsymbol{x}}^{\rm T}(t-\tau_M)Y_2\dot{\boldsymbol{x}}(t-\tau_M)-\\ & (1-\eta_2)\dot{\boldsymbol{x}}^{\rm T}(t-\tau(t))Y_1\dot{\boldsymbol{x}}(t-\tau(t))+\\ & (1-\eta_1)\dot{\boldsymbol{x}}^{\rm T}(t-\tau(t))Y_2\dot{\boldsymbol{x}}(t-\tau(t))\end{aligned} \tag{13}$$

$$\begin{aligned}\dot V_4(\boldsymbol{x}(t)) =\ & d_M^2\dot{\boldsymbol{x}}^{\rm T}(t)Z_1\dot{\boldsymbol{x}}(t)-d_M\int_{t-d_M}^{t}\dot{\boldsymbol{x}}^{\rm T}(s)Z_1\dot{\boldsymbol{x}}(s){\rm d}s +\\ & \tau_M^2\dot{\boldsymbol{x}}^{\rm T}(t)Z_2\dot{\boldsymbol{x}}(t)-\tau_M\int_{t-\tau_M}^{t}\dot{\boldsymbol{x}}^{\rm T}(s)Z_2\dot{\boldsymbol{x}}(s){\rm d}s\end{aligned} \tag{14}$$

For the terms with $\dot\tau(t)$ in (11), by using some matrices $S_1=S_1^{\rm T}>0$, $S_2=S_2^{\rm T}>0$, and $S_3=S_3^{\rm T}>0$, there is the following inequality:

$$\begin{aligned}& -2\dot\tau(t)\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{x}(t-\tau(t))\\ \boldsymbol{x}(t-\tau_M)\end{bmatrix}^{\rm T}\begin{bmatrix}P_{12}\\ P_{22}\\ P_{23}^{\rm T}\end{bmatrix}\dot{\boldsymbol{x}}(t-\tau(t)) \le\\ & \eta_2\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{x}(t-\tau(t))\\ \boldsymbol{x}(t-\tau_M)\end{bmatrix}^{\rm T}\begin{bmatrix}S_1 & 0 & 0\\ 0 & S_2 & 0\\ 0 & 0 & S_3\end{bmatrix}\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{x}(t-\tau(t))\\ \boldsymbol{x}(t-\tau_M)\end{bmatrix}+\\ & \eta_2\dot{\boldsymbol{x}}^{\rm T}(t-\tau(t))\Big(P_{12}^{\rm T}S_1^{-1}P_{12}+P_{22}S_2^{-1}P_{22}+\\ & P_{23}S_3^{-1}P_{23}^{\rm T}\Big)\dot{\boldsymbol{x}}(t-\tau(t))\end{aligned} \tag{15}$$

To utilize the information that was ignored in previous results, we apply the following two polynomials:

$$\rho_1 = d(t)\int_{t-d_M}^{t}\dot{\boldsymbol{x}}^{\rm T}(s)Z_1\dot{\boldsymbol{x}}(s){\rm d}s-\tau(t)\int_{t-\tau_M}^{t}\dot{\boldsymbol{x}}^{\rm T}(s)Z_2\dot{\boldsymbol{x}}(s){\rm d}s \tag{16}$$

$$\rho_2 = \tau(t)\int_{t-\tau_M}^{t}\dot{\boldsymbol{x}}^{\rm T}(s)Z_2\dot{\boldsymbol{x}}(s){\rm d}s-d(t)\int_{t-d_M}^{t}\dot{\boldsymbol{x}}^{\rm T}(s)Z_1\dot{\boldsymbol{x}}(s){\rm d}s \tag{17}$$

where polynomials ρ_1 and ρ_2 are named as PTVD compensation terms. It is clear that $\rho_1+\rho_2=0$. Then, using PTVD compensation terms ρ_1 and ρ_2, Jensen′s inequality[24], and Leibniz-Newton formula, $\dot V_4(\boldsymbol{x}(t))$ can be rewritten as follows

