# $H_\infty$ Control with Multiple Delays in Measurements[1)]

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**Abstract** Based on an innovation analysis method in the Krein space, a sufficient and necessary condition is given for the existence of the solution of $H_\infty$ control problem for a linear continuous-time system with multiple delays. By introducing a re-organized innovation sequence, the $H_\infty$ control problem with delayed measurements is converted into a linear quadratic (LQ) problem and a delay-free $H_2$ estimation problem in the Krein space. The controller is given in terms of two forward Riccati equations and a backward Riccati equation.

**Key words** Continuous-time systems, $H_\infty$ control, re-organized innovation analysis, multiple time delays

## 1 Introduction

$H_\infty$ control for system with delays has been recognized as a challenging problem. Time delays are encountered in many practical systems, such as aircraft, progress control and so on. In this sense, the control problem for time delay systems possesses the theoretical and practical significance. Usually, there are two classical ways of solving the $H_\infty$ control problem for delayed systems. One is to design a controller by linear matrix inequalities (LMIs)[1,2], but only sufficient conditions can be obtained for the sake of the conservation of this method. In [3,4], the controllers were given by the solution of related Riccati equation using dynamic programming. It should be noted that only the single delay case is considered by dynamic programming method. Throughout this paper, $\langle\cdot,\cdot\rangle$ denotes the inner product and "$*$" stands for the transpose of vector or matrix.

## 2 Problem statement and preliminaries

Consider the following continuous-time system with multiple delayed measurements

$$\dot{\boldsymbol{x}}(t) = F(t)\boldsymbol{x}(t) + G_1(t)\boldsymbol{w}(t) + G_2(t)\boldsymbol{u}(t) \tag{1}$$

$$\boldsymbol{y}(t) = H_i(t)\boldsymbol{x}(t_i) + \boldsymbol{v}_i(t), \quad i = 0,1,\cdots,l \tag{2}$$

$$\boldsymbol{s}(t) = L(t)\boldsymbol{x}(t) \tag{3}$$

where $\boldsymbol{x}(t)\in R^n$, $\boldsymbol{w}(t)\in R^p$, $\boldsymbol{u}(t)\in R^r$ and $\boldsymbol{s}(t)\in R^q$ are respectively the state, process noise, control input and signal. $\boldsymbol{y}_i(t)\in R^m (i=0,1,\cdots,l)$ is the delayed measurement observed by the $l+1$th system and $\boldsymbol{v}_i(t)\in R^m$ is the measurement noise. In (2), we have $t_i = t-\bar{d}_i$ and $\bar{d}_i = \sum_{k=1}^{i} d_k (d_k > 0)$. The matrices $F(t), G_1(t), G_2(t), H_i(t)$ and $L(t)$ are known matrices of appropriate dimensions. Let $Y(t)$ and $V(t)$ be respectively the measurement and measurement noise for system (1)$\sim$(3) at time $t$. We have

$$Y(t) = \begin{cases} [\boldsymbol{y}_0^*(t)\ \cdots\ \boldsymbol{y}_{i-1}^*(t)]^*, & \bar{d}_{i-1}\leqslant t<\bar{d}_i \\ [\boldsymbol{y}_0^*(t)\ \cdots\ \boldsymbol{y}_l^*(t)]^*, & t\geqslant \bar{d}_l \end{cases}, \quad V(t) = \begin{cases} [\boldsymbol{v}_0^*(t)\ \cdots\ \boldsymbol{v}_{i-1}^*(t)]^*, & \bar{d}_{i-1}\leqslant t<\bar{d}_i \\ [\boldsymbol{v}_0^*(t)\ \cdots\ \boldsymbol{v}_l^*(t)]^*, & t\geqslant \bar{d}_l \end{cases} \tag{4}$$

$Y(t)$ and $V(t)$ satisfy the following relationship

$$Y(t) = H(t)\boldsymbol{X}(t) + V(t) \tag{5}$$

where

$$H(t) = \begin{cases} \mathrm{diag}\{H_0(t), H_1(t),\cdots,H_{i-1}(t)\}, & \bar{d}_{i-1}\leqslant t<\bar{d}_i \\ \mathrm{diag}\{H_0(t), H_1(t),\cdots,H_l(t)\}, & t\geqslant \bar{d}_l \end{cases}$$