$$\begin{aligned}\dot V_4(\boldsymbol{x}(t)) =\ & \dot V_4(\boldsymbol{x}(t))+\rho_1+\rho_2 =\\ & d_M^2\dot{\boldsymbol{x}}^{\rm T}(t)Z_1\dot{\boldsymbol{x}}(t)+\tau_M^2\dot{\boldsymbol{x}}^{\rm T}(t)Z_2\dot{\boldsymbol{x}}(t)-\\ & d(t)\int_{t-d(t)}^{t}\dot{\boldsymbol{x}}^{\rm T}(s)Z_1\dot{\boldsymbol{x}}(s){\rm d}s-\\ & (d_M-d(t))\int_{t-d(t)}^{t}\dot{\boldsymbol{x}}^{\rm T}(s)Z_1\dot{\boldsymbol{x}}(s){\rm d}s-\\ & (d_M-d(t))\int_{t-d_M}^{t-d(t)}\dot{\boldsymbol{x}}^{\rm T}(s)Z_1\dot{\boldsymbol{x}}(s){\rm d}s-\\ & d(t)\int_{t-d_M}^{t-d(t)}\dot{\boldsymbol{x}}^{\rm T}(s)Z_1\dot{\boldsymbol{x}}(s){\rm d}s-\\ & \tau(t)\int_{t-\tau(t)}^{t}\dot{\boldsymbol{x}}^{\rm T}(s)Z_2\dot{\boldsymbol{x}}(s){\rm d}s-\\ & (\tau_M-\tau(t))\int_{t-\tau(t)}^{t}\dot{\boldsymbol{x}}^{\rm T}(s)Z_2\dot{\boldsymbol{x}}(s){\rm d}s-\\ & (\tau_M-\tau(t))\int_{t-\tau_M}^{t-\tau(t)}\dot{\boldsymbol{x}}^{\rm T}(s)Z_2\dot{\boldsymbol{x}}(s){\rm d}s-\\ & \tau(t)\int_{t-\tau_M}^{t-\tau(t)}\dot{\boldsymbol{x}}^{\rm T}(s)Z_2\dot{\boldsymbol{x}}(s){\rm d}s \le\\ & d_M^2\dot{\boldsymbol{x}}^{\rm T}(t)Z_1\dot{\boldsymbol{x}}(t)+\tau_M^2\dot{\boldsymbol{x}}^{\rm T}(t)Z_2\dot{\boldsymbol{x}}(t)-\\ & \left[\int_{t-d(t)}^{t}\dot{\boldsymbol{x}}(s){\rm d}s\right]^{\rm T}Z_1\left[\int_{t-d(t)}^{t}\dot{\boldsymbol{x}}(s){\rm d}s\right]-\\ & \left[\int_{t-d_M}^{t-d(t)}\dot{\boldsymbol{x}}(s){\rm d}s\right]^{\rm T}Z_1\left[\int_{t-d_M}^{t-d(t)}\dot{\boldsymbol{x}}(s){\rm d}s\right]-\\ & \left[\int_{t-\tau(t)}^{t}\dot{\boldsymbol{x}}(s){\rm d}s\right]^{\rm T}Z_2\left[\int_{t-\tau(t)}^{t}\dot{\boldsymbol{x}}(s){\rm d}s\right]-\\ & \left[\int_{t-\tau_M}^{t-\tau(t)}\dot{\boldsymbol{x}}(s){\rm d}s\right]^{\rm T}Z_2\left[\int_{t-\tau_M}^{t-\tau(t)}\dot{\boldsymbol{x}}(s){\rm d}s\right]-\\ & \frac{d_M-d(t)}{d(t)}\left[\int_{t-d(t)}^{t}\dot{\boldsymbol{x}}(s){\rm d}s\right]^{\rm T}Z_1\times\\ & \left[\int_{t-d(t)}^{t}\dot{\boldsymbol{x}}(s){\rm d}s\right]-\frac{d(t)}{d_M-d(t)}\times\\ & \left[\int_{t-d_M}^{t-d(t)}\dot{\boldsymbol{x}}(s){\rm d}s\right]^{\rm T}Z_1\left[\int_{t-d_M}^{t-d(t)}\dot{\boldsymbol{x}}(s){\rm d}s\right]-\\ & \frac{\tau_M-\tau(t)}{\tau(t)}\left[\int_{t-\tau(t)}^{t}\dot{\boldsymbol{x}}(s){\rm d}s\right]^{\rm T}Z_2\times\\ & \left[\int_{t-\tau(t)}^{t}\dot{\boldsymbol{x}}(s){\rm d}s\right]-\frac{\tau(t)}{\tau_M-\tau(t)}\times\\ & \left[\int_{t-\tau_M}^{t-\tau(t)}\dot{\boldsymbol{x}}(s){\rm d}s\right]^{\rm T}Z_2\left[\int_{t-\tau_M}^{t-\tau(t)}\dot{\boldsymbol{x}}(s){\rm d}s\right] \le\\ & d_M^2\dot{\boldsymbol{x}}^{\rm T}(t)Z_1\dot{\boldsymbol{x}}(t)+\tau_M^2\dot{\boldsymbol{x}}^{\rm T}(t)Z_2\dot{\boldsymbol{x}}(t)-\boldsymbol{\zeta}^{\rm T}(t)Z_0\boldsymbol{\zeta}(t)-\\ & \frac{d_M-d(t)}{d_M}\boldsymbol{\zeta}^{\rm T}(t)e_1^{\rm T}Z_1e_1\boldsymbol{\zeta}(t)-\\ & \frac{d(t)}{d_M}\boldsymbol{\zeta}^{\rm T}(t)e_2^{\rm T}Z_1e_2\boldsymbol{\zeta}(t)-\\ & \frac{\tau_M-\tau(t)}{\tau_M}\boldsymbol{\zeta}^{\rm T}(t)e_3^{\rm T}Z_2e_3\boldsymbol{\zeta}(t)-\\ & \frac{\tau(t)}{\tau_M}\boldsymbol{\zeta}^{\rm T}(t)e_4^{\rm T}Z_2e_4\boldsymbol{\zeta}(t)\end{aligned} \tag{18}$$

where $\boldsymbol{\zeta}^{\mathrm{T}}(t) = [\boldsymbol{x}^{\mathrm{T}}(t) \ \ \boldsymbol{x}^{\mathrm{T}}(t-d(t)) \ \ \boldsymbol{x}^{\mathrm{T}}(t-d_M) \ \ \boldsymbol{x}^{\mathrm{T}}(t-\tau(t)) \ \ \boldsymbol{x}^{\mathrm{T}}(t-\tau_M) \ \ \dot{\boldsymbol{x}}^{\mathrm{T}}(t-\tau(t)) \ \ \dot{\boldsymbol{x}}^{\mathrm{T}}(t-\tau_M)]$ and

$$Z_0 = \begin{bmatrix} Z_1+Z_2 & -Z_1 & 0 & -Z_2 & 0 & 0 & 0 \\ -Z_1 & 2Z_1 & -Z_1 & 0 & 0 & 0 & 0 \\ 0 & -Z_1 & Z_1 & 0 & 0 & 0 & 0 \\ -Z_2 & 0 & 0 & 2Z_2 & -Z_2 & 0 & 0 \\ 0 & 0 & 0 & -Z_2 & Z_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Thus, according to (11) ~ (13), (15), and (18), $\dot{V}(\boldsymbol{x}(t))$ can be rewritten as follows

$$\dot{V}(\boldsymbol{x}(t)) \le \boldsymbol{\zeta}^{\mathrm{T}}(t)\Big[\Phi + \bar{A}^{\mathrm{T}}(Y_1 + d_M^2 Z_1 + \tau_M^2 Z_2)\bar{A} - \frac{d_M - d(t)}{d_M} e_1^{\mathrm{T}} Z_1 e_1 - \frac{d(t)}{d_M} e_2^{\mathrm{T}} Z_1 e_2 - \frac{\tau_M - \tau(t)}{\tau_M} e_3^{\mathrm{T}} Z_2 e_3 - \frac{\tau(t)}{\tau_M} e_4^{\mathrm{T}} Z_2 e_4\Big]\boldsymbol{\zeta}(t) \quad (19)$$