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$$X(t)=\begin{cases}[\boldsymbol{x}^*(t_0)\ \cdots\ \boldsymbol{x}^*(t_{i-1})]^*, & \bar{d}_{i-1}\leqslant t<\bar{d}_i\\ [\boldsymbol{x}^*(t_0)\ \cdots\ \boldsymbol{x}^*(t_l)]^*, & t\geqslant \bar{d}_l\end{cases}\tag{6}$$

**Problem CM.** Consider system (1)$\sim$(3) and a positive scalar $\gamma$. Find an $H_\infty$ measurement feedback control strategy $\boldsymbol{u}(t)=\mathcal{F}(Y(s),0\leqslant s\leqslant t)$ ($\mathcal{F}$ is a linear function) such that

$$\sup_{x(0),w(\cdot)}\frac{\boldsymbol{x}^*(T)P^c(T)\boldsymbol{x}(T)+\int_0^{\mathrm{T}}\boldsymbol{u}^*(t)\boldsymbol{u}(t)\mathrm{d}t+\int_0^{\mathrm{T}}\boldsymbol{s}^*(t)\boldsymbol{s}(t)\mathrm{d}t}{\boldsymbol{x}^*(0)\Pi_0^{-1}\boldsymbol{x}(0)+\int_0^{\mathrm{T}}\boldsymbol{w}^*(t)\boldsymbol{w}(t)\mathrm{d}t+\int_0^{\mathrm{T}}V^*(t)V(t)\mathrm{d}t}<\gamma^2\tag{7}$$

where $\Pi_0$ and $P^c(T)$ are given positive definite weighting matrices.

In terms of [5], it is easy to know that, for nonzero initial values $\boldsymbol{x}(0)$ and $\boldsymbol{w}(\cdot)$, the $H_\infty$ performance in (7) can be written as the following quadratic form

$$\begin{aligned}J(T)=&\boldsymbol{x}^*(0)(\Pi_0^{-1}-\gamma^{-2}P^c(0))\boldsymbol{x}(0)+\int_0^{\mathrm{T}}(\boldsymbol{w}(t)-\hat{\boldsymbol{w}}(t))^*(\boldsymbol{w}(t)-\hat{\boldsymbol{w}}(t))\mathrm{d}t+\\ &\int_0^{\mathrm{T}}\begin{bmatrix}\boldsymbol{u}(t)-\hat{\boldsymbol{u}}(t)\\ Y(t)-H(t)X(t)\end{bmatrix}^*\begin{bmatrix}-\gamma^{-2}I_r & 0\\ 0 & I_{m\times(l+1)}\end{bmatrix}\begin{bmatrix}\boldsymbol{u}(t)-\hat{\boldsymbol{u}}(t)\\ Y(t)-H(t)X(t)\end{bmatrix}\mathrm{d}t\end{aligned}\tag{8}$$

where

$$\begin{bmatrix}\hat{\boldsymbol{u}}(t)\\ \hat{\boldsymbol{w}}(t)\end{bmatrix}=\begin{bmatrix}K_u(t)\\ K_w(t)\end{bmatrix}\boldsymbol{x}(t)=\begin{bmatrix}-G_2^*(t)P^c(t)\\ \gamma^{-2}G_1^*(t)P^c(t)\end{bmatrix}\boldsymbol{x}(t)\tag{9}$$

where $P^c(t)$ is the unique solution of the following backward time Riccati differential equation

$$\begin{aligned}\dot{P}^c(t)=&F^*(t)P^c(t)+P^c(t)F(t)+H^*(t)H(t)-P^c(t)G_2(t)G_2^*(t)P^c(t)-\\ &\gamma^{-2}P^c(t)G_1(t)G_1^*(t)P^c(t),\quad P^c(T)\end{aligned}\tag{10}$$