Obviously, if matrix inequality $\Phi + \bar{A}^{\mathrm{T}}(Y_1 + d_M^2 Z_1 + \tau_M^2 Z_2)\bar{A} - \frac{d_M-d(t)}{d_M} e_1^{\mathrm{T}} Z_1 e_1 - \frac{d(t)}{d_M} e_2^{\mathrm{T}} Z_1 e_2 - \frac{\tau_M-\tau(t)}{\tau_M} e_3^{\mathrm{T}} Z_2 e_3 - \frac{\tau(t)}{\tau_M} e_4^{\mathrm{T}} Z_2 e_4 < 0$, then $\dot{V}(\boldsymbol{x}(t)) < 0$. Based on Lemma 2, the above-mentioned matrix inequality is equivalent to the following matrix inequalities

$$\Phi + \bar{A}^{\mathrm{T}}(Y_1 + d_M^2 Z_1 + \tau_M^2 Z_2)\bar{A} - e_1^{\mathrm{T}} Z_1 e_1 - \frac{\tau_M - \tau(t)}{\tau_M} e_3^{\mathrm{T}} Z_2 e_3 - \frac{\tau(t)}{\tau_M} e_4^{\mathrm{T}} Z_2 e_4 < 0 \quad (20)$$

and

$$\Phi + \bar{A}^{\mathrm{T}}(Y_1 + d_M^2 Z_1 + \tau_M^2 Z_2)\bar{A} - e_2^{\mathrm{T}} Z_1 e_2 - \frac{\tau_M - \tau(t)}{\tau_M} e_3^{\mathrm{T}} Z_2 e_3 - \frac{\tau(t)}{\tau_M} e_4^{\mathrm{T}} Z_2 e_4 < 0 \quad (21)$$

when $d(t) = 0$ and $d(t) = d_M$, respectively. Then, applying Lemma 2 again, (20) and (21) are equivalent to (6) ~ (9), i.e., $\Phi + \bar{A}^{\mathrm{T}}(Y_1 + d_M^2 Z_1 + \tau_M^2 Z_2)\bar{A} - \frac{d_M-d(t)}{d_M} e_1^{\mathrm{T}} Z_1 e_1 - \frac{d(t)}{d_M} e_2^{\mathrm{T}} Z_1 e_2 - \frac{\tau_M-\tau(t)}{\tau_M} e_3^{\mathrm{T}} Z_2 e_3 - \frac{\tau(t)}{\tau_M} e_4^{\mathrm{T}} Z_2 e_4 < 0$ is equivalent to (6) ~ (9). Thus, if (6) ~ (9) are satisfied, then $\dot{V}(\boldsymbol{x}(t)) < 0$, i.e., system (1) is asymptotically stable. □

Remark 2. In Theorem 1, there are two points which are different from the previous results for linear neutral systems with time-varying delays.

1) An augmented Lyapunov-Krasovskii functional V_1 is used to deal with the stability problem of linear neutral systems with the time-varying delays. Meanwhile, the functionals used in previous literature are listed as follows:

$$\boldsymbol{x}^{\mathrm{T}}(t)P\boldsymbol{x}(t) = \boldsymbol{\delta}^{\mathrm{T}}(t)\begin{bmatrix} P & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\boldsymbol{\delta}(t) \quad (22)$$

$$(\boldsymbol{x}^{\mathrm{T}}(t) - \boldsymbol{x}^{\mathrm{T}}(t-\tau_M)C^{\mathrm{T}})P(\boldsymbol{x}(t) - C\boldsymbol{x}(t-\tau_M)) = \boldsymbol{\delta}^{\mathrm{T}}(t)\begin{bmatrix} P & 0 & -PC \\ 0 & 0 & 0 \\ -C^{\mathrm{T}}P & 0 & C^{\mathrm{T}}PC \end{bmatrix}\boldsymbol{\delta}(t) \quad (23)$$

Obviously, (22) and (23) are just the special cases of functional V_1. That is to say, compared with results employing (22) and (23), the criterion using functional V_1 has a larger solution set.

2) Theorem 1 not only depends on delay $d(t)$, but also depends on neural-type delay $\tau(t)$. Since the delay-dependent criteria are less conservative than delay-independent ones, Theorem 1 is less conservative than the criteria independent of neural-type delay. The numerical example in Section 3 can also verify this point.

Remark 3. When Jensen′s inequality is used to deal with $-d_M\int_{t-d_M}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)Z_1\dot{\boldsymbol{x}}(s)\mathrm{d}s$ and $-\tau_M\int_{t-\tau_M}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)Z_2\dot{\boldsymbol{x}}(s)\mathrm{d}s$ in (14), it can be dealt with as follows:

$$-d_M\int_{t-d_M}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)Z_1\dot{\boldsymbol{x}}(s)\mathrm{d}s - \tau_M\int_{t-\tau_M}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)Z_2\dot{\boldsymbol{x}}(s)\mathrm{d}s \le -\left[\int_{t-d(t)}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right]^{\mathrm{T}} Z_1\left[\int_{t-d(t)}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right] - \left[\int_{t-d_M}^{t-\tau(t)}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right]^{\mathrm{T}} Z_1\left[\int_{t-d_M}^{t-\tau(t)}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right] - \left[\int_{t-\tau(t)}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right]^{\mathrm{T}} Z_2\left[\int_{t-\tau(t)}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right] - \left[\int_{t-\tau_M}^{t-\tau(t)}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right]^{\mathrm{T}} Z_2\left[\int_{t-\tau_M}^{t-\tau(t)}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right] \quad (24)$$

According to (18), it is meant that the terms $-d(t)\times\int_{t-d_M}^{t-d(t)}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)Z_1\dot{\boldsymbol{x}}(s)\mathrm{d}s$, $-(d_M - d(t))\int_{t-d(t)}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)Z_1\dot{\boldsymbol{x}}(s)\mathrm{d}s$, $-\tau(t)\int_{t-\tau_M}^{t-\tau(t)}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)Z_2\dot{\boldsymbol{x}}(s)\mathrm{d}s$, and $-(\tau_M - \tau(t))\times\int_{t-\tau(t)}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)Z_2\times\dot{\boldsymbol{x}}(s)\mathrm{d}s$ could be ignored at the process of estimating the upper bound of $\dot{V}(\boldsymbol{x}(t))$ in the previous literature. Obviously, it leads to the increase of conservativeness in stability results. In this paper, we apply the novel PTVD compensation technique shown as (18) to compensate these ignored terms. Thus, a new stability criterion using this method can be obtained. Furthermore, since there are two irrelevant time-varying delays $d(t)$ and $\tau(t)$, PTVD compensation technique can be used only for function $d(t)$ or $\tau(t)$. Thus, on the premise of reducing the load of calculation, the satisfactory stability results can be derived.

In Theorem 1, the stability result depends not only on retarded-type delay but also on neutral-type delay. Then, the stability criterion only dependent on retarded-type delays will be proposed. Especially, the case of fast-varying neutral-type delay will be first considered in the following theorem.

Theorem 2. System (1) with time-varying delays $d(t)$ and $\tau(t)$ satisfying (2) and (3) is asymptotically stable, for the given scalar parameters d_M, μ, and $\eta_1 \le \eta_2 < 1$ or $1 < \eta_1 \le \eta_2$, if there exist some matrices:

$$P = P^{\mathrm{T}} = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{12}^{\mathrm{T}} & P_{22} & P_{23} \\ P_{13}^{\mathrm{T}} & P_{23}^{\mathrm{T}} & P_{33} \end{bmatrix} > 0$$