## 3 Main results

### 3.1 The equivalent problem in Krein space

Denote $\bar{K}_u(t)=[K_u(t)\quad 0\quad \cdots 0]$. For $\bar{d}_{i-1}\leqslant t<\bar{d}_i$ and $t\geqslant\bar{d}_l$, the number of columns of $\bar{K}_u(t)$ is the same as the number of rows of $X(t)$. From (8), the Krein space model (the variables in Krein space are denoted by block letters) related with system (1)$\sim$(3) is given by

$$\dot{\boldsymbol{x}}(t)=(F(t)+\gamma^{-2}G_1(t)G_1^*(t)P^c(t))\boldsymbol{x}(t)+G_1(t)(\boldsymbol{w}(t)-\hat{\boldsymbol{w}}(t))+G_2(t)\boldsymbol{u}(t)\tag{11}$$

$$\begin{bmatrix}\boldsymbol{u}(t)\\ Y(t)\end{bmatrix}=\begin{bmatrix}\bar{K}_u(t)\\ H(t)\end{bmatrix}X(t)+V_s(t)\tag{12}$$

where $\langle\boldsymbol{x}(0),\boldsymbol{x}(0)\rangle=(\Pi_0^{-1}-\gamma^{-2}P^c(0))^{-1}$, $\langle\boldsymbol{w}(t)-\hat{\boldsymbol{w}}(t),\boldsymbol{w}(r)-\hat{\boldsymbol{w}}(r)\rangle=I_p\delta_{tr}$ and $\langle V_s(t),V_s(r)\rangle=Q_{v_s}\delta_{tr}$. The variables $V_s(t)=[\boldsymbol{v}_u^*(t)\quad \boldsymbol{v}^*(t)]^*$ and $\boldsymbol{v}(t)=[\boldsymbol{v}_0^*(t)\quad\cdots\quad \boldsymbol{v}_l^*(t)]^*$. In order to express clearly, we denote the measurement at time $t$ of the Krein space model (11), (12) as

$$\bar{Y}(t)=\begin{cases}[\bar{\boldsymbol{y}}^*(t)\ \cdots\ \boldsymbol{y}_{i-1}^*(t)]^*, & \bar{d}_{i-1}\leqslant t<\bar{d}_i\\ [\bar{\boldsymbol{y}}^*(t)\ \cdots\ \boldsymbol{y}_{l-1}^*(t)]^*, & t\geqslant\bar{d}_l\end{cases}\quad\text{and}\quad \bar{\boldsymbol{y}}(t)=\begin{bmatrix}\boldsymbol{u}(t)\\ \boldsymbol{y}_0(t)\end{bmatrix}\tag{13}$$

Note that the above measurements form the linear space $\mathcal{L}\{\bar{Y}(\tau),0\leqslant\tau\leqslant t\}$.

Further, we know that the quadratic form $J(T)$ in (8) has the minimum $J_m(T)$ if and only if the innovation $\bar{\boldsymbol{w}}_d(t)=\bar{Y}(t)-\bar{Y}(t|t)$ exists, *i.e.*,

$$\begin{aligned}J_m(T)=&\int_0^{\mathrm{T}}\bar{\boldsymbol{w}}_d^*(t)Q_{\bar{w}_d}(t)\bar{\boldsymbol{w}}_d(t)\mathrm{d}t=\int_0^{\mathrm{T}}(\bar{Y}(t)-H(t)\hat{X}(t|t))^*(\bar{Y}(t)-H(t)\hat{X}(t|t))\mathrm{d}t-\\ &\gamma^{-2}\int_0^{\mathrm{T}}(\boldsymbol{u}(t)\bar{K}_u(t)\hat{X}(t|t))^*(\boldsymbol{u}(t)-\bar{K}_u(t)\hat{X}(t|t))\mathrm{d}t\end{aligned}\tag{14}$$

where $Q_{\bar{w}_d}(t)=\mathrm{diag}\{Q_{v_u},Q_{v_0},\cdots,Q_{v_i}\}=\mathrm{diag}\{-\gamma^{-2}I_r,I_m,\cdots,I_m\}$ and $\hat{X}(t|t)=[\hat{\boldsymbol{x}}^*(t_0|t)\ \cdots\ \hat{\boldsymbol{x}}^*(t_i|t)]^*$ for $\bar{d}_{i-1}\leqslant t<\bar{d}_i$, $Q_{\bar{w}_d}(t)=\mathrm{diag}\{Q_{v_u},Q_{v_0},\cdots,Q_{v_l}\}=\mathrm{diag}\{-\gamma^{-2}I_r,I_m,\cdots,I_m\}$ and $\hat{X}(t|t)=[\hat{\boldsymbol{x}}^*(t_0|t)\cdots\ \hat{\boldsymbol{x}}^*(t_l|t)]^*$ for $t\geqslant\bar{d}_l$. Note that $\hat{\boldsymbol{x}}(t_i|t)(i=0,1,\cdots,l)$ is the projection of $\boldsymbol{x}(t_i)$ onto the linear space $\mathcal{L}\{\bar{Y}(\tau),0\leqslant\tau\leqslant t\}$. So our next aim is to achieve the optimal estimator $\hat{X}(t|t)$.

### 3.2 Re-organized innovation sequence and optimal estimator

By re-organizing the measurements $\{\bar{Y}(\tau), 0 \leqslant \tau \leqslant t\}$, we obtain the measurements $\mathcal{Y}_i(\tau) = [\bar{\boldsymbol{y}}^*(\tau) \quad \boldsymbol{y}_1^*(\tau+\bar{d}_1) \quad \cdots \quad \boldsymbol{y}_{i-1}^*(\tau+\bar{d}_{i-1})]^*$, $i > 1$($\mathcal{Y}_1(\tau) = \bar{\boldsymbol{y}}(\tau)$ for $i = 1$) and the linear space $\mathcal{L}\{\mathcal{Y}_{l+1}(\tau)|_{0\leqslant\tau\leqslant t_l}; \cdots; \mathcal{Y}_k(\tau)|_{t_k<\tau\leqslant t_{k-1}}; \cdots; \mathcal{Y}_i(\tau)|_{t_i<\tau<s}\}$. The reorganized measurement $\mathcal{Y}_i(\tau)$ satisfies the following relationship

$$\mathcal{Y}_i(\tau) = \mathcal{H}_i(\tau)x(\tau) + \mathcal{V}_i(\tau), \quad i = 1, \cdots, l+1 \tag{15}$$

where $\mathcal{H}_1(\tau) = [K_0^*(\tau) \quad H_0^*(\tau)]^*$ and $\mathcal{V}_1(\tau) = [\boldsymbol{v}_u^*(\tau) \quad \boldsymbol{v}_0^*(\tau)]^*$ for $i = 1$; and for $i > 1$, we have

$$\mathcal{H}_i(\tau) = [\mathcal{H}_1^*(\tau) \quad H_1^*(\tau+\bar{d}_1) \quad \cdots \quad H_{i-1}^*(\tau+\bar{d}_{i-1})]^*, \ \mathcal{V}_i(\tau) = [\mathcal{V}_1^*(\tau) \quad \boldsymbol{v}_1^*(\tau+\bar{d}_1) \quad \cdots \quad \boldsymbol{v}_{i+1}^*(\tau+\bar{d}_{i-1})]^*$$

The covariance matrix of the measurement noise $\mathcal{V}_i(\tau)$ is given by $Q_{\mathcal{V}_i}(\tau) = \mathrm{diag}\{Q_{\mathcal{V}_i}(\tau), Q_{v_1}(\tau+\bar{d}_1), \cdots, Q_{v_{i-1}}(\tau+\bar{d}_1)\}$ ($Q_{\mathcal{V}_1}(\tau) = \mathrm{diag}\{Q_{v_u}, Q_{v_0}\}$ for $i = 1$). Denote $\hat{\boldsymbol{\xi}}(s,i)(s > t_i)$ as the projection of $\boldsymbol{\xi}(s)$ onto the linear space $\mathcal{L}\{\mathcal{Y}_{l+1}(\tau)|_{0\leqslant\tau\leqslant t_l}; \cdots; \mathcal{Y}_k(\tau)|_{t_k<\tau\leqslant t_{k-1}}; \cdots; \mathcal{Y}_i(\tau)|_{t_i<\tau<s}\}$. When $i = 1, \cdots, l$ and $\tau > 0$, the reorganized innovation is given by