$$Q_1 = Q_1^{\mathrm{T}} > 0, Q_2 = Q_2^{\mathrm{T}} > 0, R_1 = R_1^{\mathrm{T}} > 0, R_2 = R_2^{\mathrm{T}} > 0$$
$$Y_1 = Y_1^{\mathrm{T}} > 0, Y_2 = Y_2^{\mathrm{T}} > 0, Z_1 = Z_1^{\mathrm{T}} > 0$$
$$S_1 = S_1^{\mathrm{T}} > 0, S_2 = S_2^{\mathrm{T}} > 0, S_3 = S_3^{\mathrm{T}} > 0$$

such that the following matrix inequalities hold:

$$\bar{\Phi} + \bar{A}^{\mathrm{T}}(Y_1 + d_M^2 Z_1)\bar{A} - e_1^{\mathrm{T}} Z_1 e_1 < 0 \quad (25)$$
$$\bar{\Phi} + \bar{A}^{\mathrm{T}}(Y_1 + d_M^2 Z_1)\bar{A} - e_2^{\mathrm{T}} Z_1 e_2 < 0 \quad (26)$$

where $\bar{\Phi}$ is shown at the bottom of this page,

$$\bar{\Phi}_1 = P_{11}A + A^{\mathrm{T}}P_{11} + Q_1 + Q_2 + R_1 - Z_1 + \eta_2 S_1$$
$$\bar{\Phi}_4 = -(1-\eta_2)R_1 + (1-\eta_1)R_2 + \eta_2 S_2$$

and the other parameters are the same as those defined in Theorem 1.

Proof. Construct the following Lyapunov-Krasovskii functional:

$$\bar{V}(\boldsymbol{x}(t)) = V_1(\boldsymbol{x}(t)) + V_2(\boldsymbol{x}(t)) + V_3(\boldsymbol{x}(t)) + \bar{V}_4(\boldsymbol{x}(t)) \quad (27)$$

where $V_1(\boldsymbol{x}(t))$, $V_2(\boldsymbol{x}(t))$, and $V_3(\boldsymbol{x}(t))$ are the same as the definitions in Theorem 1, and

$$\bar{V}_4(\boldsymbol{x}(t)) = d_M \int_{-d_M}^{0}\int_{t+\theta}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)Z_1\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta$$

The process of proof is similar to Theorem 1. □

Remark 4. The systems with fast-varying neutral-type delays (i.e., $\dot{\tau}(t) > 1$) can be first considered in Theorem 2, which is achieved by using condition (3) and functional V_3. So far, there has been no literature referring to this case.

Remark 5. For Theorems 1 and 2, only by setting $Q_1 = 0$, the criteria independent of derivative of delay function $d(t)$ can be derived.

3 Numerical example

In this section, an example will be given to verify the proposed criteria.

Consider linear neutral system (1) with the following parameters[9]:

$$A = \begin{bmatrix} -0.9 & 0.2 \\ 0.1 & -0.9 \end{bmatrix}, \quad B = \begin{bmatrix} -1.1 & -0.2 \\ -0.1 & -1.1 \end{bmatrix}$$
$$C = \begin{bmatrix} -0.2 & 0 \\ 0.2 & -0.1 \end{bmatrix}$$

and delay functions $d(t)$ and $\tau(t)$ satisfy conditions (2) and (3).

Applying LMI Toolbox of Matlab, we can solve the maximum allowable upper bounds d_M by setting τ_M, μ, η_1, and η_2. With $\tau_M = 0.1$, the stability results obtained by [8, 19−20, 22], and this paper for different μ, η_1, and η_2, respectively, are shown in Table 1. It is clear that the stability results by using our method are less conservative. When $\eta_1 \le \eta_2 < 1$, the upper bound d_M obtained by using Theorem 1 is larger than that by using Theorem 2. Meanwhile, Theorem 2 is more effective and suitable to deal with linear systems with fast-varying neutral-type delays. Since only two matrix inequalities need to be solved, Theorem 2 is of less calculation load.