$$W_i^{t_i+\tau} \triangleq \mathcal{Y}_i(t_i+\tau) - \hat{\mathcal{Y}}_i(t_i+\tau, i) = H_{l+1}(\tau)\boldsymbol{e}_{l+1}^{\tau} + \mathcal{V}_{l+1}(\tau), \ \boldsymbol{e}_i^{t_i+\tau} = \boldsymbol{x}(t_i+\tau) - \hat{\boldsymbol{x}}(t_i+\tau, i) \tag{16}$$

Especially, for $i = l+1$ and $\tau \geqslant 0$

$$W_{l+1}^{\tau} \triangleq \mathcal{Y}_{l+1}(\tau) - \hat{\mathcal{Y}}_{l+1}(\tau, l+1) = H_i(t_i+\tau)\boldsymbol{e}_i^{t_i+\tau} + \mathcal{V}_i(t_i+\tau), \ \boldsymbol{e}_i^{t_i+\tau} = \boldsymbol{x}(t_i+\tau) - \hat{\boldsymbol{x}}(t_i+\tau, i) \tag{17}$$

**Theorem 1.** Let $\Phi(t) = (F(t) + \gamma^{-2}G_1(t)G_1^*(t)P^c(t))$. The cross-covariance matrices $\mathcal{P}_{l+1}^{r} \triangleq \langle \boldsymbol{x}_{l+1}^{\tau}, \boldsymbol{e}_{l+1}^{\tau}\rangle, \tau \geqslant 0$ and $\mathcal{P}_i^{t_i+\tau} \triangleq \langle \boldsymbol{x}_i^{t_i+\tau}, \boldsymbol{e}_i^{t_i+\tau}\rangle, \tau \geqslant 0$ are given by

$\mathcal{P}_{l+1}^{\tau}$ is the solution of the following Riccti equation

$$\begin{aligned}\frac{\mathrm{d}\mathcal{P}_{l+1}^{\tau}}{\mathrm{d}\tau} =& \Phi(\tau)\mathcal{P}_{l+1}^{\tau} + \mathcal{P}_{l+1}^{\tau}\Phi^*(\tau) - \mathcal{K}_{l+1}^{\tau}Q_{\mathcal{V}_{l+1}}(\tau)[\mathcal{K}_{l+1}^{\tau}]^* + G_1(t)G_1^*(t) - \\ & \gamma^{-2}G_2(t)G_2^*(t), \quad \mathcal{P}_{l+1}^0 = (\Pi_0^{-1} - \gamma^{-2}P^c(0))^{-1}\end{aligned} \tag{18}$$

where $\mathcal{K}_{l+1}^{\tau} = \mathcal{P}_{l+1}^{\tau}\mathcal{H}_{l+1}^*(Q_{\mathcal{V}_{l+1}}(\tau))^{-1}$.

$\mathcal{P}_i^{t_i+\tau}(i = l, \cdots, 1)$ is the solution of the following Riccti equation

$$\begin{aligned}\frac{\mathrm{d}\mathcal{P}_i^{t_i+\tau}}{\mathrm{d}\tau} =& \Phi(t_i+\tau)\mathcal{P}_i^{t_i+\tau} + \mathcal{P}_i^{t_i+\tau}\Phi^*(t_i+\tau) - \mathcal{K}_i^{t_i+\tau}Q_{\mathcal{V}_{l+1}}(t_i+\tau)[\mathcal{K}_i^{t_i+\tau}]^* + \\ & G_1(t_i+\tau)G_1^*(t_i+\tau) - \gamma^{-2}G_2(t_i+\tau)G_2^*(t_i+\tau), \quad \mathcal{P}_i^{t_i} = \mathcal{P}_{i+1}^{t_i}\end{aligned} \tag{19}$$

where $\mathcal{K}_i^{t_i+\tau} = \mathcal{P}_i^{t_i+\tau}\mathcal{H}_i^*(t_i+\tau)(Q_{\mathcal{V}_{l+1}}(t_i+\tau))^{-1}$.

**Proof.** Omitted.

### 3.3 The solution to $H_\infty$ control problem

**Theorem 2.** Consider system (1)~(3) and performance index (7). For any positive scalar $\gamma$, there exists an $H_\infty$ measurement feedback control strategy $\boldsymbol{u}(t) = \mathcal{F}(Y(s), 0 \leqslant s \leqslant t)$ that achieves the performance index (7) iff the matrices $\mathcal{P}_i^{t_i+\tau}(0 < \tau \leqslant d_i, i = 1, \cdots, l)$ and $\mathcal{P}_{l+1}^{\tau}(0 \leqslant \tau \leqslant t_l)$ for $0 \leqslant t \leqslant T$ are bounded and the Riccati equation in (10) has a unique solution $P^c(t)$. The matrices $\mathcal{P}_i^{t_i+\tau}$ and $\mathcal{P}_{l+1}^{\tau}$ are given by Theorem 1. Then the control strategy is given by

$$\boldsymbol{u}(t) = K_u(t)\hat{\boldsymbol{x}}(t, 1) \tag{20}$$

where $K_u(t)$ is as in (9) and the optimal estimator $\hat{\boldsymbol{x}}(t_i|t)$ is computed by the following steps.

**Step 1.** Calculating $\mathcal{P}_i^{t_i+\tau}$ and $\hat{\boldsymbol{x}}(\tau, l+1)$ for $\tau = t_l$, where $\mathcal{P}_{l+1}^{\tau}$ is as in (18) and $\hat{\boldsymbol{x}}(\tau, l+1)$ is given by

$$\frac{\mathrm{d}\hat{\boldsymbol{x}}(\tau, l+1)}{\mathrm{d}\tau} = \Phi(\tau)\hat{\boldsymbol{x}}(\tau, l+1) + \mathcal{K}_{l+1}^{\tau}[\mathcal{Y}_{l+1}(\tau) - \mathcal{H}_{l+1}(\tau)\hat{\boldsymbol{x}}(\tau, l+1)], \ \hat{\boldsymbol{x}}(0, l+1) = 0 \tag{21}$$

**Step 2.** Calculating $\mathcal{P}_i^{t_i+\tau}$ and $\hat{\boldsymbol{x}}(t_i+\tau, i), (0 < \tau \leqslant d_i, i = l, \cdots, 1)$ where $\mathcal{P}_i^{t_i+\tau}$ is as in (19) and $\hat{\boldsymbol{x}}(t_i+\tau, i)$ is given by

$$\frac{\mathrm{d}\hat{\boldsymbol{x}}(t_i+\tau, i)}{\mathrm{d}\tau} = \Phi(t_i+\tau)\hat{\boldsymbol{x}}(t_i+\tau, i) + \mathcal{K}_i^{t_i+\tau}[\mathcal{Y}_{l+1}(t_i+\tau) - H_{l+1}(t_i+\tau)\hat{\boldsymbol{x}}(t_i+\tau, i)] \tag{22a}$$

$$\hat{\boldsymbol{x}}(t_i, i) = \hat{\boldsymbol{x}}(t_i, i+1) \tag{22b}$$

**Step 3.** The estimator $\hat{\boldsymbol{x}}(t,1)$ is computed from Step 2 when $i=1$ and $\tau=d_1$.

**Proof.** Omitted.

## 4 Conclusion

In this paper, we have studied the $H_\infty$ control problem for linear continuous-time with multiple delays in measurements. Since the continuous-time innovations Gramian matrices are diagonal, the conditions for the existence of $H_\infty$ controller and the form of the $H_\infty$ controller are simpler than in discrete-time system.

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