4 Conclusion

A class of linear neutral systems with time-varying retarded-type delays and time-varying neutral-type delays is investigated in this paper. Since a new Lyapunov-Krasovskii functional and novel PTVD compensation technique are introduced, the less conservative stability criterion is proposed. The gain using the new functional is that the stability of linear neutral systems with fast-varying neutral-type delays (i.e., $\dot{\tau}(t) > 1$) can be obtained, which is for the first time considered in the stability criteria. Some useful terms can be considered by using the PTVD compensation technique, which are usually ignored at the process of estimating the upper bound of $\dot{V}(\boldsymbol{x}(t))$. The numerical example has proved that the proposed criteria are effective.

Appendix

The proof of Lemma 2. 1) (5)⇒(4)

Since variable z satisfies the following condition at interval $[0,\ \beta]$,

$$\Delta + zX_1 + (\beta - z)X_2 < 0$$

$$\bar{\Phi} = \begin{bmatrix} \bar{\Phi}_1 & P_{11}B + Z_1 & 0 & A^{\mathrm{T}}P_{12} & A^{\mathrm{T}}P_{13} & P_{11}C + P_{12} & P_{13} \\ * & \Phi_2 & Z_1 & B^{\mathrm{T}}P_{12} & B^{\mathrm{T}}P_{13} & 0 & 0 \\ * & * & \Phi_3 & 0 & 0 & 0 & 0 \\ * & * & * & \bar{\Phi}_4 & 0 & P_{12}^{\mathrm{T}}C + P_{22} & P_{23} \\ * & * & * & * & -R_2 + \eta_2 S_3 & P_{13}^{\mathrm{T}}C + P_{23}^{\mathrm{T}} & P_{33} \\ * & * & * & * & * & \Phi_6 & 0 \\ * & * & * & * & * & * & -Y_2 \end{bmatrix}$$

Table 1 Maximum allowable upper bounds d_M for different μ and $\tau_M = 0.1$

	Methods	$\mu = 0.7$	$\mu = 0.8$	$\mu = 0.9$	Unknown μ
$\eta_1 = \eta_2 = 0$ $(\tau(t) = \tau_M)$	[8]	$d_M = 0.3890$	$d_M = 0.2547$	$d_M = 0.1253$	–
	[19]	$d_M = 1.0071$	$d_M = 0.9201$	$d_M = 0.8347$	$d_M = 0.7603$
	[22]	$d_M = 1.0425$	$d_M = 0.9515$	$d_M = 0.8596$	$d_M = 0.7652$
	[20]	$d_M = 1.0628$	$d_M = 0.9642$	$d_M = 0.8642$	$d_M = 0.7652$
	Theorem 2	$d_M = 1.1281$	$d_M = 1.0941$	$d_M = 1.0882$	$d_M = 1.0882$
	Theorem 1	$d_M = 1.1642$	$d_M = 1.1294$	$d_M = 1.1210$	$d_M = 1.1208$
$\eta_1 = 0$, $\eta_2 = 0.5$	[22]	$d_M = 0.9938$	$d_M = 0.9083$	$d_M = 0.8224$	$d_M = 0.7345$
	Theorem 2	$d_M = 1.0448$	$d_M = 1.0160$	$d_M = 1.0093$	$d_M = 1.0093$
	Theorem 1	$d_M = 1.1071$	$d_M = 1.0755$	$d_M = 1.0688$	$d_M = 1.0688$
$\eta_1 = 1.1$, $\eta_2 = 2.0$	Theorem 2	$d_M = 1.2170$	$d_M = 1.1824$	$d_M = 1.1774$	$d_M = 1.1774$

Thus, when variable $z = \beta$ and $z = 0$, the two following inequalities hold

$$\Delta + \beta X_1 < 0, \quad \Delta + \beta X_2 < 0$$

2) (4)⇒(5)

Let matrices Δ_1 and Δ_2 satisfy the following conditions

$$\Delta_1 = \Delta + \beta X_1 < 0, \quad \Delta_2 = \Delta + \beta X_2 < 0$$

We can get the following result:

$$z\Delta_1 + (\beta - z)\Delta_2 < 0$$

i.e.,

$$\beta(\Delta + zX_1 + (\beta - z)X_2) < 0$$

Since $\beta > 0$, the following inequality holds

$$\Delta + zX_1 + (\beta - z)X_2 < 0$$

□

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